Institute for Operations Research
and Computational Finance

University of St.Gallen

20th IFORS Conference, Barcelona, 2014

A spot-forward model for electricity prices

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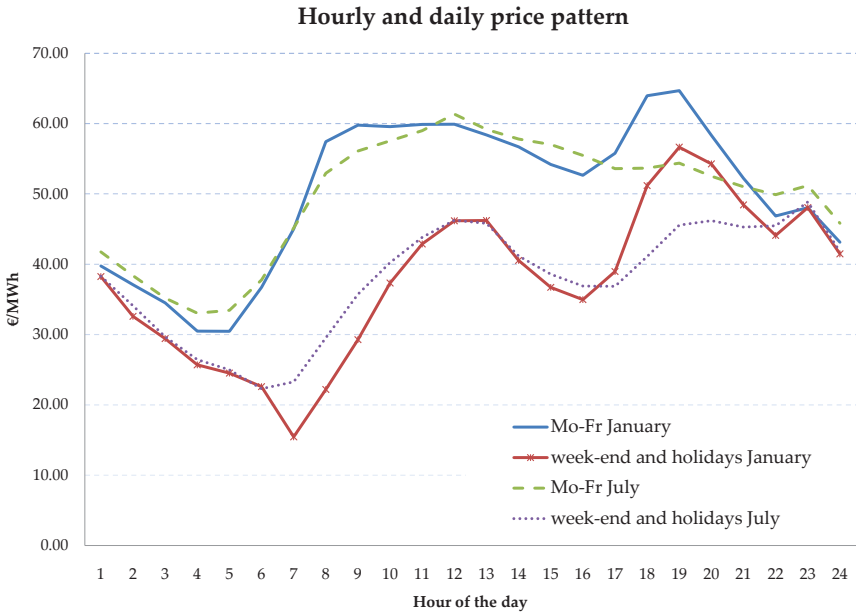
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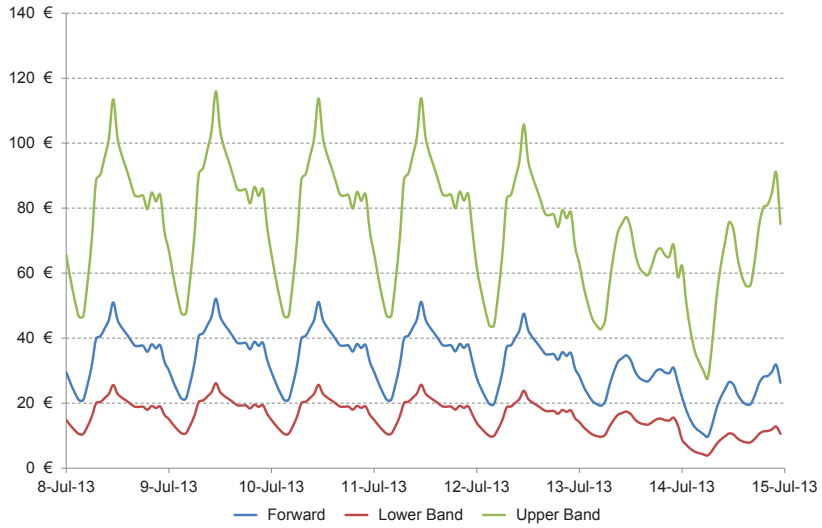
Motivation

- Consistency of simulated electricity spot prices with the observed forward prices is of great importance for risk management applications: hedging, valuation
- Classical simulation models ignore the information about expected future spot prices reflected in forward curves
- We aim at a novel regime-switching model based on the relation spot-forward in electricity prices

Model outlook

Electricity prices show characteristic seasonal patterns:



