Appendices to

Stochastic labour market shocks, labour market programmes, and human capital formation: a theoretical and empirical analysis

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This version: June, 2007
Date this version has been printed: 07 July 2007

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Appendix 1

A1.1 Proof of Lemma 1

Suppose that for a given compact space X_t for some agent i (this index will be suppressed in this section) at time t employment is the preferred labour market regime for some value $\pi = \pi'$.[1] This particular choice of the agent implies the following:

$$V_{vt}^{s}(a_{vt}, h_{vt}, \pi'_{vt} \mid I^{w} = 1) > V_{vt}^{s}(a_{vt}, h_{vt}, \pi'_{vt} \mid I^{w} \neq 1) \tag{L.1}$$

Since a larger value of the shock strictly increases future human capital while working (something that does not happen in the other states) and in turn this (strictly) increases future earnings and thus future consumption possibilities, and because the period's returns from wages increase as well, for any larger value of the shock ($\pi'' \geq \pi'$), the person works as well:

$$\begin{aligned} & V_{vt}^{s}(a_{vt}, h_{vt}, \pi''_{vt} \mid I^{w} = 1) > V_{vt}^{s}(a_{vt}, h_{vt}, \pi'_{vt} \mid I^{w} = 1) > V_{vt}^{s}(a_{vt}, h_{vt}, \pi'_{vt} \mid I^{w} \neq 1) \\ & \Rightarrow \\ & V_{vt}^{s}(a_{vt}, h_{vt}, \pi''_{vt} \mid I^{w} = 1) > V_{vt}^{s}(a_{vt}, h_{vt}, \pi''_{vt} \mid I^{w} \neq 1) \end{aligned} \tag{L.2}$$

This establishes that there is a value of π, say π', beyond which the agent will always choose employment *(w)* among all other labour market options. But then there is a range of values in the distribution of π below which contemporaneous and future earnings from employment are so low that the agent's optimal choice would be non-employment. Say this happens at $\pi = \pi^{*}$. Then for any lower value ($\pi^{**}, \pi^{**} < \pi^{*}$), the individual won't work either, because when the value of the shock declines employment becomes less attractive compared to the non-employment options. Thus, a threshold π_b^R defined in terms of X_{vt} exists that completely

characterizes the decision between choosing employment or not. The threshold π_b^R depends on assets and human capital accumulated so far as well as on state of nature (i.e. the realisations of the shock), and determines the circumstances upon which the agent is willing to work.

For the case $\pi \leq \pi_b^R$, it remains to analyse the choice between the two non-employment alternatives. From the financial capital accumulation equation we see that the shock does not influence current period physical returns for the non-employment states. If there would be no effect of the shock on human capital accumulation, then individuals would all choose state $I_{it}^n = 1$. However, the larger shock, the less attractive alternative '*n*' becomes in terms of human capital, because the depreciation is increasing in the shock. Suppose there is a value π_a^R ($\pi_a^R \leq \pi_b^R$) such that individuals are just indifferent between *q* and *n*. Because of Assumption 8 (i.e., positive prices including $P_{st} > 0$), if π decreases below π_a^R alternative *n* become more valuable, i.e., *further loss of human capital declines* such that below π_a^R: $h_{it} \mid I_{it}^n \rightarrow h_{it} \mid I_{it}^q$ and $P_{st} \mid I_{it}^n = 0$. If the shock increases above π_a^R, the alternative '*q*' gains in value. Thus the monotone reservation policy is proved.

Proof of Lemma 2

This proof extends that in Lemma 2 Costa-Dias (2002) to cover a third labour market regime. In both cases the proof uses backward induction starting with the valued function at age T and showing similar properties for ages 0 to $T-1$ (the index i is suppressed for simplicity, so that for any i, $W_{s(i)t} = W_{st}$, etc.)

