Insurance Demand and the Mitigation of Default Risk

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January 4, 2017

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Mossin’s Demand Model as Starting Point

Mossin, J. (1968). Aspects of Rational Insurance Purchasing. Journal of Political Economy, 76, 533-568 → *Full coverage is optimal iff premium is actuarially fair*

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Mossin (1968) (default-free setting)

$$W := w - c_I(\alpha_I) + (-1 + \alpha_I)L, \qquad L = \begin{cases} l, & if\ loss\ occurs, prob\ p \\ 0, & else, prob\ 1-p \end{cases}$$

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Modified Demand Model: The Case of Non-Performance

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↳ **Doherty N., Schlesinger H. (1990)**. Rational Insurance Purchasing: Consideration of Contract Nonperformance. Quarterly Journal of Economics, 105, 243-253 → *Given default risk, over- or under-insurance may be optimal even though the premium is actuarially fair*

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Doherty & Schlesinger (1990)

$$W := w - c_I(\alpha_I) + \{-1 + \alpha_I \boldsymbol{(1 - \chi_D(1 - r))}\}L, \quad \chi_D = \begin{cases} 1, & if\ insurer\ fails, prob\ q \\ 0, & else, prob\ 1-q \end{cases}$$

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Mahul O., Wright B. D. (2007). Optimal Coverage for Incompletely Reliable Insurance, Economic Letters 95, 456-461 → *Optimality of under- or over-insurance depends on size of recovery rate*

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Optimal insurance demand under the risk of contract nonperformance, if default risk can be mitigated?

→ *explicit risk-management measure; diversification*

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Our Model Framework

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Our model framework

n co-insurers: each co-insurer holds $\frac{1}{n}$ in premium and losses

$$W := w - c_I(\alpha_I, \boldsymbol{n}) - \boldsymbol{c_R(\alpha_R, n)} + \left\{-1 + \alpha_I - (\alpha_I - \boldsymbol{\alpha_R})\frac{\boldsymbol{F}}{\boldsymbol{n}}(1-r)\right\}L,$$

α_I and α_R are our decision variables

- $F \in \{0, \dots, n\}$ is the random number of failed insurers
- $F \sim BB\left(q\frac{1-\theta}{\theta}, (1-q)\frac{1-\theta}{\theta}\right)$, θ is the joint default correlation factor
- θ = 0 → F has a binomial distribution (independent defaults)

Our Model Framework: The Premium & Fee Principle

<u>Assumed premium/fee principle</u>: *Expected Payoffs* x *Proportional Cost Loading*

Premium for insurance coverage:

$$c_I(\alpha_I, n) = E[Default\ Adjusted\ Indemnification](1+\lambda_I)$$
$$= \alpha_I E\left[1 - \frac{F}{n}(1-r)\right] L(1+\lambda_I)$$
$$= \alpha_I L p\{1 - q(1-r)\}(1+\lambda_I) = c_I(\alpha_I)$$

Independent of n and θ!

Each insurer receives $\frac{1}{n} c_I(\alpha_I) \rightarrow 0$, as n becomes large: fixed running costs?

Fee for risk-management measure:

$$c_R(\alpha_R, n) = E[Payoff](1+\lambda_R)$$
$$= \alpha_R E\left[\frac{F}{n}(1-r)\right] L(1+\lambda_R)$$
$$= \alpha_R L p q(1-r)(1+\lambda_I) = c_R(\alpha_R)$$

