

Limited Information and its Impact on a Policyholder's Optimal Choice on Deductibles

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Research Questions

- Policyholders typically face two sources of uncertainty:
 - Uncertainty arising from the randomness of future losses
 - Limited information about the functional forms of the loss distribution and the utility function
- How can we incorporate limited information about the functional forms in traditional models for the optimal deductible choice?
- Which information sources can the policyholder leverage to approximate the functional forms?
- Given limited information about the functional forms: Which characteristics should good decision rules for the optimal deductible choice have?

Motivation

Wealth of policyholder

$$Y_D(L) = \begin{cases} W - R(D) & \text{if } L = 0 \\ W - R(D) - L & \text{if } 0 < L \leq D \\ W - R(D) - D & \text{if } D < L \leq N \end{cases}$$

Optimal deductible choice

$$D_K^* = \arg \max_{D \in \{D_1, \dots, D_K, N\}} \mathbb{E}[u(Y_D(L))]$$

Motivation

- Majority of employees enrolled in a health plan of a Fortune 100 firm choose strictly dominated health plans; average amount saved by changing to non-dominated plan: \$375 (Bhargava et al., 2017)
- Customers in the homeowners insurance market purchase low deductibles despite costs significantly above the expected value (Sydnor, 2010)
- Although it is financially profitable, only 11 percent of Dutch health insurance customers choose policies with a deductible feature (Van Winssen et al., 2015, 2016)
- ...

Related Literature

Deductible policies

- Optimality of deductible policies: Arrow (1963), Raviv (1979), Gollier (2013)
- Optimal deductible choice: Schlesinger (1981), Schlesinger (2013), Moffet (1977), Mossin (1968)

Decision making under limited information

- Decision making under uncertainty: Kofler and Menges (2013)
- Decision rules under uncertainty: Bamberg et al. (2019)
- Heuristics and decision rules: Gigerenzer and Gaissmaier (2011)

Contribution of the Paper

Modeling limited information

- Modeling sources of information in an expected utility framework
- Deriving approximations of the loss distribution
- Deriving approximations of the utility function

Deriving suitable heuristics for choosing an optimal deductible

- Defining desirable characteristics for decision rules
- Analyzing the performance of different decision rules under limited information

Model

- Loss L with $\mu = \mathbb{E}[L]$, $\sigma^2 = \text{Var}[L]$ and cumulative distribution function

$$F(x) = \mathbb{P}(L \leq x) = \begin{cases} 0 & \text{if } x < 0 \\ p & \text{if } x = 0 \\ p + \int_0^x f(z) dz & \text{if } 0 < x < N \\ 1 & \text{if } x \geq N \end{cases}$$

- Premium $R(D) = (1 + \lambda) C(D)$ with $C(D) = \int_D^N (z - D) f(z) dz$
- Utility function u which is everywhere twice differentiable with $u' > 0$ and $u'' \leq 0$
- Optimal Deductible: $D_K^* = \arg \max_{D \in \{D_1, \dots, D_K, N\}} \mathbb{E}[u(Y_D(L))]$

Model

- Cost function $c : \{D_1, \dots, D_K, N\} \rightarrow \mathbb{R}$ for which $c(D)$ is defined as the value $c \in \mathbb{R}$ which solves

$$\mathbb{E}[u(Y(D_K^*) - c)] = \mathbb{E}[u(Y(D))]$$

- Expected costs for a decision rule $\hat{\theta}_{m,n}^{(I)}$:

$$\mathbb{E}[c(\hat{\theta}_{m,n}^{(I)})] = \sum_{i=1}^K \mathbb{P}(\hat{\theta}_{m,n}^{(I)} = D_i) c(D_i) + \mathbb{P}(\hat{\theta}_{m,n}^{(I)} = N) c(N)$$

- Sources of information:

- Loss observations $L_1, \dots, L_n$ for which the policyholder knows μ and σ^2
- Functional values of utility function $u(x_1), \dots, u(x_m)$
- Deductible policies with deductibles $\mathbf{D}_K = (D_1, \dots, D_K)$ and prices $\mathbf{R}_K = (R(D_1), \dots, R(D_K))$ for which the policyholder knows λ