At age T the agent maximizes the contemporaneous utility only as function of consumption such that $c_T^* = (1+r_T)a_T + I_T^w \pi_T h_T W_{sT} + I_T^q (B_{sT} - P_{sT}) + I_T^n B_{sT}$; the agent decides to work

[1] The first part of this proof is similar to Costa-Dias (2002), but allows for a third labour market regime. The second

or not according to the realization of π_T conditional on past labour market history and characteristics. Whatever labour market regime the agent decides to select, $E_\pi V_{T+1}^s(.)=0$ and each of the (partitioned) value functions are characterized by the utility of final time period resources:

$$
\begin{aligned}
&V_T^s(a_T,h_T,\pi_T)=u\left((1+r_t)a_T+\pi_T h_T W_{sT}(1-\tau_T)\right) \qquad if \quad I_T^w=1;\\
&V_T^s(a_T,h_T,\pi_T)=u\left((1+r_t)a_T+\left(B_{sT}-P_{sT}\right)h_T\right) \qquad if \quad I_T^q=1; \qquad \text{(L.4)}\\
&V_T^s(a_T,h_T,\pi_T)=u\left((1+r_T)a_T+B_{sT}h_T\right) \qquad if \quad I_T^n=1.
\end{aligned}
$$

Allow for Assumption 2 at age T: the same properties for the utility function carry through for the value function for all the three labour market regimes. Allow for Assumptions in 3.3 and use the conditions in Lemma 1. Let $V_T^s(\cdot\,|\,I_T^j=1)$ be the short hand notation of the conditional (on $j=w,n,q$) value function:

$$
\begin{aligned}
E_\pi V_T^s(a_T,h_T)&=V_T^s(\cdot\,|\,I_T^w=1)P(I_T^w=1)+V_T^s(\cdot\,|\,I_T^q=1)P(I_T^q=1)+V_T^s(\cdot\,|\,I_{Tt}^n=1)P(I_T^n=1)=\\
&=\int_{\underline{\pi}}^{\pi_a^R} V_T^s(a_T,h_T\,|\,I_T^w=1)\,f(\pi)d\pi+\int_{\pi_a^R}^{\pi_b^R} V_T^s(a_T,h_T\,|\,I_T^q=1)\,f(\pi)d\pi+\int_{\pi_b^R}^{\bar{\pi}} V_T^s(a_T,h_T\,|\,I_T^n=1)\,f(\pi)d\pi
\end{aligned}
\qquad \text{(L.5)}
$$

But (L.4) implies that $V_T^s(\cdot\,|\,I_T^j=1)$ is strictly increasing, twice differentiable and concave in assets for any of the $j\in\{w,q,n\}$ labour market alternative, therefore, so is the expectation $E_\pi V_T^s(a_T,h_T)$; notice that this is also taking into account that at any point in the lifetime of individuals, including at T, the reservation thresholds depend on past information and not in the present levels of assets (as determined in Lemma 1).

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part of the proof refers to the third regime explicitly.

At ages 0 to $T-1$**:** The proof has four steps (following Costa-Dias (2002) and adapting Stokey and Lucas (1989) to be applicable to any number of labour market regimes)

Let $E_\pi V^s_{\tilde{t}+j}(a,h) = E_\pi V^s_T(a,h)$: The previous step shows that given Lemma 2, the RHS is strictly increasing, twice differentiable and a concave function in assets (a_t).

Step 1: We show that the conditional value functions $V^s_t(\cdot \mid I^j_t = 1)$ are increasing, twice differentiable and concave in (physical) assets. Given that $u(c_{\tilde{t}+j})$ is concave (Assumption 2) and $E_\pi V^s_{\tilde{t}+j}(\cdot|)$ are strictly increasing, concave and twice differentiable in $c_{\tilde{t}+j}$ and $a_{\tilde{t}+j}$, standard recursive methods show that for bounded objective functions, $V^s_{\tilde{t}+j-1}(\cdot|)$ has identical properties that $E_\pi V^s_{\tilde{t}+j}(\cdot|)$.

The proof can be found in Stokey and Lucas (1989), Chapter 9, page 261. Furthermore, take expectations on $V^s_{\tilde{t}+j-1}(\cdot|)$ over the support so that we define $E_\pi V^s_{\tilde{t}+j-1}(\cdot|)$. The latter could be represented as $E_\pi V^s_{t+1}(\cdot|)$ for any t in the working life of an individual. Then, the same standard recursive methods in Stokey and Lucas (1989) imply that with $u(c_{t+1})$ and $E_\pi V^s_{t+1}(\cdot|)$ strictly increasing, twice differentiable and concave in c_{t+1} and a_{t+1}, respectively, the value function $V^s_t(k,h,\pi|.)$ is strictly increasing, twice differentiable and a concave function in assets (a_t).

