School of Finance

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Working Papers on Finance No. 2018/23

Swiss Institute of Banking and Finance (S/BF – HSG)

November 15, 2018
this Version: November 5, 2021 (New Title)

Liquidity in the Global Foreign Exchange Market*

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November 5, 2021

Abstract

We study the global market liquidity of foreign exchange rates by analysing the price impact of trading. We propose a simple theoretical framework that refines the popular Amihud (2002) illiquidity measure and term our new estimator *realized Amihud*. We highlight our measure's consistency and higher accuracy, and these properties are validated numerically and empirically using CLS data representative of global FX trading. The price impact of FX trading increases with transaction costs, money market stress, uncertainty, and risk aversion. Furthermore, currencies with smaller price impact are less prone to deviations from triangular no-arbitrage parity. Our results are corroborated by an instrumental variables approach and a quasi-natural experiment using the shock of the Swiss franc's uncapping in January 2015.

Keywords: Foreign Exchange, Global Market, Liquidity, Price Impact, Arbitrage.

J.E.L. classification: C15, F31, G12, G15

*We are grateful to Tim Bollerslev, Nicola Borri, Massimiliano Caporin, Filippo Cavaleri, Alain Chaboud, Federico Carlini, Nina Karnaukh, Peteris Kloks, Edouard Mattille, Lukas Menkhoff, Federico Nucera, Paolo Pasquariello, Mark Podolskij, Roberto Renó, Fabricius Somogyi and Vladyslav Sushko for their relevant remarks on our work. We would also like to thank the participants at the 2020 American Finance Association meetings, the 7th Workshop on Financial Determinants of FX Rates at the Norges Bank, the 2018 SoFiE conference, the 2018 DEDA conference, the 2019 Stern Microstructure Meeting, and at seminars at City Hong Kong University, LUISS, Aarhus University, and the University of the Balearic Islands for useful comments. Angelo Ranaldo acknowledges financial support from the Swiss National Science Foundation (SNSF grant 182303). An earlier version of this paper circulated under the title "Trading Volume, Illiquidity and Commonalities in FX Markets."

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1 Introduction

Since the demise of the post-war Bretton Woods system in the 1970s, the international financial system has witnessed growing capital mobility and wider movements of foreign exchange (FX) rates. In such a regime of floating FX rates and open economies, international investors pay close attention to the liquidity of exchange rate markets. Thus some natural questions arise: How should one measure market liquidity? What are its determinants? And what are its effects on FX rates?

We address these questions by analyzing high-frequency trading volume data from CLS Group, which are representative of the global FX market. To date, attempts to study global FX liquidity have been focused on one liquidity component: transaction cost. In addition to transaction cost, we conduct a systematic study of another fundamental dimension of liquidity: the price impact of trading. To do this, we provide theoretical support for the popular Amihud (2002) illiquidity measure and propose a refinement of it that we call *realized Amihud*.[1] First, we develop a simple theory highlighting its straightforward interpretation as the inverse of market depth and number of market participants. Second, the numerical analysis validates its higher precision in measuring illiquidity and its robustness to bid-ask contamination. Third, we empirically analyze the variation of trading price impact across a relatively large set of currencies and over time. Finally, we analyze the relationship between market liquidity and price efficiency in terms of deviations from the no-arbitrage condition. We use the *granular* instrumental variables (GIV) approach of Gabaix and Koijen (2020) and the quasi-natural experiment of the Swiss franc shock in January 2015 to validate our results and hone our understanding of the liquidity-efficiency relationship.

Analyzing FX liquidity is important for at least three reasons. First, the FX market is the world's largest financial market, with a daily traded volume of USD 6.6 trillion (Bank of International Settlements, 2019). It is often considered to be verging on market efficiency given its size and the prevalence of professional traders such as global dealers and sophisticated (financial) firms. However, an in-depth understanding of FX liquidity is still missing. Second, FX rates are commonly traded in over-the-counter (OTC) markets, which are opaque and increasingly fragmented (Chaboud et al., 2021). This OTC nature and the paucity of comprehensive trading volume data on a global scale has made it difficult to examine liquidity issues such as market depth and price impact. Third, distressed markets, such as those during currency crises, are characterized by sudden drops in liquidity with adverse consequences for international investors and policy implementation, such as conducting (unconventional) monetary policy and FX interventions.

We proceed in three steps. First, we provide an intuitive theoretical underpinning for measuring market illiquidity. Our realized Amihud measure is analogous to that of Amihud (2002), i.e. the ratio of absolute asset return to its dollar volume, which is a common measure of trading price impact (e.g. Chordia et al., 2009 and Foucault et al., 2013). Rather than using the daily absolute return, we enhance it by using high frequency (intraday) return variations so as to gain a

[1] By the end of August 2021, Amihud (2002) had been cited about 10,000 times according to Google Scholar, including a number of papers published in top-tier academic journals.

more accurate measurement of return volatility and to give us the ability to ascertain whether the trading price impact is symmetrical, i.e. if buying or selling one currency for another generates the same impact. Furthermore, our simple theory illustrates that, in a frictionless market, the ratio between volatility and volume carries information about market depth.

Second, we conduct a numerical analysis showing a consistent decrease of the realized Amihud measure of illiquidity with market depth and traders' participation. The simulation analysis quantifies that the realized Amihud is about 10 times more precise than the traditional Amihud and immune to bid-ask contamination unless volatility is computed at very high sampling frequencies shorter than 30 seconds. This result confirms what was anticipated when the Amihud measure was proposed, namely that "there are finer and better measures of illiquidity ... that require a lot of microstructure data that are not available in many stock markets" (Amihud, 2002). Fortunately, the availability of granular, high-frequency data has increased significantly in recent decades.

Third, we empirically analyze FX liquidity across a relatively large set of currency pairs over an extended period using high-frequency data on trading volume and FX rates. To do this, we access unique hourly data of FX trading volume from CLS Group (CLS) which are representative of the global FX market. CLS operates the largest payment-versus-payment (PVP) settlement service in the world and CLS volume data covers around 50% of global FX turnover compared to Bank for International Settlements (BIS) triennial surveys (Cespa et al., 2021). The combination of realized volatility and CLS volume allows us to approximate the price impact of global FX trading.

Some new findings arise from the empirical analysis: cross-sectionally, we quantify the price impact of trading and document that the least traded (cross) rates such as CADJPY and USDDKK generate up to 200 times more impact than the most frequently traded pairs such as EURUSD and USDJPY. On average, the price impact for an international investor is smallest if she trades in U.S. dollars whereas it increases with the euro, the yen, and the pound, sealing the dominance of the dollar as the main reserve and "vehicle" currency. In general, the price impact is symmetrical with respect to the sign of the return. Therefore buying the dollar or other safe haven currencies does not seem to trigger a greater impact, suggesting that the market is equally deep or elastic on both sides of an FX trade. Economically, this translates into a relatively small amount of volume moving FX returns. For example, to move one standard deviation of FX returns, it takes as much as $140 billion for EURUSD while only less than half a billion for CADJPY. The time series analysis indicates that the price impact of FX trading systematically increases with transaction costs and proxies of funding constraints, uncertainty, and risk aversion. For instance, a 1% increase in transaction costs is associated with a 0.16% increase of price impact. Moreover, we utilize Electronic Broking Services (EBS) data on ultra-high-frequency orders and trades to compute benchmark measures of market illiquidity such as the effective spread and order flow price impact. We find that our illiquidity measure achieves very high correlations with the benchmark measures, suggesting that our measure is effective in sizing up the FX global liquidity and in particular, the trading price impact.

Finally, we shed light on the relationship between liquidity and price efficiency. Our premise is that a market is efficient when an asset can be traded at a unique price reflecting its fundamental value. On the contrary, violations of *the law of one price* indicate insufficient arbitrage activity and frictions such as illiquidity, undermining market efficiency. To do this, we analyze violations of the "triangular" no-arbitrage condition. That is, we compare the EURUSD rate with the rate derived by replicating the trade indirectly, for example by combining a EURGBP and a GBPUSD trade (or EURJPY and USDJPY, etc.). Two main results stand out from the analysis of all the EURUSD triplets: First, the lower the price impact of trading indirect currencies, the more the "triangular" no-arbitrage condition is satisfied. Second, in general the price impact of currency pairs involving the U.S. dollar is critical for maintaining price efficiency while that involving the Euro plays a crucial role for currency pairs of continental Europe. Overall, by isolating the trading impact and controlling for transaction costs, our analysis sheds new light on the contribution of market depth and price resilience in preserving price efficiency in the global FX market.

The liquidity - efficiency relationship is an equilibrium outcome. Therefore, regressing one variable on the other might lead to biased OLS estimates due to endogeneity stemming from simultaneity, as well as possible omitted variables and measurement errors. To overcome or mitigate these issues, we conduct two analyses. First, following the work by Gabaix and Koijen (2020), we construct our GIV as the difference between the size- and equal-weighted average of daily liquidity measures, thus benefiting from the wide heterogeneity in the cross-section of currency pairs' liquidity. The GIV method allows us to capture idiosyncratic events of market illiquidity and to establish the elasticity of price efficiency to trading impact. Second, we study the quasi-natural experiment of the Swiss franc shock in 2015. We find consistent results corroborating our main theoretical and empirical findings, thus supporting the overall idea that liquidity begets price efficiency.

We contribute to prior research on liquidity in financial markets. Most previous studies on the Amihud measure have mainly focused on stocks, proposing it as an illiquidity proxy capturing the price impact of trading[2] and analyzing pricing implications (see, e.g., Lou and Shu, 2017; Amihud and Noh, 2021). Although there is a growing literature on FX liquidity, the lack of transaction volume and flow data covering the entire FX market has led prior research to study trading volume and related liquidity issues solely for different, individual segments of the FX market at a time. Specifically, the studies that have taken this direction have focused on the FX interdealer segment for which volume and order flow data is available, since it mostly relies on electronic limit order book platforms such as EBS and Reuters/Refinitiv. (see, e.g., Evans and Lyons, 2002; Bjønnes and Rime, 2005; Chaboud et al., 2007; Mancini et al., 2013).[3] Other papers analyze customers' flow for a specific bank (see, e.g., Evans and Lyons, 2006; Osler et al., 2011; Menkhoff et al., 2016). The analysis of market liquidity for the entire FX global market has been

[2]See Foucault et al. (2013) (p. 56-9) for an in-depth discussion of why the Amihud measure captures the price impact of trading.

[3]Among others, order flow is studied in Berger et al. (2008), Frömmel et al. (2008), Breedon and Ranaldo (2013), Evans (2002), Payne (2003), and Rime et al. (2010).

limited to transaction cost measures or estimates based on (indicative) bid and ask quotes (see e.g., Karnaukh et al., 2015).[4]

Only with the recent availability of CLS data has research on FX global volume at relatively high frequencies (e.g. hourly or daily) become possible. Apart from CLS, the only source of global FX trading volume is the triennial survey of central banks conducted by the BIS providing a snapshot of FX market volume on a given day once every three years. Using CLS data, Fischer and Ranaldo (2011) look at global FX trading around central bank decisions while Hasbrouck and Levich (2021) analyze the network structure of the FX spot market. Ranaldo and Somogyi (2021) study the information content of CLS flow data to highlight asymmetric information risk in FX markets. Cespa et al. (2021) analyze the profitability of FX trading strategies exploiting the predictive ability of CLS volume. Hasbrouck and Levich (2019) is the first study proposing an Amihud measure for FX trading. Specifically, they compute the classical Amihud proxy using all settlement submissions to CLS during April of 2010, 2013, and 2016. They show that the CLS-based measures are similar to their EBS-based counterparts, but not for those currencies which are not predominantly traded on EBS. We are the first to study global FX liquidity both in terms of trading price impact and transaction costs for a relatively large cross section of FX rates over a prolonged period. To do so, we propose a refinement of the Amihud (2002) measure to improve its accuracy, and highlight how FX illiquidity affects FX (mis)pricing. Finally, we contribute to prior research that assess price efficiency by looking at violations of the triangular arbitrage condition. For instance, Foucault et al. (2017) provide a microstructure analysis of the market maker's risk of trading at stale quotes. We are the first to analyze deviations from this arbitrage condition in connection with illiquidity of (indirect) currency pairs.

This paper is organized as follows: Section 2 outlines the theoretical setting and simulation analysis. Section 3 presents the data sets. Section 4 describes the empirical analysis. Section 5 concludes the paper.

2 A theory of FX illiquidity measurement

Let us consider a world with two currencies, x (base) and y (quote). We assume that the market consists of a finite number $J \geq 2$ of active participants, who trade on the FX rate $x|y$. Within a given trading period of unit length (e.g. one hour, day, or week), the market for the currency pair $x|y$ passes through a sequence of $i = 1, \ldots I$ equilibria. The evolution of the equilibrium price is motivated by the arrival of new information to the market. At intra-period i, the desired position of the j-th trader ($j = 1, \ldots, J$) on the FX rate $x|y$ is given by

$$q_{i,j}^{x|y} = \xi^{x|y}(p_{i,j}^{x|y;*} - p_i^{x|y}), \quad \xi^{x|y} > 0, \tag{1}$$

[4]Bid-ask spreads in FX spot markets are also studied in e.g. Bessembinder (1994), Bollerslev and Melvin (1994), Christiansen et al. (2011), Hartmann (1999), Huang and Masulis (1999), and Hsieh and Kleidon (1996).

where $p_{i,j}^{x|y;*}$ is the reservation price of the j-th trader and $p_i^{x|y}$ is the current market price (both measured in logs). The equilibrium function in (1) is analogous to the theory of Clark (1973), and Tauchen and Pitts (1983), which provides a stylized representation of the supply/demand mechanism on the market. Although this is not the conventional microstructure theoretical framework and does not offer the straightforward interpretation of the price impact due to asymmetric information in the spirit of Kyle's model (Chordia et al., 2009), our theoretical framework has two features: First, the reservation price of each trader might broadly reflect some of the following aspects: individual preferences, liquidity issues, asymmetries in information sets, and different expectations about the fundamental values of the FX rate. In general, the reservation price can deviate from the market price because of idiosyncratic reason, inducing the j-th trader to trade. Second, it allows us to build a theory for the formation of the trading volume (rather than the order flow), which appears at the denominator of the realized Amihud index.

The term $\xi^{x|y}$ is a positive constant capturing the market depth: The larger $\xi^{x|y}$, the larger quantities of x can be exchanged for y (and viceversa) for a given difference $p_{i,j}^{x|y;*} - p_i^{x|y}$. In other words, $\xi^{x|y}$ measures the capacity of the market to allow large quantities to be exchanged at the intersection between demand and supply, thus recalling the concept of market depth and resilience that reduces the price impact of trading. If $(p_{i,j}^{x|y;*} - p_i^{x|y}) > 0$, the j-th trader believes that the equilibrium trading price of $x|y$ is too low, i.e. currency x should be more expensive relatively to y. Therefore, she will buy x and sell y. On the contrary, if $(p_{i,j}^{x|y;*} - p_i^{x|y}) < 0$, the j-th trader will buy y and sell x. The quantity exchanged for a unit change of $p_{i,j}^{x|y;*} - p_i^{x|y}$ is given by the slope $\xi^{x|y}$.