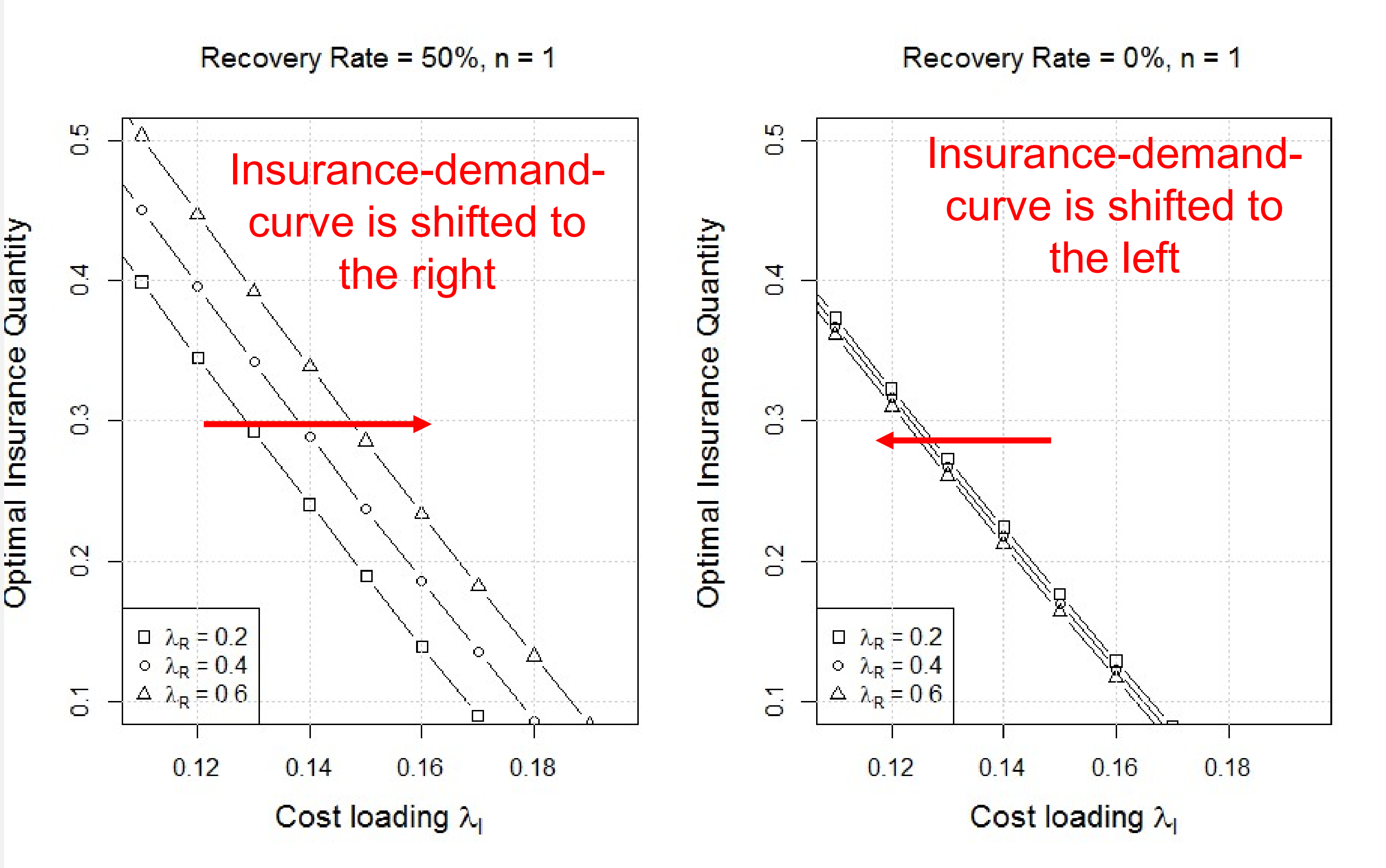
Independent of n and θ!

Question I: Price Effects on Insurance-Demand-Curve

Researching price effects

What happens to the optimal insurance coverage α_n^* if the **costs for hedging increase** ($\boldsymbol{\lambda_R}$ ↑)?

Is insurance coverage a **substitute** or **complement** of the risk-management measure?



Insurance coverage as substitute **Insurance coverage as complement**

Question II: Effect of Diversification

- Assumed the insurer can increase the number of co-insurers from *n* to n+1 → Natural question: Is it optimal to **increase** or to **decrease** insurance **coverage**?
- First intuition: Given two policies, it seems to be nearby that it is optimal to take up more of the policy that provides higher utility.