Step 2: We show that the reservation value π^R_b for the labour market shock π_t is continuous in assets (a_t). The monotonic relation between π^R_a and π^R_b implies that both reservation values are continuous and differentiable (at least once) in assets (a_t).

The reservation values π^R_a and π^R_b both solve the equalities between the three value-functions determined by the three labour market choices. Furthermore, Step 1 implies the continuous differentiability (with respect to assets) of the value functions for any given labour

market regime. Since assets are an increasing, continuous and differentiable function of human capital h_{vt}, the value functions are also strictly increasing, twice differentiable, concave functions with respect to human capital. Take, for example, the threshold π_b^R. We know from Lemma 1 that this threshold solves the equality given by $V_t^s(a,h,\pi_b^R \mid I_t^w=1)=V_t^s(a,h,\pi_b^R \mid I_t^q=1)$, where the latter is a function of the same arguments in the neighbourhood of π_b^R. All the above implies the following:

(a) The partial derivatives $V_h(\mid I),\ V_a(\mid I),\ and\ V_\pi(\mid I)$ exist. That is, Assumption 1 and Step 1 guarantee the existence of these partial derivatives for any labour market option (notice that for $V_h(\mid I)=V_a\cdot\left({}^{\partial a}/_{\partial h}\right)$ so that the existence of the partial derivative with respect to human capital is also guaranteed.)

(b) Suppose we can define a point (a^R,h^R,π_b^R). From Lemma 1 we know that π_b^R solves the equality $V_t^s(a,h,\pi_b^R \mid I_t^w=1)=V_t^s(a,h,\pi_b^R \mid I_t^q=1)$, therefore, this must also happen so that $V_t^s(a^R,h^R,\pi_b^R \mid I_t^w=1)=V_t^s(a^R,h^R,\pi_b^R \mid I_t^q=1)$. That is, at this point the equality is also true. Since the value function is continuous and differentiable over the support of π, and π_b^R is in the support $[\underline{\pi},\overline{\pi}]$, then the derivative $\dfrac{\partial V(a^R,h^R,\pi_b^R \mid I)}{\partial \pi}\neq 0$ in the neighbourhood of that point.

The Implicit Function Theorem says that if a function $V(a,h,\pi):D^n\to\mathbb{R}^m, m<n,$ complies with conditions (a) and (b), then, there exists a function $g(h,a)$ such that $V_t^s(a^R,h^R,g(a,h)\mid I_t^w=1)=V_t^s(a^R,h^R,g(a,h)\mid I_t^q=1)$ in the neighbourhood of (a^R,h^R,π_b^R). This function has an implicit representation, say $\pi^R=g(a,h)$, satisfies $\pi_b^R=g(a^R,h^R)$, and is continuous and at least once differentiable in its arguments. Notice also that in our model $a=a(h)$, and not the other way around. Assume both (a,h) follow monotonically the same

direction as is the case for fixed labour market regimes. Stokey and Lucas (1989, page 290) show that the model can be reformulated in terms of only one endogenous variable with the recursive solution applying identically to the reformulated problem. Thus, we can let $\pi_b^R = \pi_b^R(a)$. The one-to-one mapping is guaranteed.

The same argument can be applied to the reservation value π_a^R that solves for the equality between the value functions $V_t^s(a,h,\pi_a^R \mid I_t^q = 1) = V_t^s(a,h,\pi_a^R \mid I_t^n = 1)$. In both cases we have shown that Assumptions 1 and Step 1 allow for the application of the Implicit Function Theorem, and this ensures that both reservation policies are continuous differentiable functions (at least once) of assets (a_t). This is to be used in further steps.