By market clearing, i.e. $\sum_j q_{i,j}^{x|y} = 0$, we have that the average of the reservation prices clears the market, that is $p_i^{x|y} = \frac{1}{J}\sum_{j=1}^{J} p_{i,j}^{x|y;*}$, and the log-return is $r_i^{x|y} := \Delta p_i^{x|y} = p_i^{x|y} - p_{i-1}^{x|y}$. Furthermore, as new information arrives, the traders adjust their reservation prices $\Delta p_{i,j}^{x|y;*}$, resulting in a change in the market price given by the average of the increments of the reservation prices. Consequently, the generated trading volume in each i-th sub-interval is

$$v_i^{x|y} = \frac{\xi^{x|y}}{2} \sum_{j=1}^{J} |\Delta p_{i,j}^{x|y;*} - \Delta p_i^{x|y}|,$$

where $\Delta p_{i,j}^{x|y;*} = p_{i,j}^{x|y;*} - p_{i-1,j}^{x|y;*}$. We assume the following dynamics for the reservation price about the $x|y$ FX rate

$$dp_j^{x|y;*}(t) = \mu_j^{x|y}(t)dt + \sigma_j^{x|y}(t)dW_j^{x|y}(t), \quad j = 1, \ldots, J \tag{2}$$

where $\{W_j(t), j = 1, \ldots J\}$ is a collection of independent Wiener processes. The term $\sigma_j^{x|y}(t) \geq 0$ is the stochastic volatility process of the j-th trader, and we assume it to be locally square integrable. The term $\mu_j(t)$ is a predictable process with finite variation, which might represent the long-run expectation of the j-th trader about the FX rate and could be a function of fundamental quantities such as interest rates differentials and long-term macroeconomic views.

By letting $\sigma_j^{x|y}$ vary across traders, we introduce heterogeneity among them. This reconciles

with many realistic features including the evidence of long-memory in volatility, obtained by the superposition of traders operating at different frequencies; see for instance the heterogeneous autoregressive models of Müller et al. (1997) and Corsi (2009). This setup is coherent with a representation of a frictionless market where each trader participates through its reservation price to the determination of a new equilibrium price by carrying new information. For ease of exposition and tractability, we assume that trades happen on an equally spaced and uniform grid, $i = 1, 2, \dots I$. On the i-th discrete sub-interval of length $\delta = \frac{1}{I}$, we have that

$$p_{i,j}^{x|y;*} = \int_{\delta(i-1)}^{\delta i} \mu_j^{x|y}(s)ds + \int_{\delta(i-1)}^{\delta i} \sigma_j^{x|y}(s)dW_j^{x|y}(s). \tag{3}$$

Based on the return on the i-th interval, we can consider the realized variance, defined as $RV^{x|y} = \sum_{i=1}^{I} (r_i^{x|y})^2$ with $\delta = 1/I > 0$, as introduced by Andersen and Bollerslev (1998). Following Barndorff-Nielsen and Shephard (2002a,b), taking the limit for $\delta \to 0$ (that is $I \to \infty$), we get $p\lim_{I\to\infty} RV^{x|y} = \frac{1}{J^2}\mathcal{V}_{x|y}$, where $\mathcal{V}_{x|y} = \sum_{j=1}^{J} \mathcal{V}_{x|y,j}$ is the variation of the FX rate on the unit interval generated by the aggregated individual components of $r^{x|y}$. The term $\mathcal{V}_{x|y,j} = \int_0^1 \left(\sigma_j^{x|y}(s)\right)^2 ds$ is the *integrated variance* associated with the j-th trader's specific component. The term $\mu_j(t)$ does not enter in the expression of $V_{x|y,j}$ since the magnitude of the drift, when measured over infinitesimal intervals, is dominated by the diffusive component of $\Delta p_j^{x|y,*}(t)$ driven by the Brownian motion.

Following Barndorff-Nielsen and Shephard (2003), for a given $\delta > 0$ we can also define the *realized power variation* of order one (or realized absolute variation) as $RPV^{x|y} = \sum_{i=1}^{I} |r_i|$. Hence, analogously to the illiquidity proxy in Amihud (2002), we can define a new version of the Amihud illiquidity measure, namely the *realized Amihud*, as

$$A^{x|y} := \frac{RPV^{x|y}}{v^{x|y}}. \tag{4}$$

This quantity gauges the price impact of trading, that is the amount of volatility on a unit interval (as measured by $RPV^{x|y}$) associated with the trading "dollar" volume $v^{x|y} = \sum_{i=1}^{I} v_i^{x|y}$ generated in the same period. In other words, $A^{x|y}$ measures the amount of FX volatility associated with a unit of trading volume. The following proposition highlights the main determinants of this illiquidity measure.

Proposition 1. *Consider the illiquidity measure defined in* (4)*. As* $I \to \infty$ *and under homogeneity of traders, i.e.* $\sigma_j^{x|y} = \sigma^{x|y} \quad \forall j = 1, 2, \dots, J$,

$$p\lim_{I\to\infty} A^{x|y} = \frac{2}{\xi^{x|y} J\sqrt{(J-1)}}. \tag{5}$$

Proof in Appendix A.1.

Proposition 1 shows that on a period of unit length (an hour, day, or month), $A^{x|y}$ is inversely related to the slope $\xi^{x|y}$ of the equilibrium function in (1). That is, for a given difference between

the reservation price and the market price, $A^{x|y}$ decreases as this slope increases. For large values of $\xi^{x|y}$, large volume would be associated with small variations between the prevailing price and the reservation price for each trader, thus signaling market depth and liquidity. Instead, when $\xi^{x|y} \longrightarrow 0^+$, i.e. in the limiting case of a flat equilibrium function in (1), the liquidity is minimal (and $A^{x|y}$ diverges), since no actual trade takes place. Under the assumption of homogeneity of the traders, $\sigma_j^2(t) = \sigma^2(t)\ \forall j = 1, \ldots, J$, Proposition 1 also highlights the inverse relationship between the number of active traders and illiquidity.[5]

The baseline assumptions of the model (linearity of the trading function, constant number of active traders, absence of frictions) are inevitably stylized. As for the form of the equilibrium function in (1), note that the trades take place on short intraday intervals of length $\delta = 1/I$ and are generally associated with small price variations. Therefore, it is not restrictive to assume the equilibrium function to be linear on small price changes and a fixed number of traders during such a short period. In particular, the assumption that J traders observe the same market price could be reconciled with a reference price provided by an electronic limit order book platform such as EBS, Refinitiv FX Matching or Hotspot FX ECN. Finally, illiquidity frictions could be introduced by combining the theory of Tauchen and Pitts (1983) with that of the demand of immediacy by Grossman and Miller (1988), see Darolles et al. (2015, 2017).[6] The next step is to verify with a numerical analysis the theoretical predictions and robustness of the realized Amihud to microstructural frictions. Then, we will show whether this theory can be successfully adopted to characterize illiquidity and mispricing in the global FX market.

2.1 A simulation study

Resorting to the continuous-time framework outlined in Barndorff-Nielsen and Shephard (2002a,b), we can precisely measure the variability of FX rates by computing RPV (or RV) over intervals of any length (e.g. hours, days, weeks and months). Furthermore, the equilibrium theory presented above allows us to relate this variability to the aggregate level of disagreement among investors, which in turns leads to the observed trading volume. An assessment of the quality of the measurement of illiquidity based on the realized Amihud is carried out by Monte Carlo simulations. Reservation prices for a different number of J traders, equilibrium prices, as well as trading volume with different values of ξ are generated according to the model illustrated above. Stochastic volatility, $\sigma^2(t)$, evolves according to the Heston model, i.e. $d\sigma_j^2(t) = \kappa(\sigma^2(t) - \omega)dt + \eta\sigma(t)dW(t)$ with parameters $\kappa = 0.2$, $\omega = 2$, $\eta_1 = 0.1$. In the simulation of price and volume trajectories, we consider $T = 1,000$ transaction days and $I^* = 5,760$ intradaily intervals corresponding to a 15-second fre-

[5] Relaxing the assumption of homogeneity would result in a ratio of two aggregated volatility measures, each estimating the weighted average of the variance carried by each trader; see equation (18) in Appendix A.1.

[6] The latter propose a parametric filtering technique to achieve a dynamic decomposition of trading volume into a component associated with other dimensions of illiquidity (e.g. the asymmetric information between informed vs uninformed traders), and a part associated with the efficient price variations as in equation (3), which are a function of the latent information flow. The methodology of Darolles et al. (2017) could be employed to achieve a refined version of the realized Amihud robust to the possible contamination arising from other liquidity frictions. This is beyond the scope of the present analysis and it is left to future research.

quency over 24 hours. RPV is then computed using different sampling frequencies: 1 hour; 30, 15, 10, 5 minutes; and 30 and 15 seconds, corresponding to $I = [24, 48, 96, 144, 288, 1440, 2880, 5760]$ subintervals, respectively.

The results of the Monte Carlo simulations are presented in Table 1. The realized Amihud provides accurate measurements of illiquidity (defined as $A_0 = \frac{2}{\xi J\sqrt{J-1}}$) in all scenarios (low, medium and high illiquidity levels) with a relative bias below 0.5% in absolute value and a small RMSE even for relatively moderate sampling frequencies (e.g. 5-10 minutes). As expected from Theorem (1), the RMSE decreases as I increases (i.e. δ decreases), and it gets very close to the RMSE obtained under the *unfeasible* Amihud estimator that replaces RPV with the unobservable integrated (rescaled) standard deviation. In general the RMSE of the realized Amihud is ten times smaller than the corresponding daily measurement proposed by Amihud (2002). The latter can be considered the extreme case of the measurement of illiquidity obtained with only one observation per trading period i.e. $I = \delta = 1$. In this case, the illiquidity measure in (4) reduces to $A^{x|y,*} = \frac{|r|^{x|y}}{v^{x|y}}$.

Low Liquidity	$J = 20, \xi = 20, A_0 = 0.1147$					
	Percentage Relative Bias			Relative RMSE		
	Realized Amihud	Unfeasible Amihud	Daily Amihud	Realized Amihud	Unfeasible Amihud	Daily Amihud
1hour	0.8108	0.2854	-1.1250	0.1730	0.0645	0.7549
30min	0.8378	0.2173	-1.2646	0.1271	0.0581	0.7535
15min	0.2764	0.0074	-1.3232	0.0978	0.0554	0.7533
10min	0.1428	-0.0155	-1.4240	0.0839	0.0545	0.7510
5min	0.0081	-0.0138	-1.4218	0.0675	0.0533	0.7518
1min	0.0365	-0.0030	-1.4083	0.0559	0.0528	0.7518
30sec	0.0139	0.0081	-1.4092	0.0538	0.0526	0.7514
15sec	0.0052	0.0130	-1.3872	**0.0534**	0.0526	0.7516
Medium Liquidity	$J = 50, \xi = 200, A_0 = 0.0029$					
	Percentage Relative Bias			Relative RMSE		
	Realized Amihud	Unfeasible Amihud	Daily Amihud	Realized Amihud	Unfeasible Amihud	Daily Amihud
1hour	0.4800	0.1707	-2.7305	0.1712	0.0560	0.7264
30min	0.4385	0.0924	-2.8985	0.1239	0.0534	0.7248
15min	0.3454	0.0581	-2.9343	0.0954	0.0526	0.7240
10min	-0.1617	0.0110	-2.9712	0.0813	0.0515	0.7240
5min	0.0570	0.0035	-2.9607	0.0696	0.0512	0.7243
1min	-0.0131	-0.0140	-2.9746	0.0547	0.0511	0.7243
30sec	-0.0064	-0.0027	-2.9647	0.0526	0.0511	0.7243
15sec	-0.0490	-0.0080	-2.9720	**0.0520**	0.0511	0.7242
High Liquidity	$J = 100, \xi = 500, A_0 = 0.0004$					
	Percentage Relative Bias			Relative RMSE		
	Realized Amihud	Unfeasible Amihud	Daily Amihud	Realized Amihud	Unfeasible Amihud	Daily Amihud
1hour	-0.6145	0.0512	-0.7340	0.1610	0.0551	0.7528
30min	-0.2781	0.0118	-0.7636	0.1188	0.0519	0.7521
15min	-0.0996	0.0159	-0.7995	0.0944	0.0515	0.7511
10min	0.0489	-0.0090	-0.8076	0.0826	0.0518	0.7513
5min	-0.0309	-0.0163	-0.8222	0.0673	0.0513	0.7514
1min	0.0141	0.0002	-0.8018	0.0542	0.0513	0.7515
30sec	-0.0635	-0.0024	-0.7947	0.0523	0.0513	0.7517
15sec	-0.0128	-0.0009	-0.7968	**0.0519**	0.0513	0.7516

Table 1: Illiquidity measurement. The table reports Monte Carlo simulations for the assessment of the quality of illiquidity through the realized Amihud. Reservation prices for a different number of J traders, equilibrium prices, as well as trading volume with different values of ξ are generated according to the model illustrated above. Table reports the Monte Carlo relative percentage bias and RMSE (both relative to A_0) for the realized Amihud reported in (4). In bold the smallest RMSE. As a reference, we also consider the *unfeasible* version of the realized Amihud, which replaces RPV with the unobservable integrated (rescaled) standard deviation ($\sqrt{\frac{2I}{\pi}}\mathcal{S}$), with $\mathcal{S} = \int_0^1 \overline{\sigma}(s)ds$, and the daily Amihud measure, i.e. $A^* = \frac{|r|}{v}$.

It should be stressed that the asymptotic results (in the limit for $I \to \infty$) behind Proposition 1 are derived by abstracting from microstructural frictions (namely *microstructural noise*), like transaction costs in the form of bid-ask spread, clearing fees or price discreteness, which are intimately related and endogenous to the trading process. In the simplified context provided by the model, the microstructural features linked to the actual cost of trading in the form of bid-ask spread are not explicitly included but they pose essentially empirical issues. From a statistical point of view, the microstructural noise dominates over the volatility signal as $I \to \infty$, thus leading to distorted measurements of the variance. In the literature on realized variance (see among many others Bandi and Russell, 2008; Liu et al., 2015), it is common practice to resort to moderate sampling frequencies, e.g. 5-minute intervals, to reduce the impact of the microstructural noise contamination on volatility measurement.