Approximating the Loss Distribution

Empirical distribution function

$$\hat{F}_n(x) = \frac{1}{n} \sum_{i=1}^n \mathbb{1}_{(-\infty, x]}(L_i)$$

Approximating the Loss Distribution

Approximation derived from the available deductible policies

$$\hat{F}_K^{ded}(x) = \begin{cases} 0 & \text{if } x < 0 \\ \tilde{F}_K^{ded}(0) & \text{if } x = 0, \\ \tilde{F}_K^{ded}(0) + \int_0^x \hat{f}_k^{ded}(y) dy & \text{if } x > 0 \end{cases}$$

where

$$\hat{f}_K^{ded}(x) = \mathbb{1}_{(\hat{x}_1, \hat{x}_2]}(x) \hat{z}_1 + \dots + \mathbb{1}_{(\hat{x}_{\hat{K}}, N]}(x) \hat{z}_K$$

Approximating the Loss Distribution

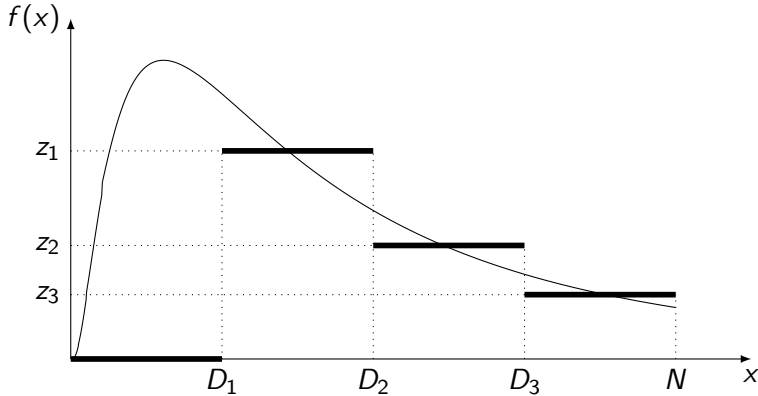


Figure 1: Illustration of a piecewise constant density function for $K = 3$ [► More details](#)

Approximating the Loss Distribution

- Weighting of the two approximations $\hat{F}_n$ and $\hat{F}_K^{ded}$ with weight $0 \leq \kappa_{n,K} \leq 1$:

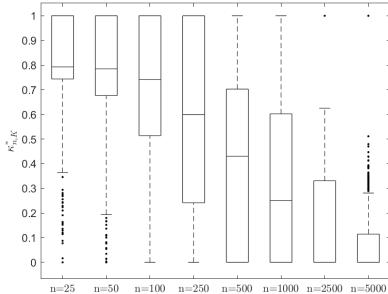
$$\hat{F}_{n,K}^* = \kappa_{n,K} \hat{F}_K^{ded} + (1 - \kappa_{n,K}) \hat{F}_n$$

- Deriving the weight $\kappa_{n,K}^*$ which minimizes the distance measure

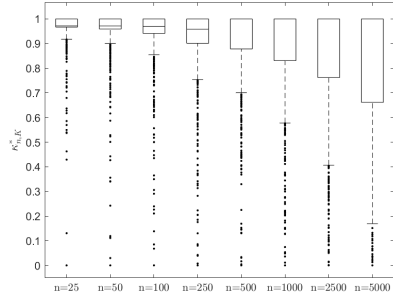
$$\begin{aligned} d_{n,K}(\kappa_{n,K}) &= \left(\kappa_{n,K} \mu_K^{ded} + (1 - \kappa_{n,K}) \bar{L} - \mu \right)^2 \\ &\quad + \beta \left(\kappa_{n,K} \sigma_{ded,K}^2 + (1 - \kappa_{n,K}) \sigma_n^2 - \sigma^2 \right)^2 \end{aligned}$$

- Properties of $\kappa_{n,K}^*$:
 - $\lim_{n \rightarrow \infty} \kappa_{n,K}^* = 0$ $\mathbb{P}$ -almost surely
 - $\lim_{K \rightarrow \infty} \kappa_{n,K}^* = 1$ under certain conditions