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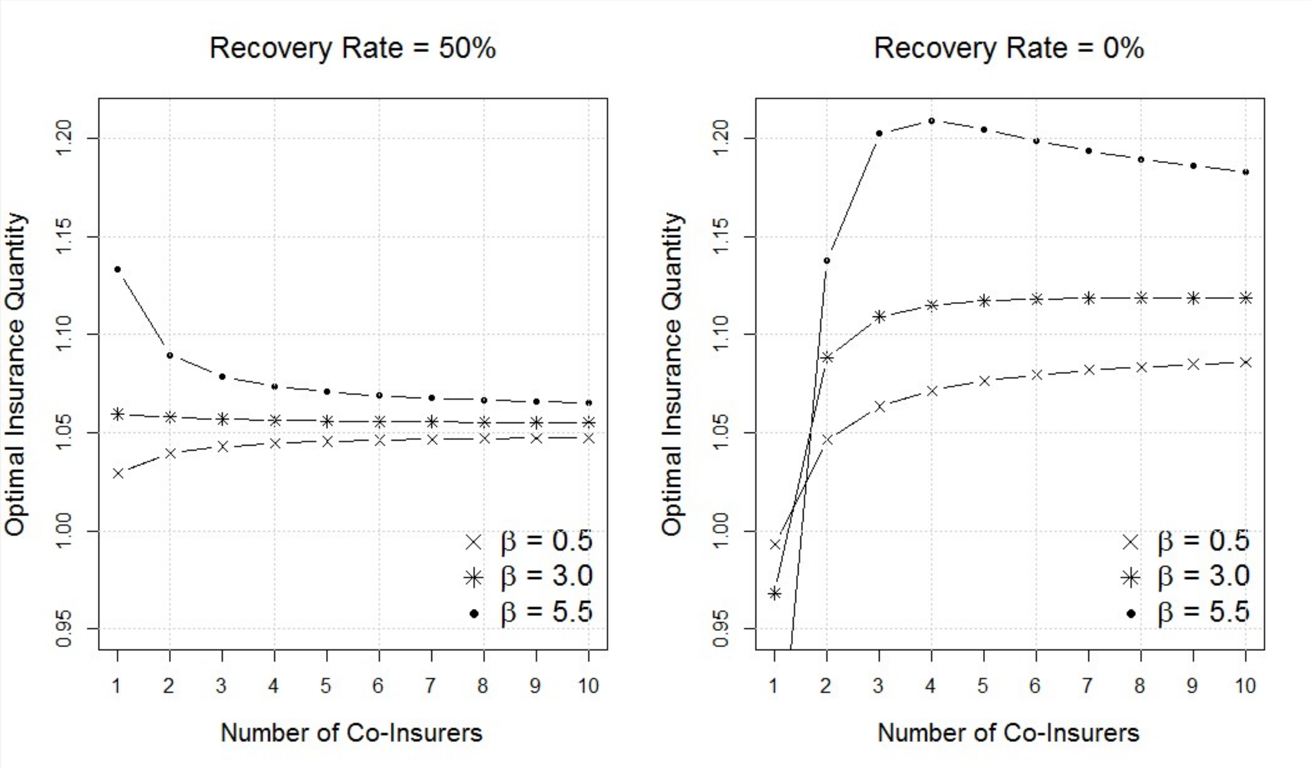

Numeric Example:

$$u(x) = -exp(-\beta x)$$

Initial wealth w	1.5
Loss prob. p	5.0 %
Loss size l	1.0
Default prob. q	1.0 %
Correlation θ	15 %
Cost loading λ_I	0.0

Monotonicity Criterion

Let $\alpha^*_{I,n}$ be the optimal insurance demand for n co-insurers and set

$$w^*_n(x) = w - l - c\left(\alpha^*_{I,n}\right) + \alpha^*_{I,n} l - \alpha^*_{I,n}(1-r)lx.$$

Then,$\alpha^*_{I,n+1} > (<) \alpha^*_{I,n}$holds true, if

$$\left(\mathbf{1} - \frac{\boldsymbol{w-l}}{\boldsymbol{w^*_n(x)}}\right)\boldsymbol{\eta}\left(\boldsymbol{w^*_n(x)}\right) < (>)\mathbf{2}, \textbf{\textit{ for all }} \boldsymbol{x} \in [\mathbf{0},\mathbf{1}],$$

where $\eta(x) := -xu'''(x)/u''(x)$ is the policyholder's measure of relative prudence.