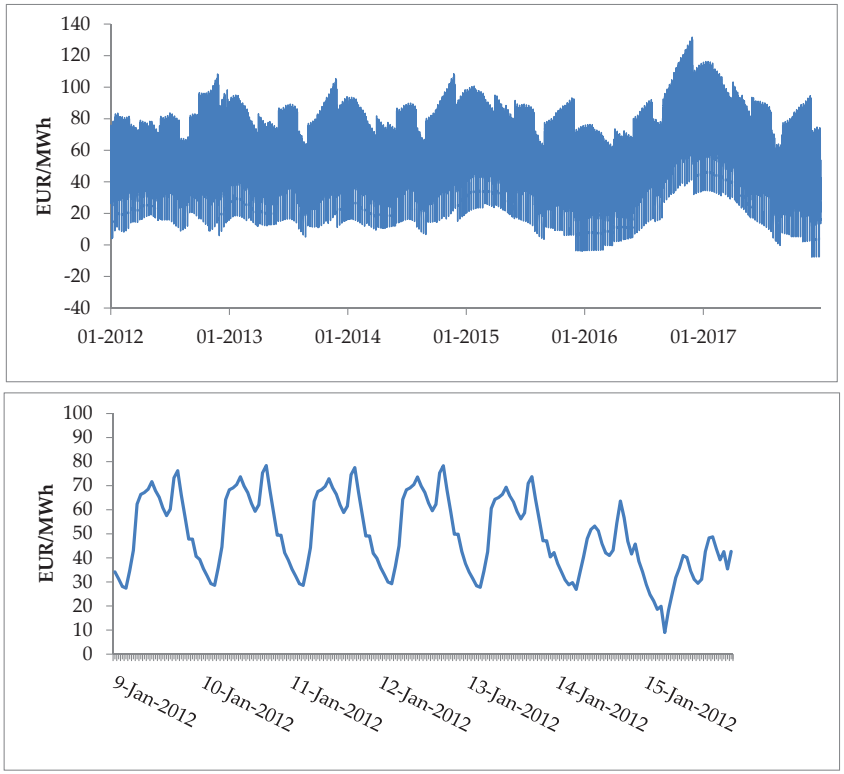


The price variation can be decomposed in a deterministic and a stochastic part:

- The deterministic part is derived as a "seasonality shape" (Blöchlinger, 2008). The shape is further incorporated in the hourly price forward curve (HPCF)
- The stochastic component is modeled as variation around the hourly price-forward curve (HPFC). We distinguish between **normal deviations (Gaussian regime)** and **extreme deviations (spike regimes)**

Derivation of HPFC

- The HPFCs are derived from market prices of traded futures (EEX Phelix futures, 01/01/2009–14/03/2013), with the procedure by Benth et al. (2007). The deviation from the seasonal component is modeled by a spline with maximum smoothness, which helps to keep the hourly seasonality shape unaffected
- Each day, future prices are observed in the market and the updated HPFC is derived
- **The spot price is thus built on the newest information on market expectation**



Model for electricity prices

$$S_t := \begin{cases} f_t^L - spike_t^- & \text{if system is in lower spike regime} \\ f_t \cdot \exp(r_t) & \text{if system is in normal regime} \\ f_t^U + spike_t^+ & \text{if system is in upper spike regime} \end{cases}$$

with

$$f_t^L := f_t \cdot e^{-\alpha_\delta^L} \qquad f_t^U := f_t \cdot e^{\alpha_\delta^U}$$

$$spike_t^- \sim \text{Exp}(\lambda_d^-), \qquad spike_t^+ \sim \text{Exp}(\lambda_d^+), \qquad d = 1, \ldots, D$$

$$r_t = a_h + \sum_{i=1}^{6} b_{i,h} \cdot \hat{r}_{t-i} + b_{24,h} \cdot \hat{r}_{t-24} + \varepsilon_t, \qquad \varepsilon_t \sim \text{N}(0,\, \sigma_h^2), \qquad h = 1, \ldots, H$$

$$\hat{r}_{t-i} = \begin{cases} \ln S_{t-i} - \ln f_{t-i} & \text{if system is in normal regime at time } t-i, \\ \text{E}(r_{t-i}) & \text{otherwise.} \end{cases}$$

Transition matrix:

$$\Pi_h = \begin{pmatrix} 1 - \pi_h^{LG} & \pi_h^{LG} & 0 \\ \pi_h^{GL} & 1 - \pi_h^{GL} - \pi_h^{GU} & \pi_h^{GU} \\ 0 & \pi_h^{UG} & 1 - \pi_h^{UG} \end{pmatrix}.$$

Model for electricity prices: Time blocks

The price volatility or the probability and magnitude of spikes may be different

- in summer and winter,
- at weekdays (Mo-Fr), Saturday and Sunday,
- at different times of the day.