Approximating the Loss Distribution



(a) $\mathbf{D}_K = (\$500, \$1,000, \dots, \$2,500)$



(b) $\mathbf{D}_K = (\$500, \$1,000, \dots, \$10,000)$

Figure 2: Distribution of $\kappa_{n,K}^*$ for different n and $\mathbf{D}_K$

Preference Uncertainty

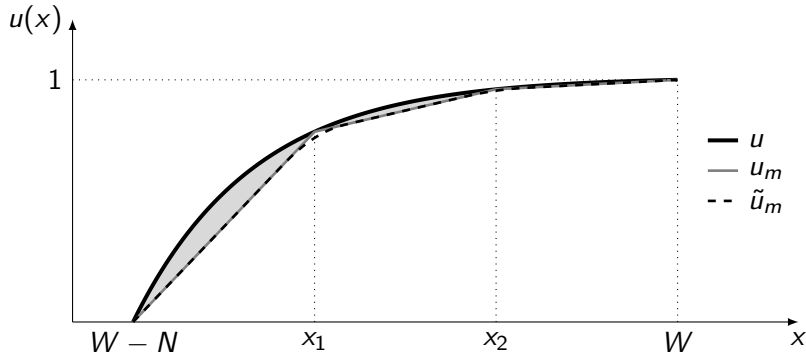


Figure 3: Illustration of the functions u_m and $\tilde{u}_m$ for $\mathbf{x}_2 = (x_1, x_2)$

Probability Equivalence (PE) Elicitation Method

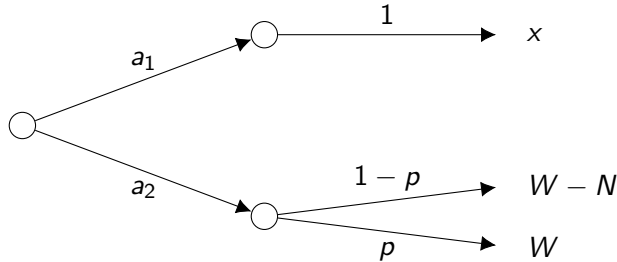


Figure 4: Illustration of the hypothetical decision situations faced by the policyholder (Source: Bamberg et al., 2019)

$$u(x) = (1 - p^*) \underbrace{u(W - N)}_{:=0} + p^* \underbrace{u(W)}_{:=1} = p^*$$

► More details

Decision Rules for the Optimal Deductible Choice

Definition

A decision rule $\hat{\theta}_{m,n}^{(I)}$ under the information vector $I = (\mathbf{D}_K, \mathbf{R}_K)$ is an estimator of the optimal deductible level D_K^* . It is called

- (i) **consistent** if $\lim_{m,n \rightarrow \infty} \hat{\theta}_{m,n}^{(I)} = D_K^*$ $\mathbb{P}$ -almost surely.
- (ii) **unbiased** if $\lim_{m \rightarrow \infty} \mathbb{E}[\hat{\theta}_{m,n}^{(I)}] = D_K^*$ holds for all $n \in \mathbb{N}$.
- (iii) **preference-independent** if $\hat{\theta}_{0,n}^{(I)} = \hat{\theta}_{m,n}^{(I)}$ $\mathbb{P}$ -almost surely for all $m \in \mathbb{N}$ for a fixed but arbitrary $n \in \mathbb{N}$.

Decision Rules for the Optimal Deductible Choice

Definition (continued)

If $u'' < 0$ and $D_K^* \neq N$, the decision rule $\hat{\theta}_{m,n}^{(I)}$ is called

(iv) **preference-consistent** if

$$\lim_{n \rightarrow \infty} \mathbb{E}[c(\hat{\theta}_{0,n}^{(I)})] - \lim_{m, n \rightarrow \infty} \mathbb{E}[c(\hat{\theta}_{m,n}^{(I)})] > 0.$$

Examples of Decision Rules

Preference-dependent decision rules

- $\hat{\theta}_{m,n}^{(l),1} = \arg \max_{D \in \{D_1, \dots, D_K, N\}} \frac{1}{n} \sum_{i=1}^n \tilde{u}_m(Y_D(L_i))$
- $\hat{\theta}_{m,n}^{(l),2} = \arg \max_{D \in \{D_1, \dots, D_K, N\}} \int_0^N \tilde{u}_m(Y_D(l)) d\hat{F}_{n,K}^*(l)$