Step 3: Allowing for Assumption 1 and the interpretation of the reservation policies in Lemma 1, the expected value function at time t can be written as follows:

$$E_\pi V_t^s(a_t,h_t) = \int_{\underline{\pi}}^{\pi_a^R} V_t^s(a_t,h_t \mid I_t^w = 1) f(\pi) d\pi + \int_{\pi_a^R}^{\pi_b^R} V_t^s(a_t,h_t \mid I_t^q = 1) f(\pi) d\pi + \int_{\pi_b^R}^{\bar{\pi}} V_t^s(a_t,h_t \mid I_t^n = 1) f(\pi) d\pi \qquad \text{(L.6)}$$

Step 1 determines that $V_t^s(\cdot \mid I_t^j = 1)$ is strictly increasing, twice differentiable and concave in physical assets for all three labour market regimes. Step 2 determines that the reservation policies are continuous differential functions of assets, and the differentiability of the joint density function of the productivity shocks is also guaranteed in Assumption 1. Therefore, $E_\pi V_t^s(a_t,h_t)$ is also twice differentiable with respect to assets a_t. This is a necessary condition for Step 4 below.

Step 4: We show that the value function $E_{\pi}V_t^s(a_t,h_t)$ is an increasing and concave function of assets a_t.

Step 3 allows for the following representation for the first derivative of $E_{\pi}V_t^s(a_t,h_t)$:

$$\frac{\partial EV_t^s(a,h)}{\partial a_t}=\int_{\underline{\pi}}^{\pi_a^R}\frac{\partial\left(V_t^s(.|I^n=1)\right)}{\partial a_t}dF(\pi)+\int_{\pi_a^R}^{\pi_b^R}\frac{\partial\left(V_t^s(.|I^q=1)\right)}{\partial a_t}dF(\pi)+\int_{\pi_b^R}^{\bar{\pi}}\frac{\partial\left(V_t^s(.|I^w=1)\right)}{\partial a_t}dF(\pi)$$

$$+\frac{\partial\pi_b^R}{\partial a}\left\{\left(V_t^s(.|I^q=1)-V_t^s(.|I^w=1)\right)\right\}dF(\pi_{b,t}^R)+ \qquad \text{(L.7)}$$

$$+\frac{\partial\pi_a^R}{\partial a}\left\{\left(V_t^s(.|I^n=1)-V_t^s(.|I^q=1)\right)\right\}dF(\pi_{a,t}^R).$$

The last two terms in the RHS vanish at the reservation value in the density function of π (the value functions are identical), while the first derivatives with respect to assets are all positive since Step 1 ensures that the conditional value function is strictly increasing. Therefore it follows that $\left(\partial EV_t^s(a,h)/\partial a_t\right)>0$. All what is needed for concavity is to show that $\left(\partial E^2V_t^s(a,h)/\partial^2 a_t\right)<0$. From (L.7), the second order derivative is given by:

$$\frac{\partial E^2V_t^s(a,h)}{\partial^2 a_t}=\int_{\underline{\pi}}^{\pi_a^R}\frac{\partial^2\left(V_t^s(.|I^n=1)\right)}{\partial^2 a_t}dF(\pi)+\int_{\pi_a^R}^{\pi_b^R}\frac{\partial^2\left(V_t^s(.|I^q=1)\right)}{\partial^2 a_t}dF(\pi)+\int_{\pi_b^R}^{\bar{\pi}}\frac{\partial^2\left(V_t^s(.|I^w=1)\right)}{\partial^2 a_t}dF(\pi)$$

$$+\frac{\partial\pi_b^R}{\partial a_t}\left\{\left(\frac{\partial\left(V_t^s(.|I^q=1)\right)}{\partial a_t}-\frac{\partial\left(V_t^s(.|I^w=1)\right)}{\partial a_t}\right)\right\}dF(\pi_{b,t}^R)+ \qquad \text{(L.8)}$$

$$+\frac{\partial\pi_a^R}{\partial a_t}\left\{\left(\frac{\partial\left(V_t^s(.|I^n=1)\right)}{\partial a_t}-\frac{\partial\left(V_t^s(.|I^q=1)\right)}{\partial a_t}\right)\right\}dF(\pi_{a,t}^R).$$