Times of the day are distinguished by the blocks:

season	night	morning	high noon	afternoon	rush hour	evening
summer	1–6	7–10	11–14	15–18	–	19–24
winter	1–6	7–10	11–14	15–16	17–20	21–24

$\rightarrow H = 33$ hourly blocks for which parameters must be estimated (normal regime).

For the spike regimes we distinguish only between days and seasons
$\rightarrow D = 6$ parameter sets

We define two functions

$$h(t): \ t \rightarrow \{1, \ldots, H\} \qquad\qquad d(t): \ t \rightarrow \{1, \ldots, D\}$$

that assign to the hour t the index h or d, respectively.

Estimation: Log-likelihood function

$$\ln \mathcal{L}(\alpha_1^L, \alpha_2^L, \alpha_1^U, \alpha_2^U) = \sum_{h'=1}^{H} \sum_{t \in \mathcal{D}^G(h')} \ln \phi(e_t \,|\, 0, \hat{\sigma}_{h'})$$

$$+ \sum_{d'=1}^{D} \sum_{t \in \mathcal{D}^L(d')} \ln \varphi(f_t^L - S_t \,|\, \hat{\lambda}_{d'}^-) + \sum_{d'=1}^{D} \sum_{t \in \mathcal{D}^U(d')} \ln \varphi(S_t - f_t^U \,|\, \hat{\lambda}_{d'}^+)$$

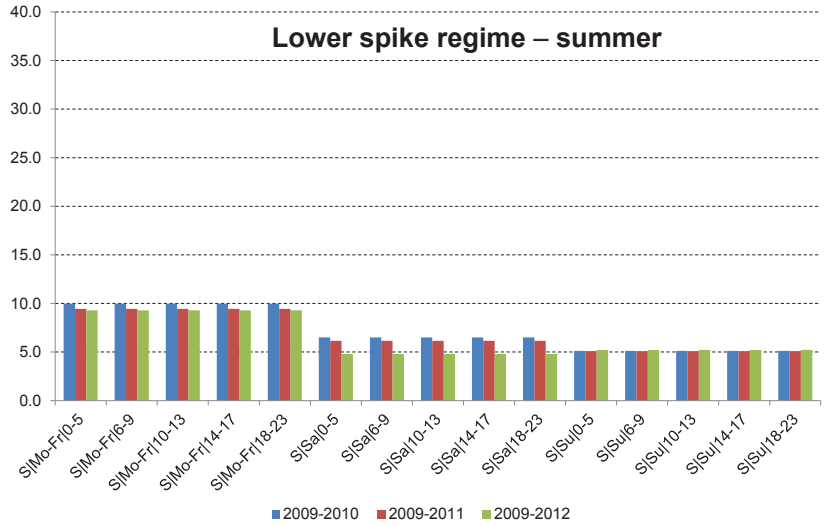
ϕ: density of the normal distribution with parameters μ, σ

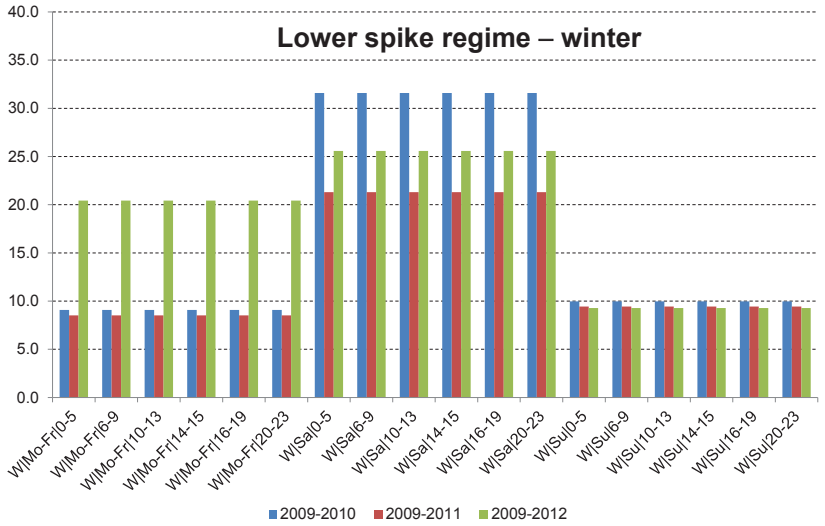
φ: density of the exponential distribution with parameter λ