Low Liquidity	$J = 20, \xi = 20, A_0 = 0.1147$					
	Percentage Relative Bias			Relative RMSE		
	Realized Amihud	Unfeasible Amihud	Daily Amihud	Realized Amihud	Unfeasible Amihud	Daily Amihud
1hour	0.5158	0.0409	4.5145	0.1721	0.0677	0.7689
30min	0.4960	0.0585	4.5081	0.1279	0.0614	0.7696
15min	0.8671	0.0875	4.4637	0.0995	0.0588	0.7661
10min	1.1110	0.0408	4.4654	0.0886	0.0575	0.7674
5min	2.0405	0.0414	4.5063	**0.0789**	0.0566	0.7686
1min	9.8298	-0.0185	4.4032	0.1147	0.0561	0.7668
30sec	18.8968	-0.0057	4.4231	0.1969	0.0560	0.7671
15sec	35.3962	0.0075	4.4452	0.3577	0.0560	0.7675
Medium Liquidity	$J = 50, \xi = 200, A_0 = 0.0029$					
	Percentage Relative Bias			Relativ RMSE		
	Realized Amihud	Unfeasible Amihud	Daily Amihud	Realized Amihud	Unfeasible Amihud	Daily Amihud
1hour	1.1610	0.0521	0.8674	0.1703	0.0613	0.7292
30min	1.3082	0.0720	0.9803	0.1281	0.0589	0.7303
15min	1.4303	0.0164	0.8910	0.1000	0.0555	0.7287
10min	2.0999	0.0099	0.8955	0.0899	0.0565	0.7295
5min	3.8031	0.0164	0.8900	**0.0857**	0.0558	0.7291
1min	19.3875	0.0126	0.8769	0.2024	0.0553	0.7285
30sec	36.1274	0.0135	0.8845	0.3653	0.0552	0.7288
15sec	64.9953	0.0063	0.8696	0.6520	0.0552	0.7286
High Liquidity	$J = 100, \xi = 500, A_0 = 0.0004$					
	Percentage Relative Bias			Relative RMSE		
	Realized Amihud	Unfeasible Amihud	Daily Amihud	Realized Amihud	Unfeasible Amihud	Daily Amihud
1hour	1.2316	0.0919	1.4268	0.1680	0.0573	0.7673
30min	1.7829	0.0230	1.2931	0.1286	0.0541	0.7647
15min	2.7392	-0.0193	1.2316	0.1033	0.0533	0.7645
10min	4.2956	0.0084	1.2794	**0.1004**	0.0528	0.7652
5min	8.6329	-0.0018	1.2516	0.1143	0.0520	0.7642
1min	38.1025	-0.0156	1.2427	0.3856	0.0522	0.7644
30sec	67.9274	-0.0047	1.2576	0.6820	0.0520	0.7646
15sec	114.6771	-0.0029	1.2568	1.1485	0.0520	0.7646

Table 2: Illiquidity measurement with transaction costs. The table reports Monte Carlo simulations for the assessment of the quality of illiquidity through the realized Amihud in presence of transaction costs. Reservation prices for a different number of J traders, equilibrium prices, as well as trading volume with different values of ξ are generated according to the model illustrated above. In this setting, equilibrium prices are contaminated by transaction costs in the form of bid-ask spread, which is set to 0.005. The key quantity of interest is the *true* illiquidity parameter, i.e. $A_0 = \frac{2}{\xi J\sqrt{J-1}}$. The table reports the Monte Carlo relative percentage bias and RMSE (both relative to A_0) for the realized Amihud reported in (4) and numbers in bold font refer to the smallest RMSE. As a reference, we also consider the *unfeasible* version of the realized Amihud, which replaces RPV with the unobservable integrated (rescaled) standard deviation ($\sqrt{\frac{2I}{\pi}}\overline{\mathcal{S}}$), with $\mathcal{S} = \int_0^1 \overline{\sigma}(s)ds$, and the daily Amihud measure, i.e. $A^* = \frac{|r|}{v}$.

This approach can be carried over to the illiquidity measurement, and its effectivness in reducing the impact of microstructure noise contamination is confirmed by the evidence reported in Table 2. In particular, we repeat the analysis of Table 1, but this time adding the bid-ask spread to the equilibrium prices and a rounding mechanism that generates price discreetness and zero returns.[7] The results suggest that in this setting we should avoid computing the realized Amihud by sampling at the highest possible frequencies (e.g from 15 to 30 seconds) because it leads to large estimation biases. Instead, sampling at moderate frequencies (e.g. from 5 to 10 minutes) drastically reduces the bias and at the same time leads to a low RMSE, especially if compared to daily Amihud, which is about eight times larger (0.086 compared with 0.729 for the medium liquidity scenario). Compared with the unfeasible Amihud measurement the loss of efficiency in terms of RMSE achieved by using returns contaminated by microstructural noise sampled at 5 (or 10) minute frequencies is moderate (0.086 compared with 0.056 for the medium liquidity scenario). This evidence suggests that the realized Amihud based on 5 minutes returns measures well the illiquidity associated with market depth and resiliency even in settings characterized by another dimension of illiquidity, i.e. transaction costs such as the bid-ask spread. In the following empirical analysis we will work assuming that sampling FX returns at 5-minute intervals is sufficient to achieve a precise measurement of FX illiquidity.

3 Data and preliminary analysis

3.1 Data sets

Our empirical analysis relies on three data sets. First, trading volume data comes from CLS, which was launched in 2002 and it is now the largest payment system for the settlement of foreign exchange transactions. By means of a payment-versus-payment mechanism, this infrastructure supports FX trading by reducing settlement risk and supporting market efficiency. For each hour of our sample period and each currency pair, we observe the settlement value and the number of settlement instructions.

Our data set covers 29 currency pairs (15 currencies) over the period from November 1, 2011 to October 19, 2016, for a total of 1,280 trading days and 30,720 hourly observations for each FX rate.[8] Following the literature (e.g. Mancini et al., 2013), we exclude observations between

[7] The proportion of zero returns in the sample of FX rates used in the empirical application ranges between 2% and 3% when returns are sampled at 5-minute frequency. The proportion of zeros generated in this simulation setup is around 30% at 5-minute frequency. This value is about twice that documented by Kolokolov and Renò (2021) for the average of the stocks belonging to S&P500. Kolokolov and Renò (2021) illustrate the detrimental effect of zeros on the quality of volatility estimates, especially when power variation measures are adopted. In circumstances where zeros are the dominant feature of the high-frequency returns (like for individual very illiquid stocks), replacing RPV with $\sqrt{RV}$ in the computation of the realized Amihud could prove beneficial since RV is expected to be more robust than RPV to the presence of zeros.

[8] The full data set contains data for 18 major currencies and 33 currency pairs. To maintain a balanced panel, we exclude the Hungarian forint (HUF), which enters the data set only on November 7, 2015. Moreover, we discard USDILS and USDKRW due to very infrequent trades. The remaining 29 currency pairs are: AUDJPY, AUDNZD, AUDUSD, CADJPY, EURAUD, EURCAD, EURCHF, EURDKK, EURGBP, EURJPY, EURNOK, EURSEK, EURUSD, GB-

Friday 10PM and Sunday 10PM since only minimal trading activity is observed during these non-standard hours.[9] In 2017, the core of CLS was composed of 60 settlement members including the top ten FX global dealers, and thousands of third parties (other banks, non-bank financial institutions, multinational corporations and funds), which are customers of settlement members.[10] The total average daily traded volume submitted to CLS was more than USD 1.5 trillion, which is around 30% of the total daily volume recorded in the last available BIS triennial survey (Bank of International Settlements, 2016). However, after adjusting for the large fraction of BIS volume originated from interbank trading across desks and double-counted prime brokered "give-up" trades, the CLS data should cover about 50% of the FX market (Cespa et al., 2021).

In our study, we focus on FX spot transactions. Except for few exceptions such as the Renminbi, the CLS spot FX rates in our sample are highly representative of the entire FX market. For instance, the currency pairs involving USD and EUR cover more than 85% (94%) of the total trading volume of the BIS triennial survey. To the best of our knowledge, only few papers have analyzed CLS volume data so far. First, Fischer and Ranaldo (2011) study the impact on FX global volume of central bank decisions using five aggregated currencies (e.g. all CLS-eligible currencies against the U.S. dollar, Euro, Yen, Sterling, and Swiss franc) rather than currency pairs. Hasbrouck and Levich (2019) analyze every CLS settlement instruction during April of 2010, 2013, and 2016. After calculating the Amihud index based on CLS data, they compare it to the corresponding estimates for the interdealer segment based on EBS and show high correlations for those currencies predominantly traded on EBS.[11] Cespa et al. (2021) use the same data set used in our study to perform an asset pricing analysis of volume. Ranaldo and Somogyi (2021) analyze the heterogeneous price impact of CLS flows decomposed by market participants and FX asymmetric information risk premium.

The second data set is obtained from Olsen Financial Technologies, which is the standard source for academic research on intraday FX rates. By compiling historical tick data from the main consolidators such as Reuters, Knight Ridder, GTIS and Tenfore, Olsen data are representative of the entire FX spot market rather than specific segments such as the interdealer FX market, which is dominated by two electronic limit order markets: EBS and Reuters. For each minute of our sample period and each currency pair, we observe the following "indicative" quotes: bid, ask, high, low, and mid-quotes. With this data, we can analyze at least four aspects of FX rates: (i) their movements at frequencies of one minute or lower; (ii) the realized volatility, realized power variation or other measures of midquote return dispersion; (iii) the quoted bid-ask spread as a

PAUD, GBPCAD, GBPCHF, GBPJPY, GBPUSD, NZDUSD, USDCAD, USDCHF, USDDKK, USDHKD, USDJPY, USDMXN, USDNOK, USDSEK, USDSGD, and USDZAR.

[9] In this paper, times are expressed in GMT.

[10] Currently, there are 72 settlement members. Most of them are large multinational banks. Furthermore, there are over 25,000 "third party" clients of the settlement members, including other banks, funds, non-bank financial institutions and corporations. CLS records the time of the transaction as if it had occurred at the first instruction being received. CLS receives confirmation for more than 90% of trade instructions from settlement members within 2 minutes of trade execution.

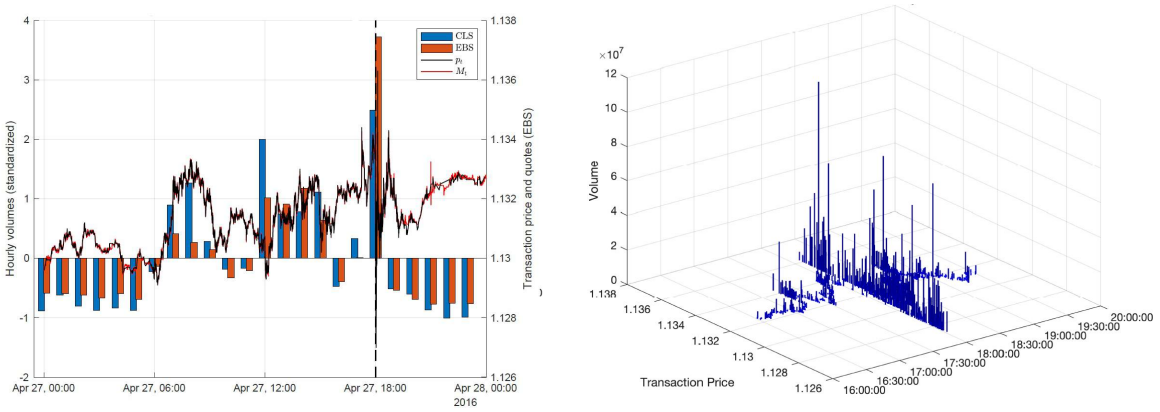
[11] In addition to the traditional Amihud measure, Hasbrouck and Levich (2019) also compute illiquidity ratios over fixed volume intervals and impact estimates based on bulk volume classification.

measure of transaction cost; and (iv) violations of triangular arbitrage conditions.

The third data set is obtained from EBS, which is the major interdealer trading platform for many currencies including EURUSD, USDJPY, and EURCHF which are studied more extensively in this paper.[12] All our CLS FX rates are also traded in EBS, which operates an order-driven electronic trading system that unites buyers and sellers of spot FX around the globe on a pre-trade anonymous central limit order book. We access trade and order data for the entire year of 2016. The EBS data set is organized in a time slice basis, that is, at the end of each 100-millisecond we observe the total amount of trades, either buys or sells, during the time slice interval. Notice that EBS provides the exact identification of whether a trade is buyer- or seller-initiated. In addition, we obtain trade price and volume (in millions of base currency). About order data, we observe the ten best "firm" bid and offer (or ask) quotes, capturing the depth of the book.

To visualize some emblematic behaviors, Panel a) of Figure 1 reports the standardized hourly EBS and CLS volume on EURUSD on April 27, 2016. On that day, the Federal Reserve hosted the Federal Open Market Committee (FOMC) and the market was expecting an interest rate hike, which did not take place. Hence, the market reacted with a sharp and immediate depreciation of the US dollar, which was subsequently reabsorbed. The figure displays a number of interesting insights. First, EBS and CLS volume follow a similar pattern. Second, trading volume increases for both EBS and CLS during the announcement hour, with EBS volume reacting slightly more than CLS during this time interval (between 6PM and 7PM). Instead, the volume of both EBS and CLS drastically reduces in the subsequent hours, and it remains below the daily average in both cases. This signals that the reaction to the news of both the interdealer segment and the customer-driven FX market was relatively short lived. Panel b) of Figure 1 provides evidence on the dynamics of FX prices and volume at a granular level around the time of the announcement by reporting volume and FX price at 15 seconds frequency. At the time of the Fed announcement (at 6:00 PM), the FX rate is at 1.33 and the volume has been relatively low since 4PM. Immediately after the announcement, the price drops to 1.30 (-2.2% variation) but the volume is still relatively low, thus indicating a sudden illiquidity episode. In other words, traders revised their beliefs in the same direction and this led to an (almost) instantaneous determination of a new equilibrium price. In the minutes after the announcement, the volume (and the volatility) of the price increased enormously until 6:30 PM, and the FX rate moved within a large range of values, that is, between 1.28 and 1.36. This signals the dynamic impact of an important news shock on prices, volatility (see Andersen et al., 2007) and hence on volume, which is associated with a large disagreement in the reservation prices of each trader. Finally, after 6:30 PM, the FX price has returned to fluctuate at its pre-announcement level with relatively low trading volume.

[12] The other main interdealer platform is Thomson Reuters. Some FX rates e.g. involving the British pound are mainly traded on it.



(a) EBS and CLS volume

(b) Volume around the announcement

Figure 1: Trading volume on April 27, 2016. Panel a) reports the hourly EBS volume (red bars) and CLS volume (blue bar). The volume series are those on the EURUSD rate and they are reported in deviation from the daily average. The dashed vertical line denotes the hour of the announcement. Panel b) plots the trading volume from the EBS data set (z-axis) and the EURUSD rate (y-axis). The time axis is centered around the Fed announcement (18:00 GMT). Sampling is at a 15-second frequency.

3.2 Descriptive analysis

We now highlight some (new) stylized facts characterizing the times series of volume, volatilities, and illiquidity measures associated with the 29 FX rates under investigation. First, we look into intraday patterns and then we study the daily time series. Turning our attention to individual FX rates, Figure 2 reports the hourly average share of total FX volume of the five most liquid FX rates. First, as expected, all the most liquid FX rates involve the USD. Second, the trading volume of these rates displays clear (intraday) seasonal patterns. For the individual FX rates, these patterns are suggestive of local effects in given geographical areas, coherent with the OTC segmented nature of FX markets. For instance, USDJPY covers around 30% of the total FX volume between 12PM and 4PM, which are the hours during which Far East markets are open. AUDUSD contributes 15% of volume during these hours, while its market share strongly declines to 7% during the central hours of the day. EURUSD is by far the most traded FX rate, with a share above 30% between 7AM to 6PM. A similar pattern also characterizes GBPUSD with an average share ranging between 5% and 10%. Finally, USDCAD is mostly traded at the opening of the business hours in North America, i.e. between 12PM and 10PM, with an approximately 10% share of total volume. These five FX rates amount to more than 70% of total global volume. In summary, the seasonal patterns are clearly discernible in two dimensions. First, on an intraday scale the trading volume follows the working time in each country or jurisdiction defining the currency pair. This means that round-the-clock, the trading volume of New Zealand dollar is the first to increase, followed by Asian, European, and American currencies. Second, the analysis of transaction volume on official banking holidays clearly shows a reduction in exchange volumes.[13]

[13]The descriptive analysis of banking holidays is available upon request.