(1) For **quadratic utility** $\eta(x) = 0, \alpha^*_{I,n}$ strictly increases in n (always)

(2) For **power utility**, with $u(x) = (x^{1-\nu} - 1)/(1-\nu)$,$\alpha^*_{I,n}$ strictly increases, if $0 < \nu < 1$

(3) If policyholder has constant absolute prudence C and if $r > r^*_1$ (i.e. over-insurance is optimal for $n = 1$, Mahul & Wright, 2007), then:

$C > (<) 2/\left\{l(r - p + pq(1-r))\alpha^*_{I,n}\right\} \Rightarrow$ decreasing (increasing) coverage is optimal

Monotonicity Criterion: Influence of Prudence

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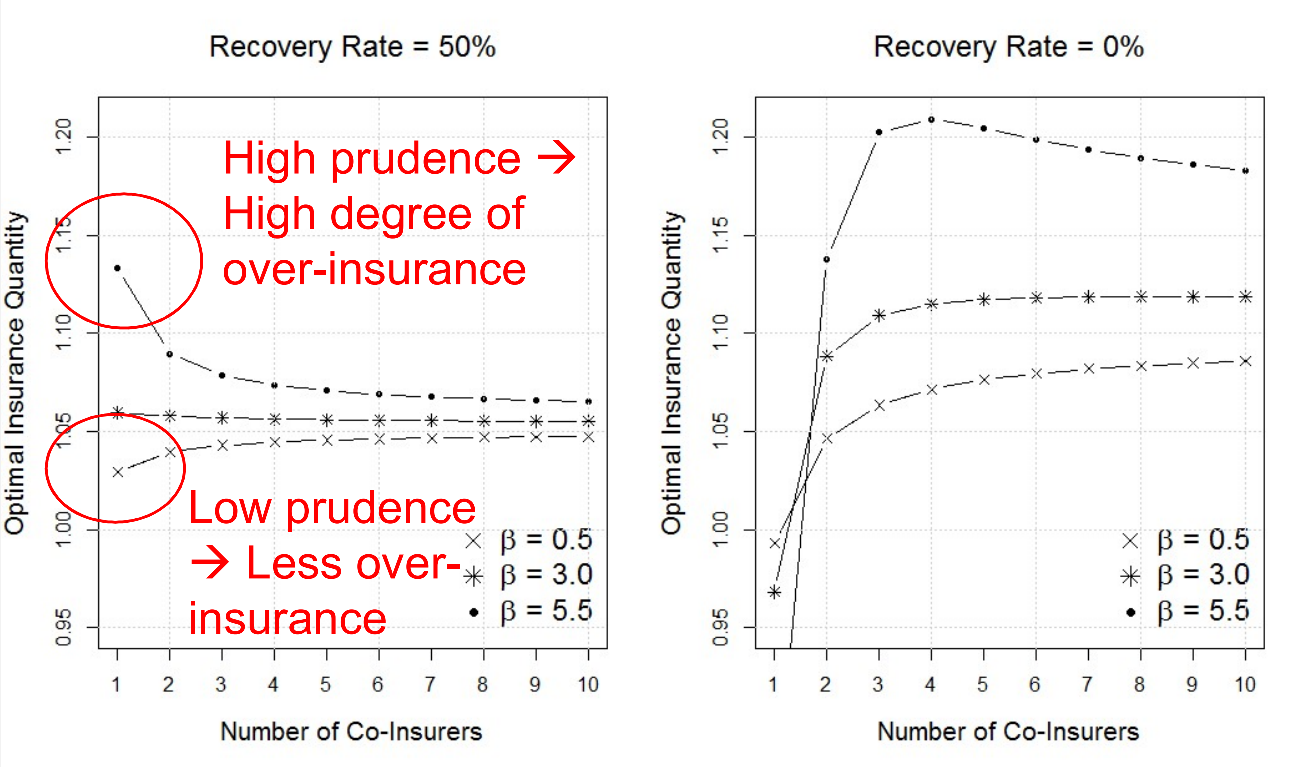

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Thank You

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