The first three terms in the RHS of (L.8) are negative because of the concavity of the conditional value functions. But the value of the last two terms in (L.8) depend on the relative

degree of concavity between paired labour market regimes (i.e. between I_t^q and I_t^w, and between I_t^n and I_t^q), and the degree of absolute risk aversion (given by the derivatives $\left(\partial\pi_b^R/\partial a\right)$ and $\left(\partial\pi_a^R/\partial a\right)$).[2] Assumption 4 states that individuals are risk averse in the sense that an increase in assets reduces the reservation policy (subjective valuation of labour market choice) thus making employment more likely than non-employment in the future for any random shock. Likewise, an increase in assets as result of non-decreased in human capital (rather than depreciation) implies that program participation becomes more likely than 'unemployment without program participation', also for any given random productivity shock. Therefore, $\left(\partial\pi_b^R/\partial a\right)<0$ and $\left(\partial\pi_a^R/\partial a\right)<0$ are implied by Assumption 4 as well as being consistent with our model (see introductory notes). But, if an increase in physical assets implies reducing the respective reservation policies through an increase in the willingness to take risk the implication is that for any given assets level, a_t, comparing the value functions between labour market regimes implies that $\left(\partial V(\cdot \mid I_t^w=1)/\partial a\right)>\left(\partial V(\cdot \mid I_t^q=1)/\partial a\right)>\left(\partial V(\cdot \mid I_t^n=1)/\partial a\right)$.[3] Decreasing absolute risk aversion

[2] That is, as stated in the introduction, individual's hold latent valuation on each of the labour market regimes that we define as 'reservation valuation policy set'. These sets depend on individual's taste for risk possible determined by individual's history, characteristics, etc: Lemma 1 embodies this idea. Each time the agent has to evaluate the labour market conditions as the shock is realized, they compare the realized shock π_t to own reservation policy that explains individual's taste for risk (π_t^R), and make a labour market choice. Since the risk attitude is given by the set of reservation policies (π_t^R), risk aversion is measured by the change on this with respect to assets, where assets includes human capital as part of the individuals wealth. This justifies that the derivates $\left(\partial\pi_t^R/\partial a\right)$ explain the concept of risk aversion (coefficient of risk aversion).

[3] That is, expected value of a choice is the weighted sum of the three possible choices so that expectations of the value function is $EV=V(\mid I=w)P(w)+V(\mid I=q)P(q)+V(\mid I=n)P(n)$, and the choice among the

and derivatives of value functions that are increasing as taste for risk increases implies that the second and third terms in the RHS of (L.8) can be positive and overtake the negative value of the first three terms. Then, concavity of the valued function can only be guaranteed if we assume 'constant absolute risk aversion' in which case $\left(\partial\pi_b^R/\partial a\right) = \left(\partial\pi_a^R/\partial a\right)$=0. This would imply that the reservation policies are not responsive to changing wealth that is neither a realistic assumption, not is it completely consistent with our structural model. Thus, Assumption 4 is required so that 'decreasing absolute risk aversion', i.e., $\left(\partial\pi_b^R/\partial a\right)$<0 and $\left(\partial\pi_a^R/\partial a\right)$<0, but by a magnitude that is 'not too large' (both values are assumed to be bounded from below in the neighbourhood of zero) guarantees that the positive terms in the last two parts of the RHS in (L.8) never overtake the negative values of the set of second derivatives. This is the only way to guarantee concavity.

A1.3 Proof of Lemma 3

Given Lemma 2 (i.e., having established the conditions for a well behaved value function), the Euler Equation is the necessary and sufficient condition for the optimal consumption decision 'for fixed labour market regimes' (since it is within labour market regimes that the value function is continuous, twice differentiable and concave function of assets). Recall the Euler Equation:

$$\left.\frac{\partial u}{\partial c_t}\right|_{I(j,t)} = E\left[\beta(1+r)\left.\frac{\partial u}{\partial c_{t+1}}\right|_{I(j,t)}\right] \qquad \forall j \tag{L.9}$$

three alternative depends on the realization of the shock that will determine the weight (probability). But independently, each of the value functions is an increasing, twice differentiable and concave function of assets, while the value of the value function for the working choice has to be steeper than for the non-employment alternatives and in turn. At this point is when we need to apply Assumption 5 (no crossing of the value functions).