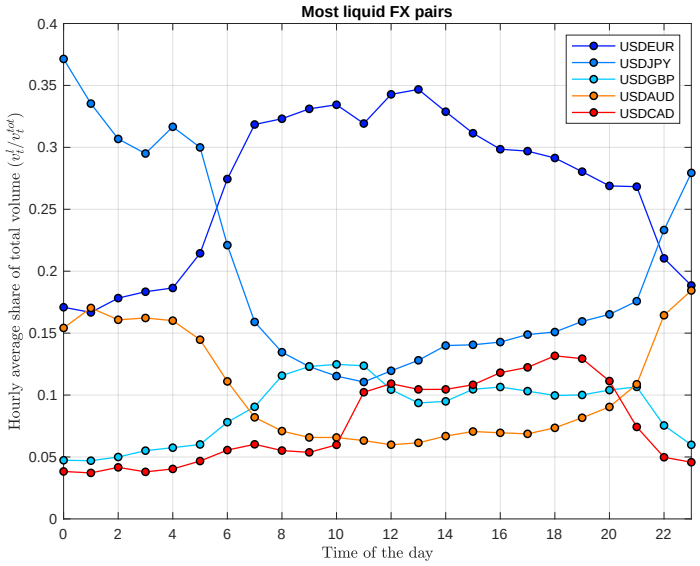


Figure 2: Averages of hourly volume (relative to the total hourly volume) of the five most liquid FX rates involving currencies trading against USD, which are (in order) EUR, JPY, GBP, USD, CAD.

Concerning the relationship between volatility and volume, Figure 3 shows that the averages of hourly RPV and hourly volume for EURUSD and USDJPY follow the same patterns. Two considerations arise: First, at the intradaily level, when FX volatility is high, so is volume, which points to a wider variation of traders' reservation prices. Thus, Figure 3 provides *prima facie* evidence that volatility and volume are mostly governed by a common latent factor, which depends on the level of heterogeneous beliefs (disagreement) between the agents. Second, the comparison between CLS data (continuous lines) and EBS data (dashed lines) in Figure 3 suggests that the FX global market and the FX interdealer segment follow the same systematic patterns both in terms of trading volume and volatility.

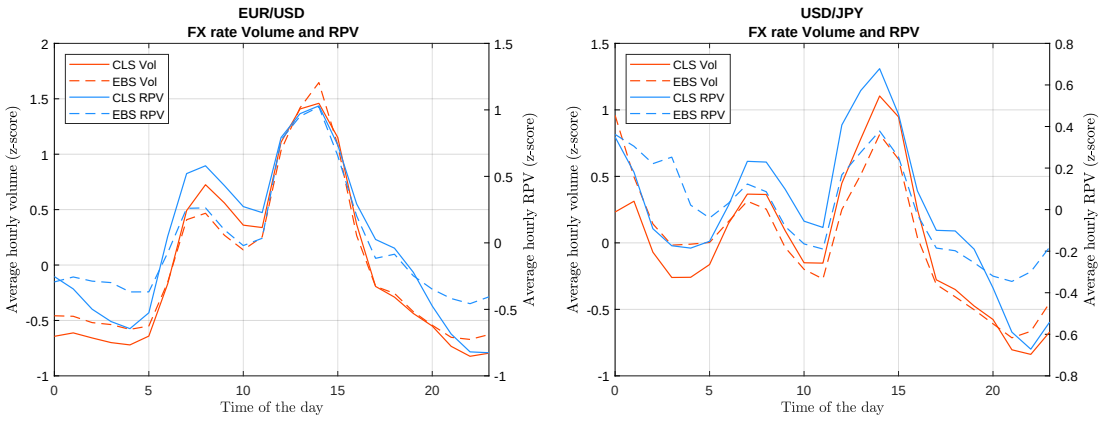


Figure 3: Averages of hourly RPV and hourly trading volume. Hourly RPV and volume are based on the sum of absolute 5-minute midquote returns and trading volume in each hour, respectively. In Panel a) EURUSD, in Panel b) USDJPY. Intraday patterns based on CLS and EBS data are depicted with continuous and dashed lines, respectively.

The separate analysis of trading volume and volatility further motivates their joint examination to obtain a measure of price impact of trading. The fact that volume and volatility move similarly as depicted in Figure 3 implies that ex ante it is unclear how illiquidity evolves over time and urges us to consider a FX illiquidity measure capturing price impact in the spirit of Amihud (2002). To take the first step in this direction, Table 3 reports the descriptive statistics of the realized Amihud for the 29 FX pairs under investigation. As expected, the most illiquid FX (cross) rates such as CADJPY and USDDKK feature the largest price impact, while trading the most liquid ones such as EURUSD and USDJPY triggers a relatively small impact of less than 2 basis points, on average. These quantities reveal a large difference in price impact ranging from a few to as much as 200 basis points. They also indicate the prohibitive costs that an investor has to bear if she wants to directly exchange two illiquid currencies compared to the indirect exchange via the U.S. dollar or euro. For example, the price impact when exchanging Canadian dollars for Japanese yens (or U.S. dollar for Danish krone) rather than passing through USDCAD and USDJPY (or EURDKK and EURUSD) is approximately 180 (90) times larger due to the liquidity pooling offered by the dollar (euro).

An advantage offered by the use of high-frequency returns is the calculation of the realized volatility conditional on the positive or negative sign of the returns during the intraday time interval. In other words, the realized power variation can be broken down into two components: the sum of the absolute returns associated with the appreciation of the base currency with respect to the quote currency and that associated with the depreciation of the base currency (denoted respectively as $RPV_+ = \sum_{|r_i|} \mathcal{I}(r_i > 0)$ and $RPV_- = \sum_{|r_i|} \mathcal{I}(r_i < 0)$. The sample averages of the *signed* realized Amihud, denoted as $A+ = RPV_+/v$ and $A^- = RPV_-/v$, are shown in the third last and penultimate column of the Table 3, respectively. Intuitively, $A+$ and $A-$ measure the impact of trading associated with positive and negative price moves. The general picture that emerges is that the trading impact is substantially symmetrical, suggesting that the purchase of the dollar or other safe haven currencies (e.g. Maggiori, 2017; Ranaldo and Söderlind, 2010) does not imply a systematically lower market depth or elasticity.

An intuitive way to approximate the impact in economic terms is the trading volume associated with a standard deviation of FX returns, which is the inverse of the Amihud measure also known as AMIVEST (e.g. Amihud, 2002). The rightmost column of Table 3 indicates that for highly liquid currencies like EURUSD or USDJPY it takes as much as $140 or $70 billion to move a standard deviation of FX returns. On the contrary, less than a billion dollars is sufficient for the same price impact for illiquid cross-currencies such as CADJPY, GBPAUD, GBPCAD, GBPCHF, or USDDKK. It is also interesting to note that even for USDHKD large volumes are needed for such a price movement. This is most likely due to its U.S. dollar peg rendering the movements of this FX rate more predictable and shielding it from volatility.

We also construct the aggregate realized Amihud measurements for the four most liquid currencies (USD, EUR, GBP and JPY). The indices are constructed as volume-weighted cross-sectional averages of the individual realized Amihud measures with either USD, EUR, GBP and JPY as base

(or quote) currency and reported at the bottom of the Table 3.[14] On average, an international investor faces the lowest (largest) price impact (market depth) when trading with USD, followed by EUR, JPY and GBP. This result is consistent with the dominant role of the dollar as the main reserve and "vehicle" currency. The cross-sectional aggregation also reduces the volatility and kurtosis of the series, making them more persistent than the realized Amihud on the individual rates. As for the individual FX rates, we note that the time series of realized Amihud are positively skewed, leptokurtic and persistent.

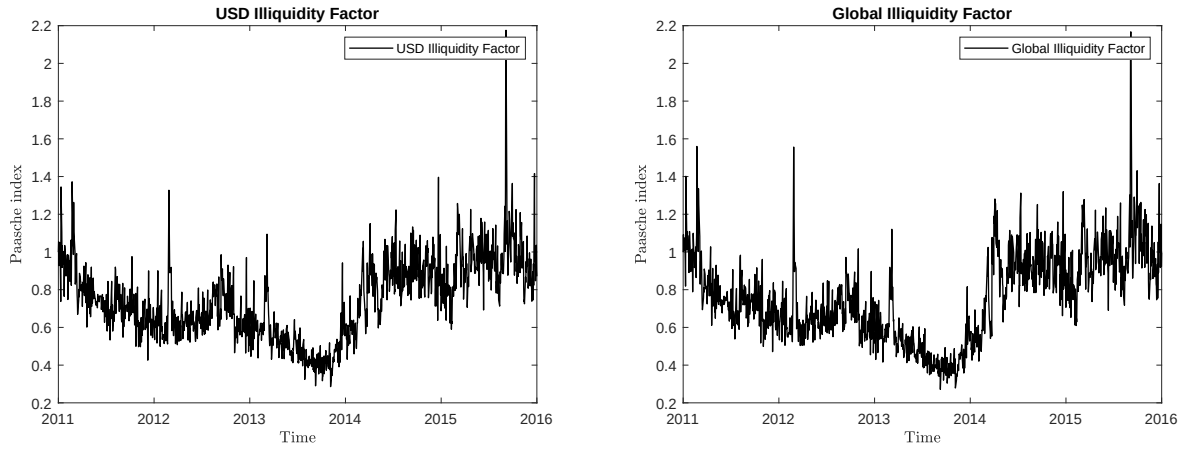
	Mean	StDev	Skewness	Kurtosis	ACF(1)	ACF(10)	ACF(22)	Mean+	Mean-	AMIVEST
AUDJPY	0.2228	0.0719	1.0496	5.6554	0.6665	0.4602	0.4001	0.1115	0.1113	2.34
AUDNZD	0.2888	0.1323	1.9332	9.6444	0.5521	0.2592	0.1346	0.1440	0.1449	1.94
AUDUSD	0.0188	0.0072	0.7148	2.9503	0.8509	0.7690	0.7190	0.0094	0.0094	28.76
CADJPY	1.8762	0.9815	1.9362	8.3112	0.5294	0.2628	0.2389	0.9400	0.9364	0.31
EURAUD	0.2270	0.0937	1.1665	4.9106	0.6392	0.4889	0.4055	0.1134	0.1136	2.42
EURCAD	0.3901	0.1392	0.9130	4.5031	0.5404	0.3572	0.2449	0.1951	0.1949	1.37
EURCHF	0.0423	0.0330	1.0445	3.4813	0.9081	0.8100	0.7310	0.0212	0.0212	24.36
EURDKK	0.0266	0.0188	4.3754	38.6626	0.3966	0.2359	0.1809	0.0133	0.0133	23.54
EURGBP	0.0370	0.0149	1.2150	5.3700	0.7557	0.6433	0.5780	0.0185	0.0185	14.73
EURJPY	0.0511	0.0247	1.9841	9.0339	0.8363	0.7411	0.6630	0.0255	0.0255	11.01
EURNOK	0.1090	0.0489	4.1337	40.5581	0.7202	0.3357	0.2755	0.0546	0.0544	4.96
EURSEK	0.0839	0.0322	3.1393	23.1042	0.6081	0.2707	0.1542	0.0420	0.0419	6.23
EURUSD	0.0037	0.0012	0.8658	4.9530	0.7700	0.6530	0.6131	0.0019	0.0019	140.13
GBPAUD	0.7737	0.4161	1.9591	9.6589	0.6844	0.4492	0.3526	0.3868	0.3869	0.77
GBPCAD	1.1291	0.7591	2.4660	12.8229	0.5731	0.4017	0.3147	0.5649	0.5647	0.60
GBPCHF	1.1251	0.6673	1.4096	5.3946	0.5886	0.4465	0.3829	0.5629	0.5623	0.58
GBPJPY	0.2091	0.0970	1.0897	4.6207	0.7614	0.6545	0.5428	0.1047	0.1044	2.80
GBPUSD	0.0109	0.0039	0.8739	4.5369	0.7567	0.6403	0.5890	0.0055	0.0054	49.19
NZDUSD	0.0863	0.0338	1.1601	5.4049	0.7400	0.6245	0.5072	0.0432	0.0432	6.34
USDCAD	0.0130	0.0048	1.9380	10.2889	0.5489	0.4854	0.4605	0.0065	0.0065	40.51
USDCHF	0.0429	0.0175	1.1907	5.3565	0.7604	0.6037	0.4829	0.0215	0.0214	12.82
USDDKK	1.4157	3.1544	17.1391	354.5686	0.4218	0.0789	0.0153	0.7085	0.7072	0.63
USDHKD	0.0056	0.0049	5.7193	60.9325	0.7149	0.4108	0.2984	0.0028	0.0028	126.86
USDJPY	0.0074	0.0024	0.8026	3.9504	0.7563	0.6545	0.5954	0.0037	0.0037	70.68
USDMXP	0.0700	0.0315	1.9595	10.2868	0.7059	0.6047	0.4910	0.0351	0.0349	7.94
USDNOK	0.3764	0.1801	1.8835	9.7242	0.4128	0.2960	0.1877	0.1888	0.1876	1.52
USDSEK	0.3162	0.1516	1.8800	8.8918	0.5159	0.3926	0.3137	0.1587	0.1575	1.79
USDSGD	0.0499	0.0195	2.5408	13.0805	0.6887	0.5217	0.4498	0.0250	0.0249	10.46
USDZAR	0.1603	0.0756	2.7487	17.2174	0.6581	0.5050	0.3997	0.0803	0.0800	3.43
USD	0.0131	0.0037	0.8086	5.2594	0.7821	0.6600	0.5923	0.0066	0.0066	
EUR	0.0222	0.0080	1.0562	4.3860	0.8038	0.6995	0.6247	0.0111	0.0111	
JPY	0.0370	0.0143	1.3374	6.8436	0.7966	0.6633	0.5792	0.0185	0.0185	
GBP	0.0544	0.0177	0.7746	4.2533	0.7192	0.5985	0.5330	0.0272	0.0272	

Table 3: Descriptive statistics of realized Amihud. Data are based on 1,235 daily observations for each currency pair. In addition, statistics for the volume-weighted averages of the largest currencies are reported separately. Realized Amihud values are re-scaled by a factor of 10^{10}. Mean+ and Mean- indicate the sample average of realized Amihud with RPV computed with only positive (or negative) returns (i.e. RPV_+ and RPV_-). AMIVEST stands for inverse Amihud and it is the average of the inverse realized Amihud, indicating how many billions of USD are necessary to move the FX rate by one standard deviation.

Finally, Figure 4 reports the time series of the realized Amihud for USD and for the entire global FX market, which are obtained as a volume-weighted Paasche index, with November 1,

[14] In Section 4.3 below, we exploit volume-weighted cross-sectional averages of the realized Amihud measure to construct instrumental variables.

2011 as base date. The evolutions of the USD and global illiquidity indices display similar patterns. After the peak related to the European sovereign debt crisis, FX liquidity improved until the end of 2014 reaching approximately 50% of its original values. After January 2015, we notice a quick reversal to the same level of illiquidity values as the beginning of the sample for both USD and global measures. The possible explanations could be (i) the shrinking interdealer market and the increasing application of internalisation[15] and (ii) regulatory pressure due for example to the Basel III and Dodd-Frank frameworks, which have since created shadow costs, thereby disincentivizing dealer liquidity provision as has been documented in other OTC and dealer markets (e.g. Adrian et al., 2017). Although regulatory and policy effects on FX liquidity go beyond the scope of this study, later in Section 4.4 we will theoretically and empirically illustrate how some of these effects can be attributed to the intervention of the Swiss central bank imposing a cap on the EURCHF rate.



(a) USD (b) Global

Figure 4: Time series evolution of USD and Global FX illiquidity measures.