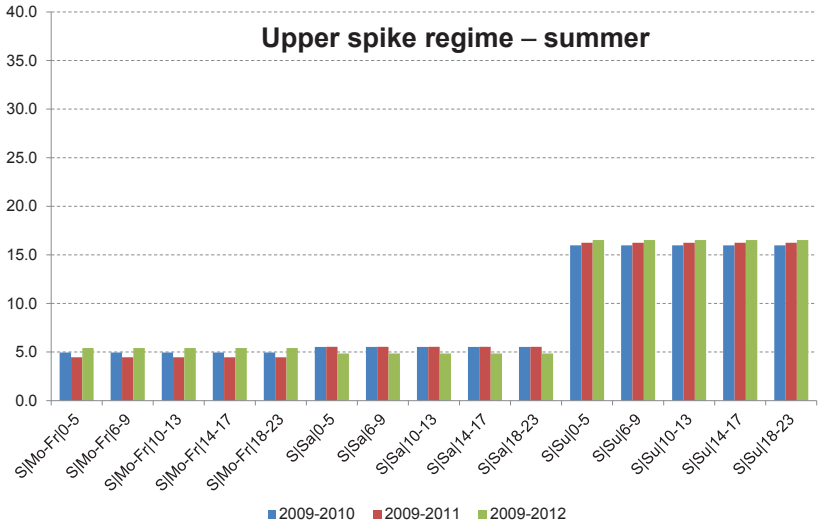
Determination of $\alpha_1^L, \alpha_2^L, \alpha_1^U, \alpha_2^U$:

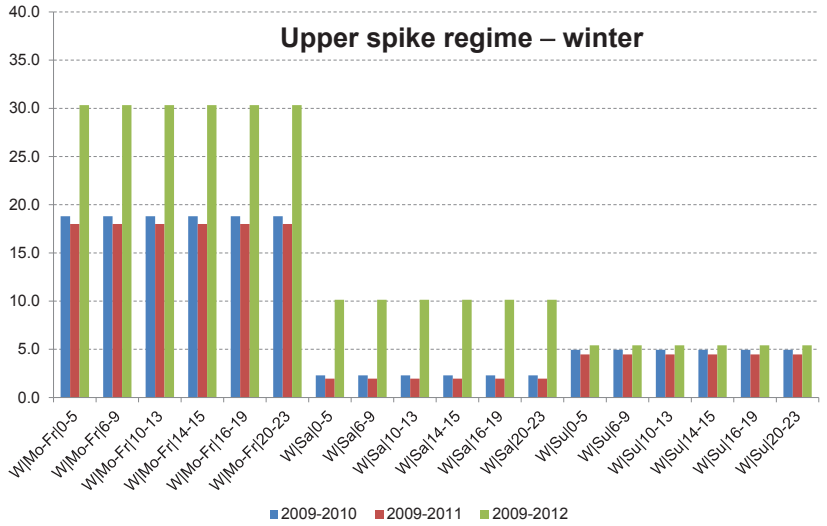
- $\ln \mathcal{L}$ is maximized to find the optimal values for the regime limits
- In each step the other parameters are estimated
- The optimal values for the limits are fixed and the log-likelihood function is maximized again with respect to the other parameters
- Probabilities of transitions and of remaining in regimes are calculated from the number of corresponding observations.