Simulation Study for the US Homeowners Insurance Market

Parameter	Notation	Value
Parameter for claims frequency	λ_P	5.64%
Parameter for claim amount	λ_E	$\frac{1}{13,814}$
Loss limit	N	\$40,000
Available deductible levels	$D_1, \dots, D_K$	\$500, \$1000, \dots, \$2500
Loading factor	λ	0.05
Weighting factor for the approximation of the loss distribution	β	0.1
Wealth	W	\$49,806
Relative risk aversion parameters	$\eta_1, \dots, \eta_n$	0.1, 0.5, 1, 2, 3, 5
Probability weighting factor for gains	γ^+	0.61
Probability weighting factor for losses	γ^-	0.69
Loss aversion parameter	λ_L	2.25

Distribution of Risk Preferences

Specification	Absolute risk aversion	Relative risk aversion
Mean estimates from insurance contexts:		
Sydnor (2006)	$1.9 \cdot 10^{-3}$	54.07
Cohen and Einav (2007)	$6.7 \cdot 10^{-3}$	97.22
Handel and Kolstad (2015): Base Model	$1.6 \cdot 10^{-3}$	60.97
Handel and Kolstad (2015): Full Model	$8.6 \cdot 10^{-5}$	3.28
Benchmark model of Cohen and Einav (2007):		
25th percentile	$2.3 \cdot 10^{-6}$	0.03
Median individual	$2.6 \cdot 10^{-5}$	0.37
75th percentile	$2.9 \cdot 10^{-4}$	4.27
90th percentile	$2.7 \cdot 10^{-3}$	39.02
95th percentile	$9.9 \cdot 10^{-3}$	143.27
Friction adjusted benchmark model of Cohen and Einav (2007):		
25th percentile	$3.0 \cdot 10^{-8}$	0.0011
Median individual	$3.3 \cdot 10^{-7}$	0.0125
75th percentile	$3.7 \cdot 10^{-6}$	0.1410
90th percentile	$3.5 \cdot 10^{-5}$	1.33
95th percentile	$1.3 \cdot 10^{-4}$	4.95

Costs of Choosing a Wrong Deductible

	\$500	\$1,000	\$1,500	\$2,000	\$2,500	\$40,000
$\eta = 0.1$	14.06	12.88	11.76	10.70	9.71	0*
$\eta = 0.5$	3.27	2.23	1.34	0.60	0*	56.40
$\eta = 1$	1.90	1.04	0.44	0.10	0*	161.73
$\eta = 2$	0.50	0.01	0*	0.48	1.42	499.16
$\eta = 3$	0.13	0*	0.60	1.92	3.95	1,177.60
$\eta = 5$	0*	0.63	2.49	5.59	9.92	5,639.40

Preference-Dependent Decision Rules

	Costs by degree of risk aversion			Population costs per capita	
	$\eta = 0.1$	$\eta = 0.5$	$\eta = 5$	$\mathbb{P}_{\mathcal{U}}$	$\mathbb{P}_{\mathcal{U}}^{CE}$
Expected costs for $\hat{\theta}_n^{(I),1}$:					
$n = 5$	2.8	45.77	4,500.3	323.61	1,330
$n = 25$	5.15	33.87	2,724.9	200.84	812.05
$n = 100$	6.27	27.25	966.74	79.88	302.24
Expected costs for $\hat{\theta}_{m,n}^{(I),1}$					
for $m = 5$:					
$n = 5$	2.8	45.82	4,517.2	324.77	1,334.8
$n = 25$	5.12	34.43	3,154.0	230.75	983.5
$n = 100$	6.2	28.45	1,423.9	112.85	440.77
Expected costs for $\hat{\theta}_{m,n}^{(I),1}$					
for $m = 20$:					
$n = 5$	2.8	45.82	4,517.2	324.77	1,334.8
$n = 25$	5.14	34.09	3,187.8	230.97	939.09
$n = 100$	6.27	27.85	1,485.9	113.59	443.94