4 Empirical analysis

4.1 Determinants of trading volume and volatility

Proposition 1 in Section 2 postulates that volatility and trading volume are proportional to the level of heterogeneity in the reservation prices between agents, that is, traders' disagreement about the fundamental asset value. We first examine how changes in daily trading volume correlate with changes in volatility (as measured by daily RPV) and other factors that have been shown to explain FX liquidity (measured as transaction costs, see e.g. Karnaukh et al., 2015) and trading activity in stock markets (e.g. Chordia et al., 2001). Although some of these variables are

[15]Internalisation is the common and growing dealers' practice of temporarily warehousing the position originated from a client's transaction until it is offset against opposing client flow. See Chaboud et al. (2021) for a detailed discussion and evidence on this issue.

likely to be mutually endogenous, attention is directed towards documenting novel correlation patterns pertaining to FX trading volume rather than causation. To this purpose, the 29 currency pairs are pooled together to examine whether daily FX volume is linked to changes in overall market conditions and its liquidity. More precisely, we consider the following linear regression model for daily trading volume

$$v_t = \beta_0 + \beta_1' x_t + \beta_2 \gamma_t + \beta_3' \delta_t + \beta_4 v_{t-1} + \varepsilon_t, \tag{6}$$

where x_t is a vector of regressors subsuming daily (realized) volatility, the relative bid-ask spread (BAS), the TED spread (the yield spread between the U.S. three-month Libor and T-bills, a common proxy of funding strains) and the FX VIX (i.e. the JP Morgan Global FX implied volatility index, a proxy for uncertainty and global fear), which can be decomposed into time-varying uncertainty and risk aversion (Bekaert et al., 2013).[16] Further, γ_t denotes a dollar appreciation dummy and δ_t includes day-of-the-week dummies. The same regression is then estimated for different dependent variables: volatility (RPV), the realized Amihud, and the relative bid-ask spread (BAS). The estimates of the regression coefficients are reported in Table 4.

Some expected as well as novel patterns emerge from this regression analysis. First, the findings from specification 1 and 2 confirm that FX trading volume increases with volatility. Hence, the common (intraday) volume-volatility relation showed in Figure 3 also arises in their temporal evolution. Second, transaction costs as measured by the bid-ask spread have an opposite effect on volume and volatility: it decreases with the former and increases with the latter, suggesting that larger transaction costs discourage trading activity and increase price dispersion. Third, uncertainty and global fear (gauged by FX VIX) also disincentivize trading activity and increase volatility. Fourth, when decoupling risk aversion from uncertainty, we observe that uncertainty has a negative impact on trading volume, while risk aversion increases with volume, perhaps due to investors' propensity to adjust their portfolio positions when becoming more risk-averse. As expected, volatility increases with uncertainty and risk aversion. Fourth, trading volume and volatility follow an inverted U-shape relation across weekdays, that is, they tend to be larger in the middle of the week.

We now turn our attention to the regression results of the daily realized Amihud, i.e. the ratio between daily RPV and daily volume. The results are exhibited in column (3) of Table 4, including the same control variables discussed above. Overall, we find that the realized Amihud illiquidity measure increases with transaction costs, the TED spread as well as implied volatility and both of its components. In addition to being statistically significant, these results are economically meaningful. For instance, an increase of transaction costs by 1% is associated (ceteris paribus) with an increase of illiquidity by 0.16%. Overall, our results appear consistent with standard empirical predictions of microstructure theories as they suggest that (i) the two main dimensions of market illiquidity, i.e. transaction cost and price impact, are positively related and (ii) the price

[16] Notice that as an uncertainty measure, the FX VIX should covary positively with disagreement (e.g. Söderlind, 2009). See also Buraschi and Jiltsov (2006) and Buraschi et al. (2014) for a theoretical and empirical analysis of the linkage between implied volatility and disagreement.

	Volume		RPV		Realized Amihud		Relative BAS	
	(1a)	(1b)	(2a)	(2b)	(3a)	(3b)	(4a)	(4b)
Volume			0.0739^a	0.0733^a			-0.0163^a	-0.0170^a
			(9.15)	(9.18)			(−7.00)	(−7.26)
RPV	0.0504^a	0.0523^a					0.0908^a	0.0929^a
	(13.62)	(14.21)					(7.50)	(7.85)
Relative BAS	-0.0563^a	-0.0582^a	0.3082^a	0.3056^a	0.0016^a	0.0016^a		
	(−17.82)	(−18.41)	(7.82)	(7.78)	(4.29)	(4.32)		
TED	−0.0054	-0.0183^c	-0.0703^a	-0.0804^a	0.0032^a	0.0040^a	0.0044	-0.0154^c
	(−0.51)	(−1.66)	(−2.75)	(−2.71)	(3.26)	(3.53)	(0.56)	(−1.88)
VXY	-0.2044^a		1.5897^a		0.0525^a		-0.3757^a	
	(−2.78)		(7.22)		(3.66)		(−5.96)	
UNC		-0.0127^a		0.0831^a		0.0030^a		-0.0226^a
		(−3.13)		(6.81)		(3.63)		(−6.10)
RA		0.0153^b		0.1058^a		0.0010^b		0.0202^a
		(2.44)		(5.23)		(2.18)		(4.41)
USD	0.0965	0.0287	−0.3510	−0.4041	0.0015	0.0063	−0.0966	−0.2003
	(0.52)	(0.15)	(−0.84)	(−0.99)	(0.19)	(0.79)	(−0.69)	(−1.49)
Monday	-3.9359^a	-3.8618^a	-5.8102^a	-5.7624^a	0.1369^a	0.1330^a	-4.7688^a	-4.6527^a
	(−13.14)	(−13.03)	(−9.29)	(−9.00)	(9.80)	(9.93)	(−18.57)	(−18.40)
Tuesday	1.5679^a	1.5992^a	-1.4439^a	-1.4036^a	-0.0860^a	-0.0867^a	-0.9349^a	-0.8852^a
	(5.86)	(6.02)	(−3.07)	(−2.95)	(−3.70)	(−3.77)	(−5.61)	(−5.36)
Thursday	−0.1172	−0.0829	0.0981	0.1279	0.0159	0.0137	−0.2722	−0.2194
	(−0.44)	(−0.31)	(0.16)	(0.21)	(1.29)	(1.16)	(−1.59)	(−1.31)
Friday	-1.8435^a	-1.7856^a	-5.7758^a	-5.7423^a	0.0317^a	0.0286^a	1.5067^a	1.5859^a
	(−6.71)	(−6.57)	(−8.86)	(−8.70)	(3.04)	(2.81)	(6.55)	(6.90)
Lagged Dep.	0.9346^a	0.9341^a	0.7092^a	0.7130^a	0.4946^a	0.4882^a	0.8840^a	0.8828^a
	(115.63)	(115.73)	(18.67)	(18.49)	(3.69)	(3.60)	(71.16)	(71.76)
Constant	3.3888^a	2.9779^a	-6.9356^a	0.6321	-0.1668^a	0.0332^b	4.0270^a	3.0817^a
	(5.84)	(7.47)	(−3.24)	(0.37)	(−3.27)	(1.96)	(5.87)	(5.05)
R^2	0.90	0.90	0.76	0.76	0.36	0.36	0.93	0.93

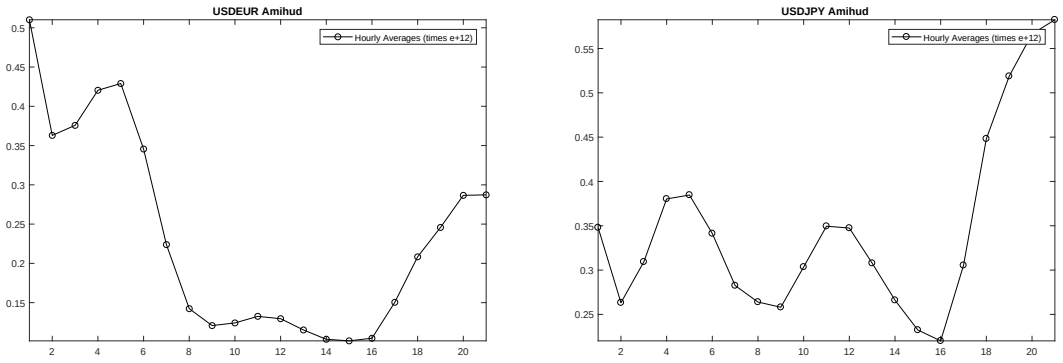
Table 4: Regressions of volume, volatility (RPV), illiquidity, and bid-ask spread. Volume and RPV are the daily trading volume and daily RPV respectively, the realized Amihud is the ratio between daily RPV and daily volume, and the bid-ask spread is the daily average of one-minute spreads. The t-statistics are in parentheses and the standard-errors are robust to heteroskedasticity and autocorrelation in the residuals. The superscripts a, b and c indicate significance at 1%, 5% and 10% significance level respectively.

impact of trading gets larger with money market strains, uncertainty, and risk aversion.

To conclude the descriptive analysis, it is interesting to compare the results exhibited in columns (3) and (4) of Table 4 as they shed light on the two main dimensions of liquidity, i.e. transaction costs and price impact measured by the realized Amihud indicator. Compared to transaction costs, the FX impact of trading reacts more to funding strains (TED) and uncertainty and is especially large at the beginning of the week. Also, the positive autocorrelation of trading price impact is less strong than that of transaction costs. The trading impact appears to be slightly larger when the U.S. dollar appreciates but this result is not statistically significant, consistent with the overall symmetrical price impact documented in Table 3.

4.2 How well does the realized Amihud measure illiquidity?

Proposition 1 in Section 2 provides a closed-form expression for illiquidity in the spirit of Amihud (2002), i.e. the ratio between volatility and trading volume. The empirical prediction is that the price impact of trading decreases with market depth and the number of active traders. The visual inspection of Figure 5, representing the intraday averages of the realized Amihud on EURUSD and USDJPY, suggests that the trading price impact tends to decrease when international financial centers are open, that is, when the FX market is deep and populated by active traders. More precisely, it is discernible that the price impact abruptly decreases at the opening of the European markets and it is minimal when both European and American markets are jointly open. After 8PM the illiquidity grows again and it is maximal during the night hours. A consistent pattern arises for USDJPY (the right-hand side of Figure 5): the price impact of trading on USDJPY reduces at the opening of the main financial markets, i.e. Tokyo, London and New York, and it substantially increases again after 4PM.



(a) EUR (b) JPY

Figure 5: Hourly averages of the realized Amihud from 1AM until 9PM for two currencies trading against USD; in Panel a) EUR, in Panel b) JPY.

So far, we have analyzed FX illiquidity on a global scale. Now, we ask the question whether our FX illiquidity measure is positively correlated with other illiquidity proxies in the FX *inter-dealer* segment. We focus on EURUSD since it is primarily traded on the EBS interdealer trading platform.[17] In the same spirit of Hasbrouck (2009), we analyze correlations between daily illiquidity measures.[18] More specifically, we compute correlations of the realized Amihud (denoted as A) with the following proxies: relative quoted spread (i.e. quoted spread, i.e. ask minus bid quotes, divided by midquote; denoted as BAS); effective cost denoted as EC (i.e. the absolute value of the difference between transaction price and midquote); high-low estimate of the effective spread (denoted as CS) (Corwin and Schultz, 2012); cost estimates implied by the Roll model (Roll, 1984), which are computed as the autocovariance between consecutive price changes with

[17] The analysis of USDJPY, which is also predominantly traded on EBS, provides similar results.
[18] Further details on the construction of the FX illiquidity measure are presented in Appendix B.

negative estimates set to zero (denoted as R); trade-by-trade order flow price impact (denoted as λ); the traditional Amihud measure (i.e. daily return in absolute value over trading volume; denoted as A^*); and a new version of the traditional Amihud measure replacing the daily absolute return with the daily high-low range as the numerator (denoted as A^{**}).[19]

	A	BAS	EC	CS	R	λ	A^*	A^{**}
A	1.0000	0.7885	0.6680	0.3877	0.7068	0.9115	-0.0671	0.2622
BAS	0.9080	1.0000	0.7944	0.5247	0.6594	0.7923	-0.0115	0.1682
EC	0.8712	0.9128	1.0000	0.5701	0.8901	0.7455	0.0687	0.1843
CS	0.5460	0.6335	0.6570	1.0000	0.4730	0.4285	-0.0784	-0.0994
R	0.6791	0.5696	0.7950	0.4361	1.0000	0.7690	0.0289	0.1931
λ	0.9041	0.7759	0.8332	0.5583	0.7523	1.0000	-0.0304	0.2510
A^*	0.4005	0.4326	0.3683	-0.0873	0.1933	0.2138	1.0000	0.5881
A^{**}	0.5390	0.5204	0.4567	-0.0679	0.2928	0.3284	0.9248	1.0000

Table 5: Correlation matrix for illiquidity measures for the EURUSD rate. Pearson (Spearman) correlations are reported in the lower (upper) triangular portion of the table. The data set comes from EBS and ranges from January 1, 2016 to December 31, 2016. A, denotes the realized Amihud, BAS the bid-ask spread, EC the effective cost, CS the Corwin-Schultz spread estimator, R the Roll measure, λ the trade-by-trade price impact coefficient, A^* the classic low-frequency Amihud measure computed with the absolute value of daily log-return, and A^{**} is an alternative low-frequency Amihud measure computed with daily range at the numerator.

Table 5 shows the correlation matrix for illiquidity measures and delivers three main messages: First, it clearly shows that our realized Amihud measure is positively correlated (Pearson) with intraday illiquidity proxies based on EBS data. In particular, it is highly correlated with the effective cost and order flow price impact, which are generally used as the benchmark measures of transaction cost and trading price impact, respectively. Similarly to what found in the stock literature (e.g. Hasbrouck, 2009), this finding corroborates the idea that the realized Amihud effectively captures the price impact of trading.

Second, the realized Amihud index is positively correlated with the traditional low-frequency Amihud indicator. On the one hand, the volatility signal is less accurate when using low frequency (daily) data, thus resulting in smaller correlation with the other proxies of FX illiquidity compared with the realized Amihud. On the other hand, one could obtain estimates of FX trading price impact even approximating volatility with daily absolute returns (as in the traditional Amihud indicator) rather than gauging it with more accurate high-frequency measures (RPV). This also applies to the other new version of the low-frequency Amihud index we have proposed whose numerator is the daily range. Actually, the range-based version seems to offer higher correlations with the intraday benchmarks than the traditional Amihud index.

[19]The daily relative quoted and effective spreads are average values of intraday data snapped at 100-millisecond intervals. Using the quoted spread gives very similar results. The Corwin & Schultz spread estimator is computed for each day using hourly maximum and minimum prices. A daily measure is then extracted by averaging out the 23 hourly spread estimates. Negative estimates are set to zero. Furthermore, the correlation coefficients of the realized Amihud with the order flow price impact based on five- or one-minute intervals are similar to the trade-by-trade order flow, although slightly smaller.

Finally, the realized Amihud index is correlated (with the expected sign) with other low-frequency estimates of effective spread, in particular the Roll measure.[20] Overall, we find that our high-frequency illiquidity measure in the spirit of the Amihud indicator covaries positively with other illiquidity proxies, suggesting that it is effective in measuring FX illiquidity and in particular, the price impact of trading. The Spearman rank correlations mostly confirm these results although the magnitudes are somewhat smaller.