Estimation results: Expected spike magnitudes

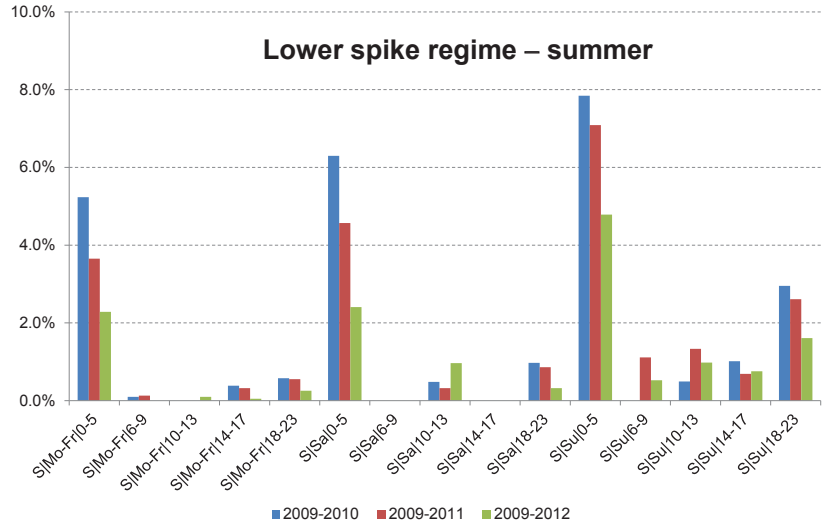


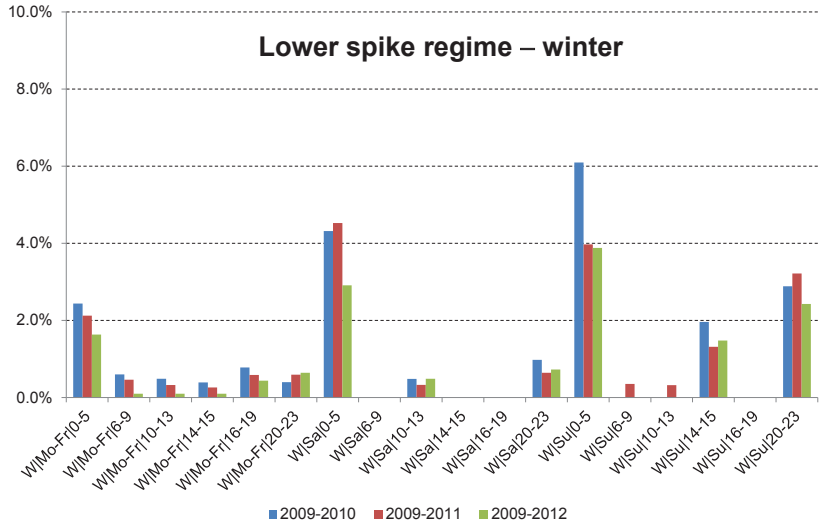


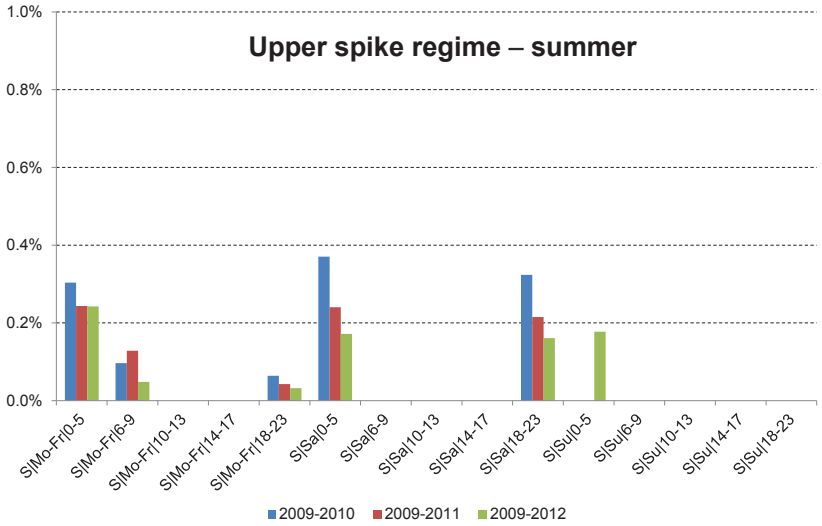


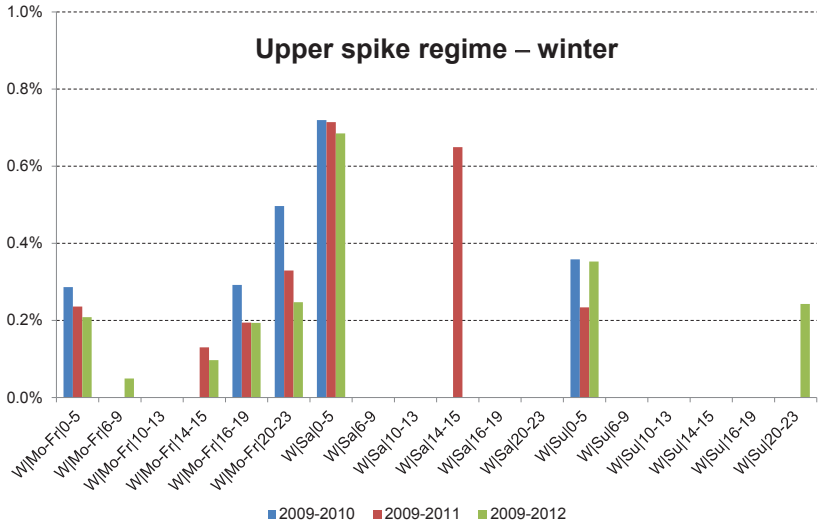