Preference-Dependent Decision Rules

	Costs by degree of risk aversion			Population costs per capita	
	$\eta = 0.1$	$\eta = 0.5$	$\eta = 5$	$\mathbb{P}_{\mathcal{U}}$	$\mathbb{P}_{\mathcal{U}}^{CE}$
Expected costs for $\hat{\theta}_n^{(I),2}$:					
$n = 5$	0.72	44.51	5.09	3.47	5.32
$n = 25$	1.28	26.33	4.87	2.85	4.13
$n = 100$	1.72	17.91	4.77	2.71	3.66
Expected costs for $\hat{\theta}_{m,n}^{(I),2}$					
for $m = 5$:					
$n = 5$	0.72	45.2	5.1	3.54	5.53
$n = 25$	1.24	30.54	5.22	3.1	4.60
$n = 100$	1.65	24.77	5.17	3.11	4.32
Expected costs for $\hat{\theta}_{m,n}^{(I),2}$					
for $m = 20$:					
$n = 5$	0.72	44.99	5.1	3.5	5.36
$n = 25$	1.28	28.73	5.22	3.01	4.41
$n = 100$	1.7	21.72	5.17	2.94	4.06

Conclusion

- Including limited information in traditional models for the optimal deductible choice
- Derivation of approximation for loss distribution from different sources of information
- Discussion of approximations for the utility function
- Identification of suitable decision rules for the optimal deductible choice
- Valuable insights for designing decision aids:
 - Today's decision aids are mostly expected costs minimizer
 - Our analysis suggests that this approach could be improved by accounting for risk preferences and the loss distribution (not only expected costs)

Appendix

Approximating the Loss Distribution

- Step 1: Solve the system of $K + 1$ linear equations given by

$$\begin{aligned}\tilde{f}_K^{ded}(x) &= \mathbb{1}_{(D_1, D_2]}(x) z_{K,1} + \dots + \mathbb{1}_{(D_K, N]}(x) z_{K,K} \\ \tilde{F}_K^{ded}(0) &= \tilde{p}_K = 1 - \int_0^N \tilde{f}_K^{ded}(z) dz \\ &= 1 - \sum_{i=1}^{K-1} z_{K,i} (D_{i+1} - D_i) - z_{K,K} (N - D_K) \\ C(D_k) &= \sum_{i=k}^{K-1} z_{K,i} \int_{D_i}^{D_{i+1}} (y - D_k) dy + z_{K,K} \int_{D_K}^N (y - D_k) dy\end{aligned}$$

- Step 2: Transform $\tilde{F}_K^{ded}$ to $\hat{F}_K^{ded}$ with the help of an algorithm

Approximating the Loss Distribution

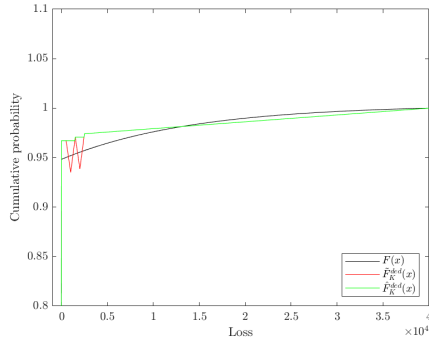


Figure 5: Example for $\tilde{F}_K^{ded}$ and $\hat{F}_K^{ded}$

► Back

Optimal Survey Strategy

- Set of all possible utility functions is described by the probability space $(\mathcal{U}, \mathcal{F}, \mathbb{P}_{\mathcal{U}})$
- Determining the set of survey questions $\mathbf{x}^{**} = (x_1^{**}, \dots, x_m^{**}) \in \mathcal{X}^m$ with the help of the following procedure:

$$\tilde{x}_1^{**} = \arg \min_{x \in \mathcal{X}} \sum_{v \in \mathcal{U}} \mathbb{P}_{\mathcal{U}}(v) \int_{W-N}^W (v(z) - \tilde{v}_1(z))^2 dz$$

$$\tilde{x}_m^{**} = \arg \min_{x \in \mathcal{X}} \sum_{v \in \mathcal{U}} \mathbb{P}_{\mathcal{U}|\tilde{\mathbf{u}}_{m-1}}(v) \int_{W-N}^W (v(z) - \tilde{v}_m^{(h)}(s((\tilde{x}_1^{**}, \dots, \tilde{x}_{m-1}^{**}, x)), z))^2 dz$$

$$\mathbf{x}^{**} = (x_1^{**}, \dots, x_m^{**}) = s((\tilde{x}_1^{**}, \dots, \tilde{x}_m^{**}))$$

► Back

Bias in the Elicitation Method

- In theory, widely used elicitation methods like the PE method and the certainty equivalence (CE) method should lead to the same utility function
- Experimental evidence for difference in the estimated utility functions (cf., e.g., Bleichrodt et al., 2001)
- Assume that the relationship between biased utility function $u^{(b)}$ and u is given by

$$u(x) = s(u^{(b)}(x)) \iff u^{(b)}(x) = s^{-1}(u(x))$$

- Deductible choice: $D_K^{(b)} = \arg \max_{D \in \{D_1, \dots, D_K, N\}} \mathbb{E} [u^{(b)}(Y_D(L))]$
- Impact on optimal deductible choice:
 - If $s' > 0, s'' < 0$, $D_K^* \leq D_K^{(b)}$.
 - If $s' > 0, s'' > 0$, $D_K^* \geq D_K^{(b)}$.