But (L.9) gives the optimal intertemporal relation for the choice variable assuming concavity of the value function only with respect to assets, when in reality the problem in (2) implies a more complex set of dynamics in the state space. Then, there must be as many optimal consumptions paths that are consistent with (L.9) as possible values of h_{t+1} that are consistent with the assets path (a_t) that underlines (L.9). Then identification/characterization of the optimal consumption path is only possible if we find an expression analogous to (L.9) such that the new expression implies restrictions for human capital. Recall Step 1 in the proof of Lemma2. This step states that under the regularity assumptions for $u(c)$ and $E_\pi V^s_{t+1}(\cdot|)$ in c_t and a_{t+1}, standard recursive methods show that $V^s_t(\cdot|I^j_t=1)$ has identical properties than $E_\pi V^s_{t+1}(\cdot|)$. First we apply the envelope theorem to $V^s_t(\cdot|I^j_t=1)$ so that at the optimal consumption choice and for fixed labour regime, a change in assets implies zero additions from future changes in the value function:

$$\begin{aligned}\left(\partial V^s_t(.|I^j)\big/\partial a\right) &= \left.\frac{\partial u}{\partial a}\right|_{c^*(t),I(j,t)} + \beta E_t \left.\frac{\partial V_{t+1}(.|I^j}{\partial a_{t+1}}\right|_{c^*(t),I(j,t)} \\ &= \left.\frac{\partial u}{\partial c}\cdot\frac{\partial c}{\partial a}\right|_{c^*(t),I(j,t)} \quad \text{since} \quad E_t \left.\frac{\partial V_{t+1}(.|I^j}{\partial a_{t+1}}\right|_{c^*(t),I(j,t)} = 0 \qquad \text{(L.10)} \\ &= u'(c_t)(1+r_t)\end{aligned}$$

Since $\left(\partial V^s_t(.|I^j)/\partial a_t\right) = (1+r_t)u'(c_t)$ and $V^s_t(\cdot|I^j_t=1)$ has identical properties than $E_\pi V^s_{t+1}(\cdot|)$, we take expectations so that $E_t\left(\partial V^s_{t+1}(.|I^j)/\partial a_{t+1}\right) = (1+r_{t+1})E_t u'(c_{t+1})$; the result is labour market regime and skill specific. The result is then applied to the Euler Equation in (L.9):

$$\left.\frac{\partial u(c_t)}{\partial c_t}\right|_{I(j)} = \beta\cdot E_t\left[\left.\frac{\partial V^s{}_{t+1}(a_{t+1},h_{t+1})}{\partial a_{t+1}}\right|_{I(j)}\right] \qquad \text{(L.11)}$$

Expression (L.11) maintains the same properties as the Euler condition in (L.9) but we have now established a relation between current consumption and the other dynamic variable in the system, human capital. We are now closer to identifying the optimal condition for consumption (optimal consumption path) taking into account the full dynamic system. Notice from the dynamics in (1) that the two endogenous state variables always follow the same direction, while the value function is concave in assets. This means that the derivative in the RHS of (L.11) is positive for any value of h_{t+1}, with this latter variables also increasing as a_{t+1} increases. At the same time (L.11) explains that any marginal change in utility today has to be matched by an equal but weighted expected marginal change in tomorrow's utility establishing a precise relation between the concavity of $u(.)$ and $EV(.)$ with respect to the variables c_t and a_t. From the dynamics in (1) we see that this must imply that we are pinning down the optimal human capital path. That is $a_{t+1} = (1+r)a_t + INC(h,W,\pi)|_{I(j)} - c_t$. Then, for fixed working conditions, any increase in assets has to be met by an increase in consumption so that (L.11) is satisfied, and this leaves no room for h_{t+1} to move other than whatever value satisfies (L.11). In other words, (L.11) can be re-written as:

$$\left.\frac{\partial u(c_t)}{\partial c_t}\right|_{I(j)} = \beta \cdot E_t\left[\left.\frac{\partial V^s{}_{t+1}(a_{t+1}, h_{t+1} \mid h_{t+1})}{\partial a_{t+1}}\right|_{I(j)}\right] \qquad \text{(L.12)}$$