4.3 Illiquidity and Mispricing

We now highlight how the no-arbitrage condition keeping FX rates tied to equilibrium relations passes through trading activity (volume), which in turn is sustained by liquidity. By triangular no-arbitrage parity it must hold that:

$$p_i^{x|y} = p_i^{x|z} + p_i^{z|y}, \tag{7}$$

where $p_i^{x|z} = \frac{1}{J}\sum_{j=1}^{J} p_{i,j}^{x|z;*}$ and $p_i^{z|y} = \frac{1}{J}\sum_{j=1}^{J} p_{i,j}^{z|y;*}$. Hence, the *synthetic* midquote return on $x|y$ results in:

$$\tilde{r}_i^{x|y;z} = \frac{1}{J}\sum_{j=1}^{J} \Delta p_{i,j}^{x|z;*} + \frac{1}{J}\sum_{j=1}^{J} \Delta p_{i,j}^{z|y;*}. \tag{8}$$

Hence, by imposing the triangular no-arbitrage relation on the FX rates, which prescribes that $r_i^{x|y} = \tilde{r}_i^{x|y;z}$, it follows that

$$\frac{1}{J}\sum_{j=1}^{J} p_{i,j}^{x|y;*} = \frac{1}{J}\sum_{j=1}^{J} p_{i,j}^{x|z;*} + \frac{1}{J}\sum_{j=1}^{J} p_{i,j}^{z|y;*}. \tag{9}$$

This means that the sum of traders' reservation prices on $x|y$ must be equal to the sum of the average traders' reservation prices of $z|y$ and $x|z$. This also implies that each trader can take a direct position on $x|y$ or operate on the synthetic rate by forming independent beliefs on $x|z$ and $z|y$, thus generating volatility and trading volume on each individual FX rate. It follows that trading volume, volatility and liquidity across FX rates are linked by no-arbitrage constraints, which lead to the commonalities across FX rates.[21]

If the triangular trade is performed with an illiquid currency (enduring larger price impacts), this prevents the flow of information from being quickly embedded into the FX rate, and the

[20] The same picture holds when the Roll estimator is augmented with the Gibbs sampling method proposed by Hasbrouck (2009).

[21] For instance, the *synthetic* realized variance on the x—z and z—y rates can be expressed as

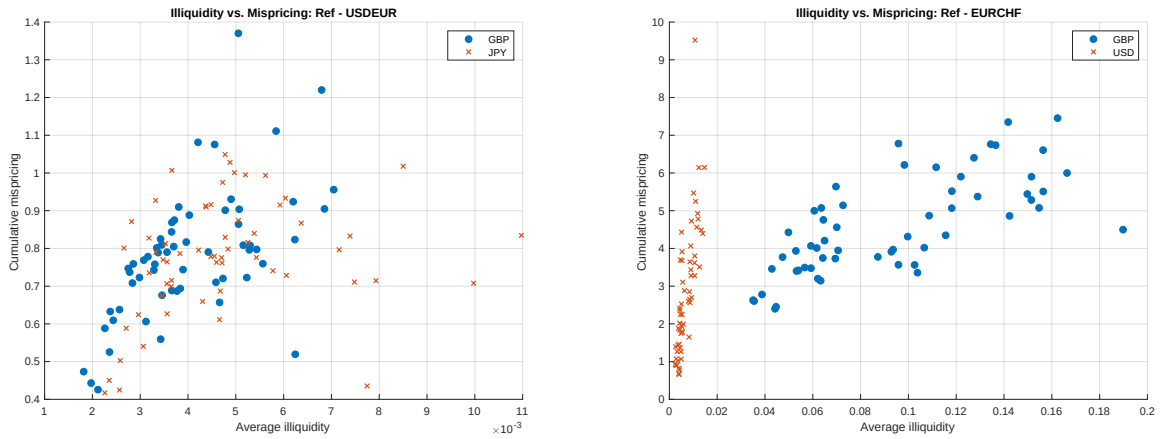
$$p\lim_{I\to\infty} \widetilde{RV}^{x|y,z} = p\lim_{I\to\infty} \sum_{i=1}^{N} \tilde{r}_i^{x|y;z^2} = \frac{\mathcal{V}_{x|z} + \mathcal{V}_{z|y} + 2C\mathcal{V}_{x|z,z|y}}{J^2}, \tag{10}$$

where $\mathcal{V}_{x|z} = \sum_{j=1}^{J}\int_0^1 \left(\sigma_j^{x|z}(s)\right)^2 ds$ and $\mathcal{V}_{z|y} = \sum_{j=1}^{J}\int_0^1 \left(\sigma_j^{z|y}(s)\right)^2 ds$ are the components of the return variation generated by the cumulative individual variations of the reservation prices on $x|z$ and $z|y$. The term $C\mathcal{V}_{x|z,z|y}$ measures the covariance between $x|z$ and $z|y$.

parity is expected to break down or to converge more slowly to equilibrium conditions.[22] Hence, for each individual FX rate, we can compute the price impact of trading on $x|z$ and $z|y$, defined as $A^{x|z} = \frac{RPV^{x|z}}{v^{x|z}}$ and $A^{z|y} = \frac{RPV^{z|y}}{v^{z|y}}$, and relate them to deviations from triangular parity. Using midquote returns, we then consider the volatility of no-arbitrage violations (*pricing* errors), defined as $pe^{x|y;z} = \tilde{r}_i^{x|y;z} - r_i^{x|y}$, to investigate whether liquidity is associated with smaller variability of the pricing errors, where the latter is measured as

$$RPVE^{x|y;z} = \sum_{i=1}^{I} |pe_i^{x|y;z}|. \tag{11}$$

We illustrate the link between pricing errors and illiquidity by looking at the dependence between the variability of pricing errors, as measured by the RPVE$^{x|y;z}$, and illiquidity of the two indirect FX rates forming the triangular arbitrage, computed as $\overline{A}^{x|y;z} = (A^{x|z} + A^{z|y})/2$. For instance, for EURUSD, EURGBP, and GBPUSD we consider the cumulative absolute deviations between the direct (EURUSD) and synthetic rates (via EURGBP and GBPUSD), and the average realized Amihud measures of the two indirect FX rates constituting the triplet for the triangular arbitrage condition (EURGBP and GBPUSD). Figure 6 plots the monthly cumulative pricing error



(a) Pricing error variation against illiquidity: EURUSD (b) Pricing error variation against illiquidity: EURCHF

Figure 6: Monthly pricing error variation (RPVE$^{x|y;z}$) against average illiquidity, $\overline{A}_t^{x|y;z}$.

variation (RPVE$^{x|y;z}$) against trading impact, $\overline{A}_t^{x|y;z}$, for the EURUSD (Panel a) and EURCHF rates (Panel b). The figures clearly display a positive relationship between mispricing and illiquidity. The next step is to understand whether more liquid currencies involved in the triangular arbitrage relation feature a steeper dependence between mispricing and illiquidity compared with the illiquid ones. For instance, the red crosses in Panel b of Figure 6) refer to the EURCHF position passing through EURUSD and USDCHF, which benefits from more liquidity compared to that passing through EURGBP and GBPCHF (blue dots). Economically, the average of the inverse Amihud (IA or AMIVEST) in Table 3 for EURUSD and USDCHF is 77, which is eleven times

[22] Furthermore, being an OTC market, at any given time there may be different FX rates (Evans and Rime, 2016).

that of EURGBP and GBPCHF. The same reasoning applies to the EURUSD position in Panel a of Figure 6) replicated with two indirect positions (i.e., GBPUSD with EURGBP marked with blue circles, and USDJPY with EURJPY marked with red crosses) that do not differ much in terms of liquidity.[23] These results suggest that the same amount of reduction of trading impact is more effective in reducing arbitrage deviations when transacting liquid currencies rather than illiquid ones.

Next, we assess the mispricing-liquidity relation by inspecting which currency pair facilitates the price adjustment process. To highlight if this is a systematic feature, we carry out a statistical analysis performing the following regression:

$$RPVE_t^{x|y;z} = \alpha + \beta_1 A_t^{x|z} + \beta_2 A_t^{z|y} + \varepsilon_t, \tag{12}$$

where $A_t^{x|z}$ and $A_t^{z|y}$ are the illiquidity measures on the same "indirect" FX rates used to calculate $\text{RPVE}_t^{x|y;z}$. We expect the parameters β_1 and β_2 to be positive and significant if the price impact of trading in the currency pairs ($x|z$ and of $z|y$) forming the triangular no-arbitrage with $x|y$ leads to pricing errors. In other words, this analysis should point out in which currency pair liquidity is critical to preserve price efficiency in terms of the triangular no-arbitrage condition. Ideally, we would like to conduct a robust inference on the elasticity parameter linking better liquidity conditions on $x|z$ and $y|z$ to the reduction of the pricing error variability. However, there are at least three reasons to expect that $A_t^{x|z}$ and $A_t^{z|y}$ may correlate with the error term, ε_t, and thereby be endogenous in regression (12), making standard OLS inference on β_1 and β_2 biased.

First, there could be omitted variables affecting pricing errors. For example, the transaction costs and idiosyncratic risk (Pontiff, 2016) might matter in the determination of no-arbitrage violations. A remedy is the inclusion of control variables in regression (12) such as the bid-ask spread and realized Amihud ($A_t^{x|y}$) of the direct FX rate. Second, there is likely a degree of measurement error in gauging illiquidity by A_t. Although the superior accuracy of the realized Amihud is proven theoretically and numerically, volatility as measured by RPV is computed with a finite number of transactions, see the discussion in Meddahi (2002) among others. This leaves open the possibility of an errors-in-variable (EIV) problem in A_t. It leads to a potential attenuation bias towards zero in the estimates of β_1 and β_2 in regression (12). The estimated standard error is biased towards zero, too, so the t-tests are less biased than the parameter estimates, but the EIV problem remains. Third, endogeneity could originate from reverse or simultaneous causation by a common factor. For instance, FX dealers normally increase the liquidity provision to arbitrageurs but decrease it when constrained as to generate common movements in liquidity and price efficiency. Indeed, arbitrage violations and illiquidity are two faces of the same coin: large no-arbitrage violations may lead traders to refrain from trading or supplying liquidity thus generating a larger price impact as measured by A_t. This drives a classic case of simultaneity, where traders tend to place orders on the most liquid FX rates, rather than on the illiquid ones, where trading is more costly and endure larger price impacts. Since this relationship induces a

[23] The IA averages of GBPUSD-EURGBP and USDJPY-EURJPY pairs are 33 and 41.

positive dependence between pricing errors and illiquidity, the resulting bias in the estimate of the expected positive effects of interest, β_1 and β_2, i.e., the elasticity of no-arbitrage violations to illiquidity, is in the same (positive) direction.

We propose two remedies. First, we include $BAS_t^{x|y}$ and $A_t^{x|y}$, i.e. the measures of transaction cost and trading price impact of the direct FX rate $x|y$, as controls to mitigate possible omitted variable bias. Second, we correct for possible endogeneity (stemming from simultaneity in particular) by instrumental variable (IV) techniques, specifically, two-stages least squares (2SLS). In particular, we follow the method of GIV by Gabaix and Koijen (2020), which is a systematic way to obtain instruments. In this context, the GIV are created as the difference between the aggregated volume-weighted and equally-weighted illiquidity measures on a given currency x, that is

$$\Gamma_t^x = \sum_{i=1}^{N} w_{i,t} A_t^{x|i} - \sum_{i=1}^{N} e_{i,t} A_t^{x|i}, \tag{13}$$

where $w_{i,t} = \frac{v_t^{x|i}}{\sum_{i=1}^{N} v_t^{x|i}}$ is the weight based on relative volume, while $e_{i,t} = \frac{1}{N}$. This choice proves to be particularly suited to solve endogeneity issues in a multidimensional context such as those originating from the trading activity on the global FX market. Indeed, GIV allows us to break down the commonality structure arising from the no-arbitrage constraints as illustrated in (10) and extract idiosyncratic illiquidity shocks from the data disentangling them from the systematic component. Then we can use GIVs as (powerful) instruments to identify the relation of interest.

Table 6 reports the parameter estimates of both OLS and TSLS regressions, together with a standard battery of test statistics relevant for the 2SLS analysis (Hausman and first stage F statistic). As a guideline, a first stage F statistic of at least 10 indicates that the IVs have adequate explanatory power for the endogenous variable (Staiger and Stock, 1994). The Wu-Hausman statistic tests whether instrumentation is needed, i.e., whether there is a significant difference between the original (OLS) and instrumented (IV) specifications. As the direct currency pair, we consider the most liquid exchange rate, i.e. EURUSD. This allows us to obtain the largest number of indirect currencies for the triangular parity, i.e. $z = \{AUD, CAD, CHF, GBP, JPY, NOK, SEK\}$.

We adopt three methods: First, the OLS linear regression (12) for the EURUSD rate, for which the first column of Table 6 reports the OLS estimated coefficients for each currency. Second, we carry out the GIV estimation and the corresponding estimates are showed in the second column of the same Table. As instruments, we consider GIVs on all currencies constituting the triangular no-arbitrage relation, that is EUR, USD and the third currency, denoted as Γ_t^{EUR}, Γ_t^{USD} and Γ_t^z. Third, we extend the GIV approach by including lags of $A_t^{x|z}$ and $A_t^{z|y}$ as further instruments to correct for the EIV problem in the realized Amihud, as suggested by Hansen and Lunde (2014) in the context of volatility measurement (*Lags*). The results of the augmented GIV approach are reported in the third column of Table 6.