Probabilities of transitions from normal to spike regimes







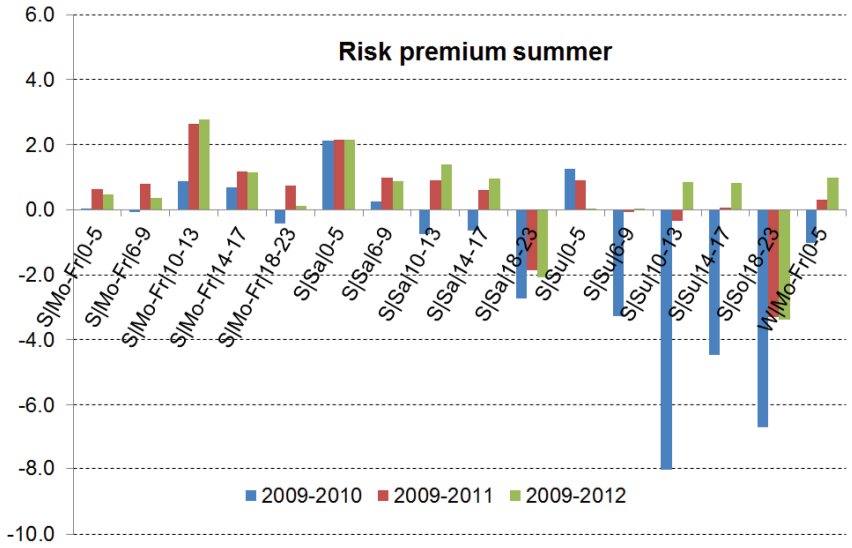


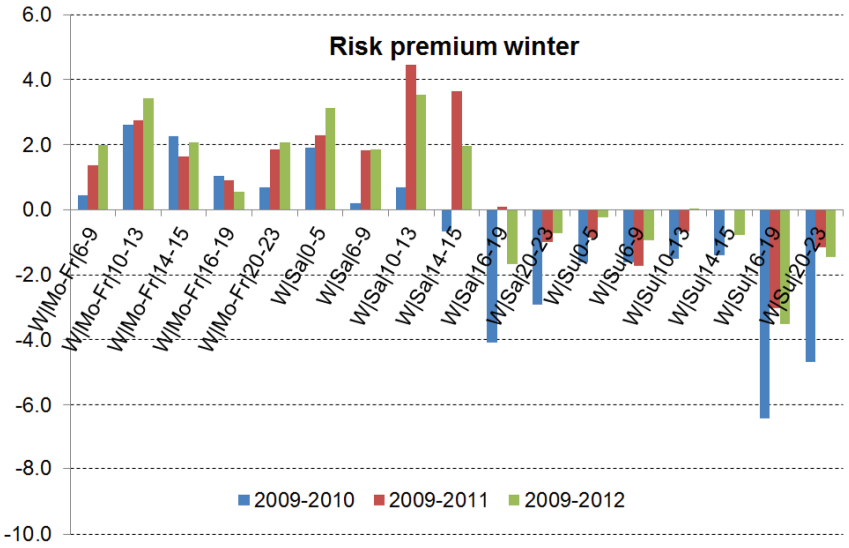
Probabilities of transitions from spike to normal regimes & regime limits