Then, given the properties of the value function, the values of the state variables and for fixed skills and working decisions, the optimal condition for consumption is given by (L.12). With this (allowing for all regularity conditions and assumptions above), the problem in (2) has a unique solution 'for fixed labour market regimes' and for given skill type. In the development of (L.12) we have seen that agents are restricted to be risk averse. Expression (L.12) places further restriction in the variables that determine the behaviour of individuals: consumption (c_t) and

savings (a_{t+1}) must both be *normal goods* in the sense that an increase in net income must be followed by an increase in both consumption and assets for fixed labour market regimes. The reason is the following: suppose 'total net income' increases (for example as result of an increase in human capital, but also as result of any other change in the state space). From the low of motion in assets (see (1)), the implication is that either (a_{t+1}) or (c_t) increase. But both u and EV are concave functions, therefore, both must increase to keep the equality in (L.12) satisfied. Another way to interpret this is as follows: allowing for $EV(a,h)$ in L.11 does not pin down a specific optimal path among all possible optimal paths given all admissible h paths, so L.11 is necessary but not sufficient. Conditioning on h implies that the Euler is now based on $EV(a,h|h)$ thus restricting the relation between assets and consumption so that the marginal intertemporal gains are now fixed for given labour market conditions. This latter is what allows to identify the optimal path but at the expense of further restrictions on the type of consumption and savings that individuals are allowed to consume and hold.

Lemmas 1, 2 and 3 complete the set of regularity conditions that allow for expression (2) to represent the individual's problem, for the problem to be well defined and for this to have a unique solution (identification of an optimal consumption path). At the same time, expression (2) is based on (1) and we have shown that the structural model as specified in (1) is well behaved. This is what allows us to use the characterization of the endogenous variables to specify the reduced form specification, and with this to estimate the parameters. In reality, what is crucial is to make sure that for fixed labour market regimes the dynamic endogenously changing variables change all monotonically in the same direction. Our specification is correct because the newly introduced labour market regime still maintains such monotonic relation. Assuming a well behaved bounded functions in a bounded support (for anyone of the three labour market

regimes), the problem boils down to ‘maximising a concave function’ subject to a set of constrain that ‘do not jump in different directions in some unspecified form’: this is also guaranteed. Because in our case these constrains also behave monotonically, the problem can be placed in the shape of a value function with behaviour that is driven by the dynamics in the model, thus the value function is also well behaved. The regularity conditions for the value function implies three constrains (risk aversion, consumption is normal and savings is also a normal good). This completes the theoretical part (the structural model and its conditions).

Appendix 2

Probit estimates for the conditional probabilities $P(I_t^w = 1 | \mathrm{K}_t^w; \gamma_{t,b})$, $P(I_{t+1}^w = 1 | \mathrm{K}_{t+1}^w; \gamma_{t,b})$ and $P(I_t^n = 1 | \mathrm{K}_t^n; \gamma_{ta})$, $P(I_{t+1}^n = 1 | \mathrm{K}_{t+1}^n; \gamma_{t+1a})$. The samples used in each of the four cases are based on the distribution from Table 1 so that estimates for period t are based on 48,653 units, and estimates for period $t+1$ are based on 45,222. The 3,431 drop in sample between periods results from those who move to be non-classified in one of the labour market regimes after period t.

Table A2.1: Probit Estimates for the outcomes working (versus not working) and unemployment without ALMP (versus working and unemployed with ALMP)