The upper panel of Table 6 shows the results from the baseline specification, i.e. without control variables. The first stage F statistic suggests that the GIV have a strong explanatory power for the endogenous variables in all cases. We find that the GIV correction matters, as the Wu-

Baseline

x=EUR, y=USD		z=SEK			z=NOK			z=JPY			z=GBP	
	OLS	GIV	Lags	OLS	GIV	Lags	OLS	GIV	Lags	OLS	GIV	Lags
α	0.108	0.066	0.037	0.099^{b}	-0.034	0.006	0.277^{a}	0.108	0.244^{a}	0.217^{a}	0.083	0.167^{a}
β_{EUR}	0.395^{a}	0.550^{b}	0.580^{a}	0.441^{a}	0.595^{a}	0.565^{a}	0.011	0.019	0.004	0.019	-0.020	-0.038
β_{USD}	-0.006	-0.119^{b}	-0.120^{b}	0.019	-0.002	-0.011	0.076^{a}	0.237^{a}	0.116^{a}	0.129^{a}	0.301^{a}	0.235^{a}
R^2	0.363	0.397	0.422	0.504	0.513	0.540	0.088	0.106	0.102	0.265	0.281	0.314
First-Stage F	–	276.0	197.8	–	62.81	111.9	–	10.84	227.8	–	12.19	104.5
Hausman test	–	10.46^{a}	7.71^{b}	–	9.73^{a}	23.56^{a}	–	30.04^{a}	26.16^{a}	–	17.27^{a}	61.81^{a}

	z=CAD			z=AUD			z=CHF		
	OLS	GIV	Lags	OLS	GIV	Lags	OLS	GIV	Lags
α	0.229^{a}	-0.359	0.070^{a}	0.297^{a}	0.872^{a}	0.295^{a}	0.037	0.001	0.051^{a}
β_{EUR}	0.028^{a}	0.335^{b}	0.090^{a}	0.081^{a}	-0.625^{a}	-0.008	-0.015	-0.007	0.000
β_{USD}	0.130^{a}	0.410^{a}	0.227^{a}	0.063^{a}	0.194^{b}	0.154^{a}	0.303^{b}	0.361^{a}	0.304^{a}
R^2	0.211	0.395	0.285	0.121	0.235	0.132	0.199	0.198	0.203
First-Stage F	–	12.37	101.8	–	36.26	130.9	–	19.57	172.93
Hausman test	–	239.5^{a}	32.33^{a}	–	149.6^{a}	13.75^{a}	–	7.40^{b}	15.53^{a}

Controls

x=EUR, y=USD		z=SEK			z=NOK			z=JPY			z=GBP	
	OLS	GIV	Lags	OLS	GIV	Lags	OLS	GIV	Lags	OLS	GIV	Lags
α	-0.056	-0.126	-0.124	-0.010	-0.009	-0.013	0.069^{a}	0.079^{a}	0.058^{a}	0.054	0.058^{b}	0.047^{a}
β_{EUR}	0.386^{a}	0.590^{a}	0.593^{a}	0.465^{a}	0.567^{a}	0.599^{a}	-0.002	-0.022	-0.006	-0.031	-0.116^{c}	-0.041^{c}
β_{USD}	-0.045^{b}	-0.159^{a}	-0.168^{a}	-0.021	-0.124^{a}	-0.133^{a}	0.067^{a}	0.057^{a}	0.085^{a}	0.089^{a}	0.054	0.136^{a}
R^2	0.414	0.344	0.340	0.555	0.529	0.519	0.603	0.601	0.604	0.656	0.657	0.662
First-Stage F	–	413.1	236.9	–	254.0	136.4	–	10.50	288.7	–	19.39	141.9
Hausman test	–	14.74^{a}	13.91^{a}	–	19.78^{a}	24.64^{a}	–	0.61	11.90^{a}	–	4.19	31.20^{a}

	z=CAD			z=AUD			z=CHF		
	OLS	GIV	Lags	OLS	GIV	Lags	OLS	GIV	Lags
α	0.045^{a}	-0.041^{c}	-0.017^{a}	0.070	-0.040	0.009	0.001	0.170	-0.038
β_{EUR}	0.019^{a}	0.115^{a}	0.050^{a}	0.033^{a}	1.067	0.000	0.050^{a}	0.167	-0.030
β_{USD}	0.077^{a}	0.161^{a}	0.249^{a}	-0.045^{b}	-2.389	-0.012	0.284	0.488	0.258^{b}
R^2	0.638	0.692	0.686	0.623	0.651	0.622	0.266	0.262	0.272
First-Stage F	–	64.85	55.52	–	1.85	158.1	–	2.13	118.7
Hausman test	–	51.84^{a}	58.73^{a}	–	63.14^{a}	4.30	–	4.60	2.31

Table 6: RPVE vs. liquidity regression estimates. The superscripts *a*, *b* and *c* indicate significance at 1%, 5% and 10% significance level respectively.

Hausman test rejects the null of consistency of the OLS estimates in all cases. Regarding the parameter estimates of β_{EUR} and β_{USD}, it is worth noting that at least one of the two is positive and significant at 1% level in all cases and this holds true even when using the (augmented) GIV methods. We detect a zonal configuration in the significance of the parameters. Overall, the dollar liquidity appears crucial for price efficiency. Specifically, β_{USD} is a positive and significant coefficient for CAD, CHF, JPY, and GBP suggesting that a greater trading impact of these currencies against the dollar is associated with larger mispricing. On the other hand, the liquidity of the euro seems to play an important role for the Scandinavian currencies (NOK and SEK). For them, the euro essentially replaces the dollar in establishing a connection between trading impact and price inefficiency. Overall, our results square well with the fact that the U.S. dollar is the predominant "vehicle" currency and that the euro is the relevant currency for some European countries.

The bottom panel of Table 6 shows that our findings remain qualitatively unaltered after controlling for bid-ask spread and realized Amihud in regression 6. This reinforces the idea that

β_{EUR} and β_{USD} capture the price impact of trading specific to the individual FX rates constituting the triangular arbitrage relation. Overall, the parameter estimates validate the findings illustrated in the scatter plots in Figure 6. In particular, by regressing daily RPVE on FX illiquidity, we find further evidence that liquidity begets price efficiency by limiting arbitrage deviations.

4.4 Central bank intervention and FX illiquidity

Another method to assess the validity of the proposed realized Amihud index as a powerful measurement of illiquidity is through a meaningful natural experiment. Through the lenses of the theory developed in Section 2, the announcement of the cap removal of the Swiss franc by the Swiss National Bank (SNB) on January 15, 2015, represents an ideal laboratory. Starting from September 6, 2011, the SNB set a minimum exchange rate of 1.20 francs to the euro (capping the franc's appreciation) stating that "the value of the franc is a threat to the economy," and that it was "prepared to buy foreign currency in unlimited quantities." This means that the SNB had a declared binding *cap* on the transaction price that was removed on January 15, 2015.[24]

In terms of our model, the SNB can be considered as the special $(J+1)$-th trader. The SNB intervention strategy of selling CHF for EUR in potentially unlimited quantities is implemented if the average of the reservation prices of the remaining J traders falls below the cap, that is if $\frac{1}{J}\sum_{j=1}^{J} p^{*}_{i,j} < \log(1.2)$.[25] Indeed, despite the cap on the transaction price, the reservation prices of individual traders might well be below the 1.20 threshold. For instance, the Thomson Reuters survey before the SNB cap removal suggests that the reservation prices of some agents were far below the threshold enforced by the SNB.[26] To implement it, the SNB essentially bought (sold) foreign (domestic) currency to guarantee that the transaction price remained above the threshold, that is:

$$p_i^{EUR|CHF} = \frac{1}{J+1}\sum_{j=1}^{J+1} p_{i,j}^{EUR|CHF,*} \geq \log(1.2), \tag{14}$$

where $p_{i,J+1}^{EUR|CHF,*} = \left(\log(1.2) - \frac{1}{J}\sum_{j=1}^{J} p_{i,j}^{EUR|CHF,*}\right) \times \mathcal{I}\left(\sum_{j=1}^{J} p_{i,j}^{EUR|CHF,*} < \log(1.2)\right)$ and $\mathcal{I}(\cdot)$ is the indicator function. The enforcement of the capping regime by the SNB generates extra trading volume. In particular, the trading volume is:

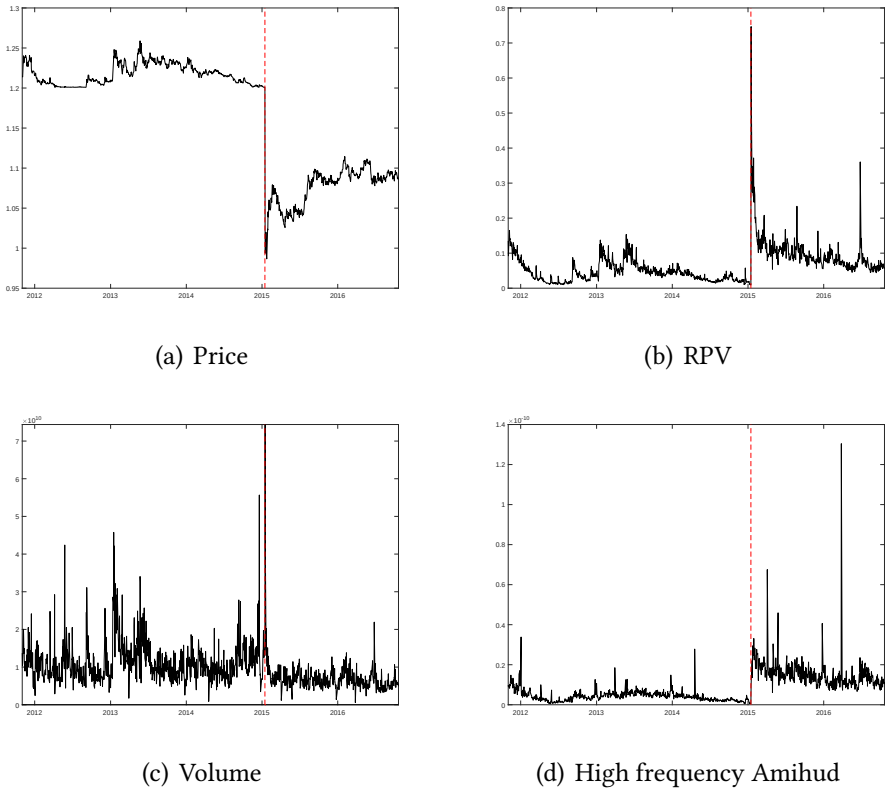
$$v_i^{EUR|CHF} = \frac{\xi^{x|y}}{2}\sum_{j=1}^{J} |\Delta p_{i,j}^{EUR|CHF,*} - \frac{1}{J}\sum_{j=1}^{J}\Delta p_{i,j}^{EUR|CHF,*}| + v_i^{SNB}, \tag{15}$$

where v_i^{SNB} is the trading volume generated by the central bank to maintain the cap on the FX

[24] The SNB announcement was mostly unanticipated by market participants, see e.g. Jermann (2017) and Mirkov et al. (2016).

[25] Breedon et al. (2018) provide empirical evidence that the SNB actually implemented this strategy by setting a considerable ask volume at 1.20. This strategy implied an enormous supply of liquidity for Swiss franc buyers at or below that threshold.

[26] The Thomson Reuters survey indicates a wide dispersion of beliefs of professional market participants around 1.20 along most of the capping period. It also indicates that before January 15, 2015, some of them believed that the exchange rate of the Swiss franc against the euro should be 1.12. These data are available on request.



(a) Price (b) RPV

(c) Volume (d) High frequency Amihud

Figure 7: Spot FX rate of EURCHF from 2012 to 2016 (a), realized power variation (b), trading volume (c) and realized Amihud (d). The announcement date of the cap removal of the Swiss franc by the SNB on January 15, 2015 corresponds to the vertical red-dashed line.

rate. Ceteris paribus, the model prescribes a low volatility of the observed returns due to the implicit constraint given by the capping and a larger volume due to FX interventions by SNB. This entails a higher market depth during the capping period, in turn implying low values of the realized Amihud before the removal of the FX capping regime and a subsequent increase once the cap is removed. This reasoning could be extended to other forms of fixed exchange rate regimes, and the results regarding the Hong Kong dollar in Table 3 are consistent with this idea.

Figure 7 provides graphical support for the prescriptions of the theoretical model. Indeed, daily volatility (RPV) is relatively low until January 15, 2015; it spikes on the day of the announcement of the un-capping, and it remains high until the end of 2016. The trading volume has the opposite behavior, being relatively high during the capping period and reverting to a lower value after January 15, 2015. Finally, the realized Amihud displays a clear upward shift after the removal of the Swiss franc cap. To provide statistical support, Table 7 reports the sample average of the main market variables before and after the cap removal. After the announcement, FX volatility significantly increases, trading volume decreases, and trading price impact measured by the realized Amihud index increases (even discarding the announcement day). Furthermore, the average trading volume size significantly decreases, suggesting a reduction in market depth. The lack of statistical significance in the change of the dispersion (standard deviations and high-low ranges, that are not tabulated) in Thomson Reuters survey of forecasts before and after the

announcement suggests that market participants did not disagree more (less) after (before) the currency cap removal. This also suggests that the liquidity dry-up after the cap removal cannot be explained by a stronger consensus regarding agents' reservation prices. All in all, the analysis

	Capping	Free floating	Test	p-value	Direction
RPV	2.4534	5.4577	22.03	0.000	⇑
Volume	1169.8	690.83	-17.07	0.000	⇓
A	0.2158	0.7990	54.74	0.000	⇑
Size	126.40	98.8660	-27.49	0.000	⇓
RPVE	0.0306	0.0423	15.19	0.000	⇑

Table 7: Sample averages of realized power variation (*RPV*), realized volatility (RV), trading volume (Volume), high-frequency Amihud illiquidity measure (*A*), and average trade size (Size) during the capping period (from November 1, 2011 to January 14, 2015) and after the capping period (from January 16, 2015 to November 30, 2016 - free floating). The un-capping announcement day (on January 15, 2015) is excluded. The variables are measured at daily frequency. RPVE is the pricing error variation on EURUSD when using CHF to construct the synthetic returns exploiting the triangular no-arbitrage relation. The variables have been rescaled. Table also reports a test for the equality of the averages in the two sub-samples, $z = \frac{m_2 - m_1}{\sqrt{v1/n1 + v2/n2}}$, and associated p-values (one-tail) calculated accounting for the auto-correlation in the data.

of this natural experiment corroborates the empirical predictions of the theory, that is, the central bank's enforcement of its reservation price leads to lower volatility, larger trading volume, and higher liquidity. By abandoning this regime, opposite patterns arise.

5 Conclusion

In this paper, we analyzed the liquidity of the global foreign exchange (FX) market. In addition to transaction costs that have already been considered in previous literature, we studied another important dimension of market illiquidity, that is, the price impact of FX trading. To do this, we proposed a refinement of the famous Amihud (2002) illiquidity measure and showed its consistency and higher accuracy theoretically and numerically. The novelty of our measure, which we call the realized Amihud index, is threefold: First, it has a straightforward interpretation as the inverse of market depth and number of market participants. Second, it utilizes realized variance and thus more accurately sizes up the return variation and price impact of trading. Third, we access a unique FX trading volume set from CLS Group representative of the global FX market. This data set allowed us to measure the transaction volume hour by hour across a relatively large set of currency pairs over a protracted period.

The following results arose from our empirical analysis: First, our measure quantifies the approximate price impact to trade one currency for another. Furthermore, our measure is highly correlated with benchmark proxies of trading price impact in the FX interdealer segment. Second, the price impact of FX trading increases with transaction cost, money market stress, uncertainty,

and risk aversion. Third, we show that currencies with smaller price impact are less prone to deviations from the triangular no-arbitrage parity. Our results are validated by the instrumental variables approach and quasi-natural experiment of the Swiss franc shock in 2015.

References

Adrian, T., Boyarchenko, N., and Shachar, O. (2017). Dealer balance sheets and bond liquidity provision. *Journal of Monetary Economics*, 89:92–109.

Amihud, Y. (2002). Illiquidity and stock returns: Cross-section and time-series effects. *Journal of Financial Markets*, 5:31–56.

Amihud, Y. and Noh, J. (2021). Illiquidity and stock returns II: Cross-section and time-series effects. *Review of Financial Studies*, 34(4):2101–2123.

Andersen, T. G. and Bollerslev, T. (1998). Answering the skeptics: Yes, standard volatility models do provide accurate forecasts. *International Economic Review*, 39(4):885–905.

Andersen, T. G., Bollerslev, T., and Diebold, F. X. (2007). Roughing it up: Including jump components in the measurement, modeling, and forecasting of return volatility. *The Review of Economics and Statistics*, 89(4):701–720.

Bandi, F. M. and Russell, J. R. (2008). Microstructure noise, realized variance, and optimal sampling. *The Review of Economic Studies*, 75(2):339–369.

Bank of International Settlements (2016). Foreign exchange and derivatives market activity in April 2016: Triennial central bank survey. Triennial Central Bank Survey.

Bank of International Settlements (2019). Triennial central bank survey of foreign exchange and OTC derivatives markets. Triennial Central Bank Survey.

Barndorff-Nielsen, O. E. and Shephard, N. (2002a). Econometric analysis of realized volatility and its use in estimating stochastic volatility models. *Journal of the Royal Statistical Society Series B*, 64:253–280.