Sample period		01/01/2009 31/12/2010		01/01/2009 31/12/2011		01/01/2009 14/03/2013	
seas.	day	$\hat{\pi}_h^{LG}$	$\hat{\pi}_h^{UG}$	$\hat{\pi}_h^{LG}$	$\hat{\pi}_h^{UG}$	$\hat{\pi}_h^{LG}$	$\hat{\pi}_h^{UG}$
S	Mo-Fr	65.40	53.85	66.77	46.67	65.74	55.00
S	Sa	60.78	87.50	64.52	87.50	64.81	87.50
S	Su	70.24	95.83	69.92	95.83	68.33	92.00
W	Mo-Fr	65.22	80.36	65.29	77.78	69.86	76.32
W	Sa	73.08	33.33	75.00	33.33	71.43	85.71
W	Su	65.00	0.00	68.92	0.00	71.00	77.78

Sample period	01/01/2009 31/12/2010		01/01/2009 31/12/2011		01/01/2009 14/03/2013	
Day index	$\delta = 1$	$\delta = 2$	$\delta = 1$	$\delta = 2$	$\delta = 1$	$\delta = 2$
$\hat{\alpha}_\delta^L$	0.50	0.99	0.50	0.85	0.69	0.91
$\hat{\alpha}_\delta^U$	0.78	1.15	0.78	1.11	0.80	1.05