Example for a Correction Function s

- Decision maker exhibits preferences which are a combination of prospect theory and expected utility theory
- Utilities for actions a_1 and a_2 are given by $u(x)$ and

$$u(x) + w^+(p)(u(W) - u(x)) - \lambda_L w^-(1 - p)(u(x) - u(W - N))$$

- Bleichrodt et al. (2001) derive the following correction function:

$$s(y) = \frac{w^+(y)}{w^+(y) + \lambda_L w^-(1 - y)}$$

Example for a Correction Function s

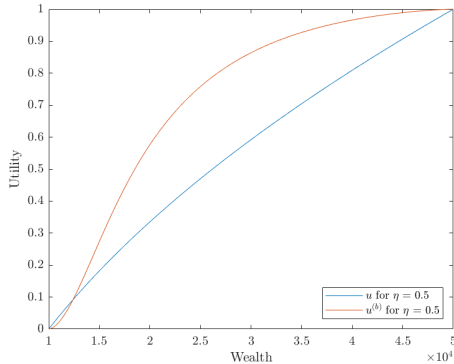


Figure 6: Impact of the correction function s of Bleichrodt et al. (2001) on a CRRA utility function with relative risk aversion η

Biased Preference-Dependent Decision Rules

	Costs by degree of risk aversion			Population costs per capita	
	$\eta = 0.1$	$\eta = 0.5$	$\eta = 5$	$\mathbb{P}_{\mathcal{U}}$	$\mathbb{P}_{\mathcal{U}}^{CE}$
Expected costs for $\hat{\theta}_n^{(I),b,1}$:					
$n = 5$	3.11	44.6	4,387.5	315.75	1,296.6
$n = 25$	8.61	23.83	2,075.3	155.41	617.57
$n = 100$	12.59	6.55	282.63	30.72	91
Expected costs for $\hat{\theta}_{m,n}^{(I),b,1}$					
for $m = 5$:					
$n = 5$	2.8	45.82	4,517.2	324.77	1,334.8
$n = 25$	5.96	30.68	3,221.5	233.88	948.91
$n = 100$	9.46	14.8	1,339.4	104.16	399.48
Expected costs for $\hat{\theta}_{m,n}^{(I),b,1}$					
for $m = 20$:					
$n = 5$	2.95	45.82	4,517.2	324.84	1,334.7
$n = 25$	7.78	28.21	2,861.1	209.02	843.25
$n = 100$	11.62	11.29	967.7	77.68	288.65

Biased Preference-Dependent Decision Rules

	Costs by degree of risk aversion			Population costs per capita	
	$\eta = 0.1$	$\eta = 0.5$	$\eta = 5$	$\mathbb{P}_{\mathcal{U}}$	$\mathbb{P}_{\mathcal{U}}^{CE}$
Expected costs for $\hat{\theta}_n^{(I),b,2}$:					
$n = 5$	11.35	1.1	0.57	8.89	5.42
$n = 25$	12.58	2.05	0.35	9.91	5.98
$n = 100$	13.21	2.54	0.42	10.45	6.32
Expected costs for $\hat{\theta}_{m,n}^{(I),b,2}$					
for $m = 5$:					
$n = 5$	10.09	0.37	5.1	8.17	6.13
$n = 25$	10.1	0.59	5.21	8.2	6.18
$n = 100$	10.3	0.7	5.15	8.36	6.27
Expected costs for $\hat{\theta}_{m,n}^{(I),b,2}$					
for $m = 20$:					
$n = 5$	11.34	0.84	5.1	9.17	6.73
$n = 25$	12.35	0.79	5.21	9.94	7.2
$n = 100$	12.74	0.83	5.15	10.25	7.36

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