Barndorff-Nielsen, O. E. and Shephard, N. (2002b). Estimating quadratic variation using realized variance. *Journal of Applied Econometrics*, 17(5):457–477.

Barndorff-Nielsen, O. E. and Shephard, N. (2003). Realized power variation and stochastic volatility models. *Bernoulli*, 9(2):243–265.

Bekaert, G., Hoerova, M., and Lo Duca, M. (2013). Risk, uncertainty and monetary policy. *Journal of Monetary Economics*, 60:771–778.

Berger, D. W., Chaboud, A. P., Chernenko, S. V., Howorka, E., and Wright, J. H. (2008). Order flow and exchange rate dynamics in Electronic Brokerage System data. *Journal of International Economics*, 75:93–109.

Bessembinder, H. (1994). Bid-ask spreads in the interbank foreign exchange markets. *Journal of Financial Economics*, 35:317–348.

Bjønnes, G. H. and Rime, D. (2005). Dealer behavior and trading systems in foreign exchange markets. *Journal of Financial Economics*, 75:571–605.

Bollerslev, T. and Melvin, M. (1994). Bid-ask spreads and volatility in the foreign exchange market. *Journal of International Economics*, 36:355–372.

Breedon, F., Chen, L., Ranaldo, A., and Vause, N. (2018). Judgement day: Algorithmic trading around the Swiss franc cap removal. Technical report, Bank of England Working Paper.

Breedon, F. and Ranaldo, A. (2013). Intraday patterns in FX returns and order flow. *Journal of Money, Credit and Banking*, 45(5):953–965.

Buraschi, A. and Jiltsov, A. (2006). Model uncertainty and option markets with heterogeneous beliefs. *The Journal of Finance*, 61(6):2841–2897.

Buraschi, A., Trojani, F., and Vedolin, A. (2014). When uncertainty blows in the orchard: Comovement and equilibrium volatility risk premia. *The Journal of Finance*, 69(1):101–137.

Cespa, G., Gargano, A., Riddiough, S. J., and Sarno, L. (2021). Foreign exchange volume. *Review of Financial Studies*, Forthcoming.

Chaboud, A., Rime, D., and Sushko, V. (2021). The foreign exchange market. In *The Research Handbook of Financial Markets*. Refet Gürkaynak, R. and Jonathan Wright.

Chaboud, A. P., Chernenko, S. V., and Wright, J. H. (2007). Trading activity and exchange rates in high-frequency EBS data. International Finance Discussion Papers, Board of Governors of the Reserve System and Harvard University.

Chordia, T., Huh, S.-W., and Subrahmanyam, A. (2009). Theory-based illiquidity and asset pricing. *The Review of Financial Studies*, 22(9):3629–3668.

Chordia, T., Roll, R., and Subrahmanyam, A. (2001). Market liquidity and trading activity. *Journal of Finance*, 56:501–530.

Christiansen, C., Ranaldo, A., and Söderlind, P. (2011). The time-varying systematic risk of carry trade strategies. *Journal of Financial and Quantitative Analysis*, 46:1107–1125.

Clark, P. (1973). A subordinated stochastic process model with finite variance for speculative prices. *Econometrica*, 41(1):135–55.

Corsi, F. (2009). A simple approximate long-memory model of realized volatility. *Journal of Financial Econometrics*, 7:174–196.

Corwin, S. A. and Schultz, P. H. (2012). A simple way to estimate bid-ask spreads from daily high and low prices. *Journal of Finance*, 67:719–759.

Darolles, S., Le Fol, G., and Mero, G. (2015). Measuring the liquidity part of volume. *Journal of Banking & Finance*, 50:92–105.

Darolles, S., Le Fol, G., and Mero, G. (2017). Mixture of distribution hypothesis: Analyzing daily liquidity frictions and information flows. *Journal of Econometrics*, 201(2):367–383.

Evans, M. D. (2002). FX trading and exchange rate dynamics. *Journal of Finance*, 57:2405–2447.

Evans, M. D. and Lyons, R. K. (2002). Order flow and exchange rate dynamics. *Journal of Political Economy*, 110:170–180.

Evans, M. D. and Lyons, R. K. (2006). Understanding order flow. *International Journal of Finance & Economics*, 11:3–23.

Evans, M. D. and Rime, D. (2016). Order flow information and spot rate dynamics. *Journal of International Money and Finance*, 69(1):45–68.

Fischer, A. and Ranaldo, A. (2011). Does FOMC news increase global FX trading? *Journal of Banking and Finance*, 35:2965–2973.

Foucault, T., Kozhan, R., and Tham, W. W. (2017). Toxic arbitrage. *The Review of Financial Studies*, 30(4):1053–1094.

Foucault, T., Pagano, M., and Röell, A. (2013). *Market Liquidity - Theory, Evidence, and Policy*. Oxford University Press.

Frömmel, M., Mende, A., and Menkhoff, L. (2008). Order flows, news, and exchange rate volatility. *Journal of International Money and Finance*, 27(6):994–1012.

Gabaix, X. and Koijen, R. S. (2020). Granular instrumental variables. Technical report, National Bureau of Economic Research.

Grossman, S. J. and Miller, M. H. (1988). Liquidity and market structure. *The Journal of Finance*, 43(3):617–633.

Hansen, P. R. and Lunde, A. (2014). Estimating the persistence and the autocorrelation function of a time series that is measured with error. *Econometric Theory*, 30(1):60–93.

Hartmann, P. (1999). Trading volumes and transaction costs in the foreign exchange market: Evidence from daily dollar-yen spot data. *Journal of Banking & Finance*, 23:801–824.

Hasbrouck, J. (2009). Trading costs and returns for US equities: Estimating effective costs from daily data. *The Journal of Finance*, 64(3):1445–1477.

Hasbrouck, J. and Levich, R. M. (2019). FX market metrics: New findings based on CLS bank settlement data. NBER Working paper 23206.

Hasbrouck, J. and Levich, R. M. (2021). Network structure and pricing in the FX market. *Journal of Financial Economics*, 141(2):705–729.

Hsieh, D. A. and Kleidon, A. W. (1996). Bid-ask spreads in foreign exchange markets: Implications for models of asymmetric information. In Frankel, J. A., Galli, G., and A., G., editors, *The Microstructure of Foreign Exchange Markets*, pages 41–67. The University of Chicago Press, Chicago.

Huang, R. D. and Masulis, R. W. (1999). FX spreads and dealer competition across the 24-hour trading day. *Review of Financial Studies*, 12:61–93.

Jermann, U. J. (2017). Financial markets' views about the euro-swiss franc floor. *Journal of Money, Credit and Banking*, 49:553–565.

Karnaukh, N., Ranaldo, A., and Söderlind, P. (2015). Understanding FX liquidity. *Review of Financial Studies*, 28(11):3073–3108.

Kolokolov, A. and Renò, R. (2021). Jumps or staleness? *Available at SSRN 3882775*.

Liu, L. Y., Patton, A. J., and Sheppard, K. (2015). Does anything beat 5-minute RV? A comparison of realized measures across multiple asset classes. *Journal of Econometrics*, 187(1):293–311.

Lou, X. and Shu, T. (2017). Price impact or trading volume: Why is the Amihud (2002) measure priced? *Review of Financial Studies*, 30(12):4481–4520.

Maggiori, M. (2017). Financial intermediation, international risk sharing, and reserve currencies. *American Economic Review*, 107:3038–3071.

Mancini, L., Ranaldo, A., and Wrampelmeyer, J. (2013). Liquidity in the foreign exchange market: Measurement, commonality, and risk premiums. *Journal of Finance*, 68:1805–1841.

Meddahi, N. (2002). A theoretical comparison between integrated and realized volatility. *Journal of Applied Econometrics*, 17(5):479–508.

Menkhoff, L., Sarno, L., Schmeling, M., and Schrimpf, A. (2016). Information flows in foreign exchange markets: Dissecting customer currency trades. *Journal of Finance*, 71:601–634.

Mirkov, N., Pozdeev, I., and Soderlind, P. (2016). Toward removal of the Swiss franc cap: market expectations and verbal interventions. Working Paper Series 10/16, Swiss National Bank.

Müller, U. A., Dacorogna, M. M., Davé, R. D., Olsen, R. B., Pictet, O. V., and Von Weizsäcker, J. E. (1997). Volatilities of different time resolutions - Analyzing the dynamics of market components. *Journal of Empirical Finance*, 4(2-3):213–239.

Osler, C. L., Mende, A., and Menkhoff, L. (2011). Price discovery in currency markets. *Journal of International Money and Finance*, 30(8):1696–1718.

Parkinson, M. (1980). The extreme value method for estimating the variance of the rate of return. *Journal of Business*, 53(1):61–65.

Payne, R. (2003). Informed trade in spot foreign exchange markets: An empirical investigation. *Journal of International Economics*, 61:307–329.

Pontiff, J. (2016). Costly arbitrage and the myth of idiosyncratic risk. *Journal of Accounting and Economics*, 42:35–52.

Ranaldo, A. and Söderlind, P. (2010). Safe haven currencies. *Review of Finance, European Finance Association*, 14(3):385–407.

Ranaldo, A. and Somogyi, F. (2021). Asymmetric Information Risk in FX Markets. *Journal of Financial Economics*, 140(2):391–411.

Rime, D., Sarno, L., and Sojli, E. (2010). Exchange rate forecasting, order flow and macroeconomic information. *Journal of International Economics*, 80:72–88.

Roll, R. (1984). A simple implicit measure of the effective bid-ask spread in an efficient market. *Journal of Finance*, 39:1127–1139.

Söderlind, P. (2009). Why disagreement may not matter (much) for asset prices. *Finance Research Letters*, 6(2):73–82.

Staiger, D. O. and Stock, J. H. (1994). Instrumental variables regression with weak instruments. *Econometrica*, 65(3):557–586.

Tauchen, G. E. and Pitts, M. (1983). The price variability-volume relationship on speculative markets. *Econometrica*, 51:485–505.

A Proofs

A.1 Proof of Proposition 1

By the properties of the super-position of independent SV processes,[27] the limit for $\delta \to 0$ (or $I \to \infty$) of $RPV^{x|y}$ is

$$p \lim_{I\to\infty} \delta^{1/2} RPV^{x|y} = \sqrt{\frac{2}{\pi}} \mathcal{S}_{x|y}, \tag{16}$$

where $\mathcal{S}_{x|y} = \int_0^1 \overline{\sigma}^{x|y}(s)ds$ is the integrated average standard deviation, where the latter is defined as $\overline{\sigma}^{x|y}(t) = \frac{1}{J}\sqrt{\sum_{j=1}^{J} {\sigma_j}^{x|y^2}(t)}$. The aggregate volume of $x|y$ on a unit (daily) interval is $v^{x|y} = \sum_{i=1}^{I} v_i^{x|y}$, and letting $I \to \infty$, we get

$$p \lim_{I\to\infty} \delta^{1/2} v^{x|y} = \frac{\xi^{x|y}}{2}\sqrt{\frac{2}{\pi}} \overline{\mathcal{S}}_{x|y}, \tag{17}$$

with $\overline{\mathcal{S}}_{x|y} = \frac{1}{J}\sum_{j=1}^{J}\int_0^1 \varsigma_j^{x|y}(s)ds$, where $\varsigma_j^{x|y}(t) = \sqrt{(J-1)^2 {\sigma_j}^{x|y^2}(t) + \sum_{s\neq j} {\sigma_s}^{x|y^2}(t)}$.

Hence, we get that

$$p \lim_{I\to\infty} A^{x|y} = \frac{2\mathcal{S}_{x|y}}{\xi^{x|y}\overline{\mathcal{S}}_{x|y}}, \tag{18}$$

which reflects the ratio of the total average standard deviation carried by each trader. Under homogeneity of the traders, i.e. $\sigma_j^{x|y}(t) = \sigma^{x|y}(t)$, $\varsigma_j^{x|y}(t)$ reduces to $\sqrt{J(J-1)}\sigma^{x|y^2}(t)$, and we get that

$$\overline{\mathcal{S}}_{x|y} = J\sqrt{J-1}\mathcal{S}_{x|y}, \tag{19}$$

so that Proposition 1 follows directly.

B Alternative illiquidity measures

Liquidity measures are computed using the EBS data set from January 1, 2016, to December 31, 2016. In a first step, we separate trades and quotes data. We then exclude data from Friday 10 PM to Sunday 10 PM GMT. Subsequently, the multi-level order book is collapsed such that we have a single observation for each snapshot. To this end, we select both the best available quotes, that is the highest bid and the lowest ask, and compute volume-weighted ask and bid quotes in order to keep track of the order book depth. Notice that after the first step, we obtain an irregularly spaced time series. This is because EBS includes new observations only whenever new information becomes available. Hence, missing values are replaced by the previous (last available) observation so as to obtain evenly space time stamps (frequency: one tenth of a second).

For each day, we compute the following set of illiquidity measures: realized Amihud, effective

[27] Similarly to Barndorff-Nielsen and Shephard (2002b), $\overline{\Delta p_i}^{x|y,*}(t) = \frac{1}{J}\sum_{j=1}^{J}\Delta p_{i,j}^{x|y,*}$ is equivalent in law to $\overline{\Delta p_i}^{x|y,*} = \int_{\delta(i-1)}^{\delta i} \overline{\sigma}^{x|y}(t)dW^{x|y,*}(t)$, where $\overline{\sigma}^{x|y}(t) = \frac{1}{J}\sqrt{\sum_{j=1}^{J} {\sigma_j}^{x|y^2}(t)}$.

cost, bid-ask spread, relative bid-ask spread, Corwin & Schlutz, Roll estimate, price impact and traditional (low-frequency) Amihud. Intra-day quoted (relative) spread and effective cost are first computed at a high frequency. Afterward, daily averages are computed. Notice that the effective cost is defined as the absolute distance between the transaction price and the prevailing mid price.

The traditional (low-frequency) Amihud measure is defined as the absolute value of the daily log-return divided by the total trading volume in that day, again using trade data. Roll's measure is computed for each day as the daily autocovariance between consecutive price changes, that is from trade to trade. The measure is computed using high-frequency trade data, which are not generally evenly spaced. No adjustments for the trade direction have been made. We also consider an alternative (low-frequency) Amihud measure, which is based on the daily range by Parkinson (1980), i.e. $A_t^{**} = \frac{\sqrt{RG_t}}{v_t}$, where $RG_t = \frac{1}{4\log(2)}(p_t^H - p_t^L)^2$ is the daily range.

The Corwin & Schultz (CS) spread estimator has been computed for each day using hourly maximum and minimum prices. Hourly closing prices have been defined as the last observation available for a given hour. A daily measure is then extracted by averaging out the 23 hourly spread estimates. Notice, however, that both measures have been initially proposed as a monthly spread estimator, which is obtained with daily max. and min. price series. Therefore, intra-day seasonalities might potentially affect the measurement. The price impact coefficient, denoted as λ, is computed for each day in a trade-by-trade fashion. We regress the signed square root of the volume against the log-returns, that is

$$r_t = \lambda \cdot sign \cdot \sqrt{vol_t}, \tag{20}$$

where *sign* takes on the value -1 or 1 depending on whether the trade was seller- or buyer-initiated.[28] This also conveys the definition adopted for order flow. Recall also that time intervals between consecutive deals are not constant.

[28] More precisely, according to the EBS data manual for deal records, a 0 indicates that the transaction is seller-initiated. Conversely, a 1 corresponds to buyer-initiated trades. To compute signed volumes, they are mapped to -1 and 1, respectively.