Risk premiums



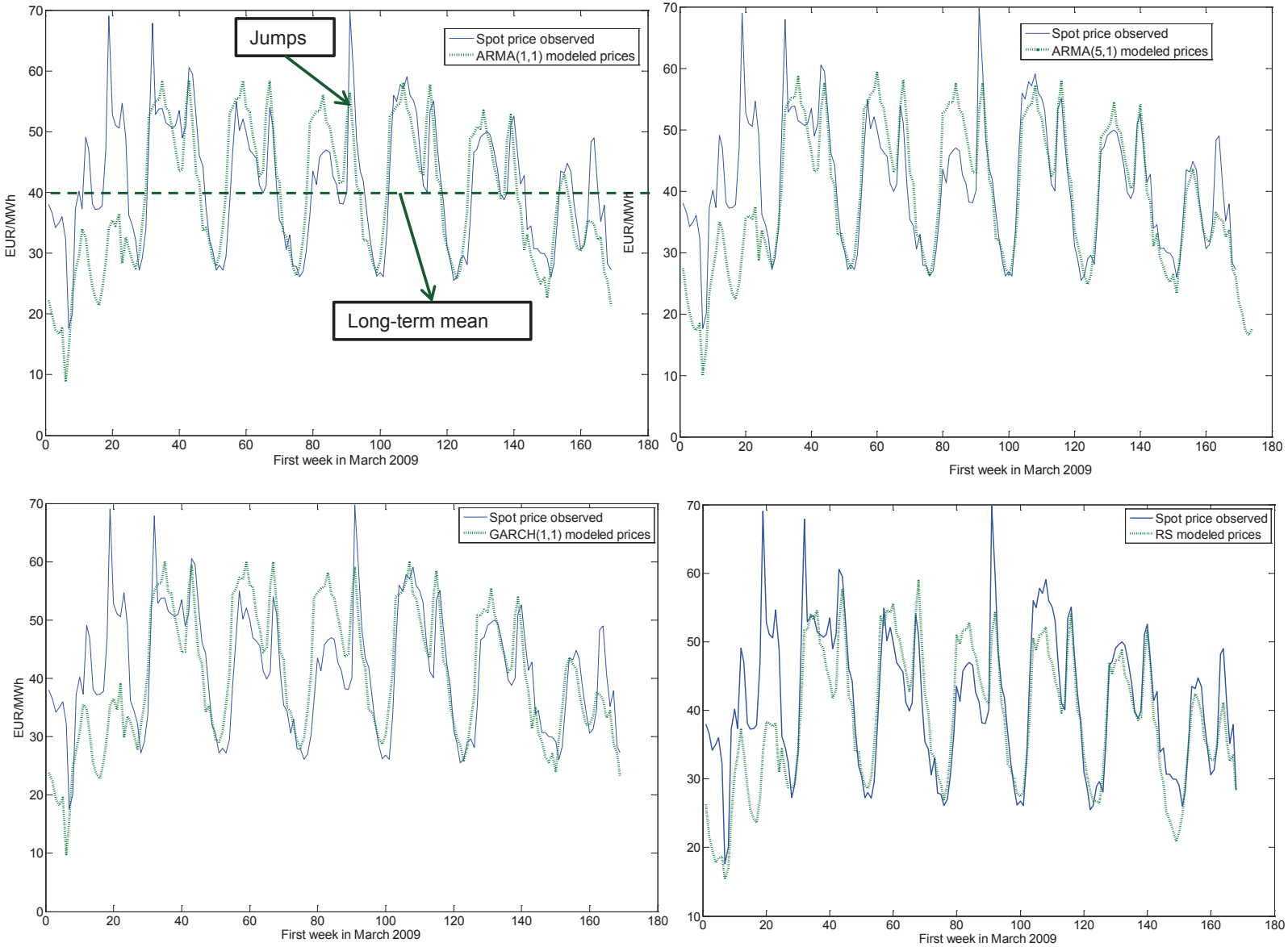


- Risk premium = difference forward minus spot price
- Derived from simulation of 1000 scenarios with regime-switching model
- Higher magnitudes in winter than in summer
- Values are positive during the week and decrease or become negative for the weekend
- Premiums vary over time

Estimation results for ARMA and GARCH

Sample		ARMA(1,1)	ARMA(5,1)	GARCH(1,1)
01/01/2009–	c	0.035	0.035	
31/12/2010	α_i	0.825*	0.763*, 0.051, -0.018, 0.009*, 0.020*	
	β_j	0.168*	0.228	
	ϕ_0			-2.167*
	ϕ_{1z}			0.046*
	ϕ_{2y}			0.900*
01/01/2009–	c	-0.08	-0.005	
31/12/2011	α_i	0.827*	1.765*, -0.903*, 0.161*, -0.062*, 0.028*	
	β_j	0.079*	-0.877*	
	ϕ_0			-2.135*
	ϕ_{1z}			0.015*
	ϕ_{2y}			0.948*
01/01/2009–	c	0.058*	0.086*	
14/03/2013	α_i	0.879*	0.409*, 0.394*, 0.026*, 0.021*, -0.031*	
	β_j	0.0003	0.470*	
	ϕ_0			1.657*
	ϕ_{1z}			0.016*
	ϕ_{2y}			0.892*
* = significant at 1% level				

Simulation of 1000 scenarios



Simulation results

- ARMA/GARCH: Stochastic component is simulated, then seasonality shape is added
- Regime-switching (RS) model: Prices are simulated directly (seasonality incorporated in HPFC)

Sample		ARMA(1,1)	ARMA(5,1)	GARCH(1,1)	RS model
01/01/2009–	$MAPE$	0.135	0.136	0.095	**0.079**
31/12/2010	R^2	0.490	0.493	0.490	**0.607**
01/01/2009–	$MAPE$	0.144	0.143	0.096	**0.084**
31/12/2011	R^2	0.442	0.442	0.439	**0.652**
01/01/2009–	$MAPE$	0.150	0.15	0.086	**0.083**
14/03/2013	R^2	0.414	0.414	0.409	**0.617**

Summary

- Classical time series models are based only on historical data, no information on future is used
- RS model is built on market view reflected by price forward curves, spot prices vary around HPFC: Higher R^2 and smaller errors than time series models
- A base and two spike regimes are distinguished:
  - Autoregressive process takes dependencies between consecutive hours into account (normal regime)
  - Negative prices and spike (clusters) can be reflected
  - Probabilities modeled by transition matrix
- Electricity prices show different volatilities and spike behavior depending on time of day, weekday or season
- Small estimated risk premiums, vary over time; consistent with literature
- Potential applications: Simulation of spot prices for medium and long term planning of electricity production, risk management, valuation of power contracts etc.