Dependent Variable	Working					Unemployed without ALMP			
Time Period	t		$t+1$			t		$t+1$	
	Coefficient	s.e	Coefficient	s.e		Coefficient	s.e	Coefficient	s.e
Constant	5.373	0.184	4.828	0.205		4.104	0.261	3.714	0.32
Age	-0.218	0.006	-0.195	0.007		-0.231	0.009	-0.191	0.011
Age square	0.003	0	0.002	0		0.003	0	0.002	0
Lives in German Canton	-0.232	0.023	-0.185	0.024		-0.19	0.032	-0.191	0.065
Household size	0.028	0.01	0.01	0.011		0.048	0.015	-0.053	0.07
Permanent Partner present	-0.463	0.029	-0.368	0.031		-0.436	0.041	0.04	0.016
Household ownership	-0.195	0.026	-0.164	0.027		0.019	0.033	-0.334	0.045
Primary Industry (Agro, fishery, mine)	0.118	0.068	0.193	0.068		-0.173	0.083	0.067	0.037
Secondary Industry (Manufacturing)	0.649	0.021	0.625	0.023		0.454	0.026	-0.089	0.087
Skill Class 1	-0.008	0.03	0.012	0.033		0.064	0.039	0.421	0.031
Skill Class 2	-0.325	0.025	-0.263	0.026		-0.404	0.032	0.018	0.047
Natural Logs, Net household income	-0.241	0.012	-0.247	0.013		-0.157	0.016	-0.3	0.036
Labour market experience <= 6 months	0.797	0.117	1.06	0.151		0.162	0.195	-0.223	0.018
Labour market experience <=12 months	0.286	0.14	-0.331	0.17		0.089	0.198	0.373	0.236
Labour market experience <=18 months	0.056	0.146	0.645	0.176		0.281	0.258	-0.285	0.255
Labour market experience <= 2 years	-0.01	0.109	-0.27	0.136		-0.038	0.216	0.225	0.245
Labour market experience <= 4 years	-0.14	0.051	0.03	0.058		-0.793	0.101	0.148	0.186
Time dummy, 1991	-0.485	0.059	-0.12	0.053		-0.408	0.072	-0.372	0.113
Time dummy, 1992	-0.218	0.05	0.045	0.048		-0.338	0.063	-0.138	0.071
Time dummy, 1993	-0.137	0.048	-0.016	0.05		-0.295	0.065	-0.108	0.072
Time dummy, 1994	-0.005	0.045	-0.04	0.049		-0.261	0.064	-0.116	0.071
Time dummy, 1995	-0.152	0.052	-0.331	0.059		-0.171	0.069	-0.092	0.074
Time dummy, 1996	-0.261	0.05	-0.216	0.053		-0.281	0.077	-0.313	0.105
Time dummy, 1997	-0.281	0.049	-0.308	0.057		-0.308	0.076	-0.347	0.098
Time dummy, 1998	-0.244	0.049	-0.392	0.059		-0.137	0.071	-0.265	0.098
Time dummy, 1999	-0.36	0.05	-0.418	0.059		-0.195	0.072	-0.135	0.082
Time dummy, 2000	-0.484	0.057	-0.453	0.064		-0.17	0.073	-0.076	0.075
Time dummy, 2001	-0.32	0.05	-0.274	0.053		-0.07	0.061	-0.08	0.072
Time dummy, 2002	-0.259	0.036	-0.129	0.039		0.031	0.049	-0.031	0.077
Unemployed for 6 or less moths						-0.29	0.086	-0.03	0.058
Unemployed for 12 or less months						-0.095	0.126	-0.317	0.085
Unemployed for 18 or less moths						0.012	0.154	0.175	0.115
Unemployed for 24 or less months						2.133	0.114	-0.133	0.143
								2.168	0.112

Note: All estimates are based on truncated probits (expressed,15, Section 4) with points $[\underline{\pi}, \overline{\pi}] = [0.000001, 23.8]$ as the points used for truncating the likelihood function. Table 1 explains the sample sizes used for estimating each of the specifications in the table. For example, estimates for the working outcome at period t, conditionally compares 45,826 working males to (980+1,847) non-working males to estimate the coefficients in columns 2, whereas for column 3 (at $t+1$) the comparison is between 42,438+322+264 working males against (545 + 607 + 236 + 128 + 97 + 585) non-working individuals who are still active labour market participants. Similarly, Table 1 shows the sizes involved in estimating the coefficients in columns 4 and 5. The omitted variables are 'lives in a non-German speaking canton', tertiary (service) sector, skill class 3 (the highest skill considered), 'has working experience greater than 48 months' and specifically for columns 4 and 5 'has been unemployed for more than 2 years'. Furthermore we omit the time dummies for 2002 and 2003. We consider significance at 5% level or below with bold coefficient suggesting such level of significance. All p-values for the diagnostics suggest rejecting overall heteroscedasticity and acceptance of the specification by means of the likelihood ratio.