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Stress-testing for portfolios of commodities

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Outlook

- We offer a case study that sheds light on the Basel III financial regulations on stress testing:
 - How to stress test a portfolio?
 - Why does Basel III criticize the overreliance on historical prices and correlations?
- We use the methods encouraged by Basel III:
 - **Extreme Value Theory (EVT)** for the risk factors
 - **Copula** analysis for joint dynamics of portfolio risk factors

Data

- We used daily logarithmic returns from 01 January 1998 to 31 December 2011.
- In the case of the reference index the provider rolls the futures contracts over four times a year, depending on the most liquid contracts trading on a particular commodity. For simplicity we used contracts with approximately one year maturity, that are rolled six months before the expiration.

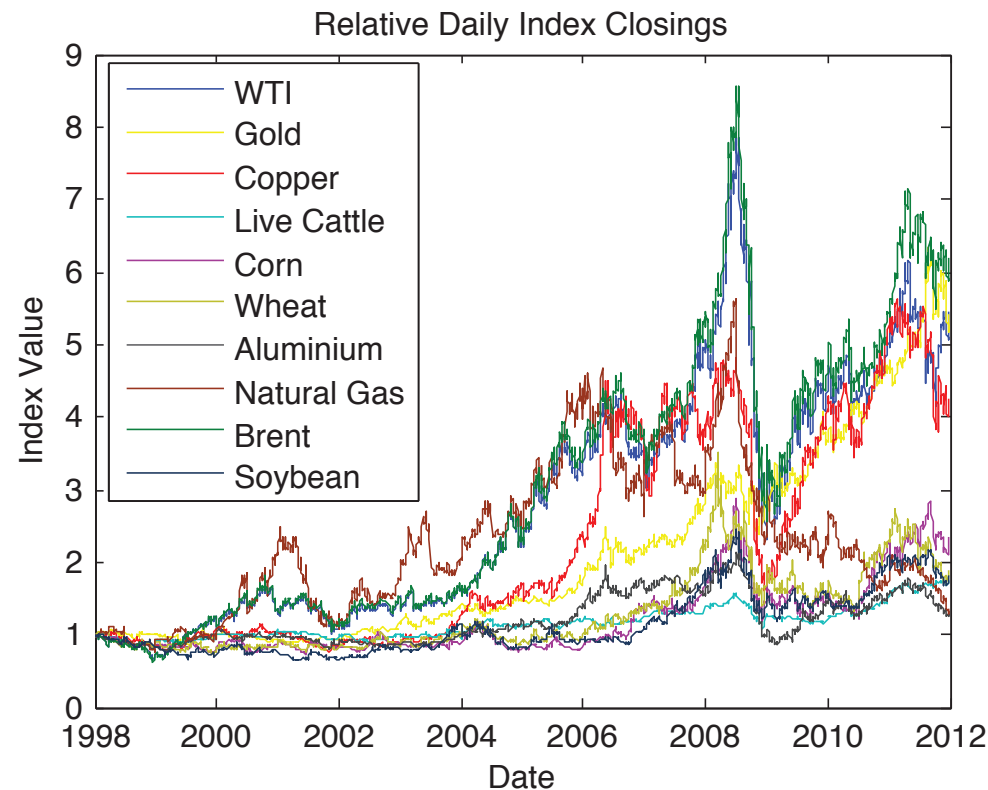


Figure 1: Historical daily price movements in relative value

Model choice for the risk factors

The descriptive statistics of commodity return series can be summarized as follows:

- Fat tails
- Positive skewness in general (“inventory effect”)
- Significantly higher kurtosis than the normal distribution
- Stationary returns
- Systematic volatility clustering pattern

The risk factors are modeled by an ARMA(1,1)-GARCH(1,1) GJR model:

$$y_t = \mu + \phi_1 y_{t-1} + \xi \varepsilon_{t-1} + \varepsilon_t \quad (1)$$

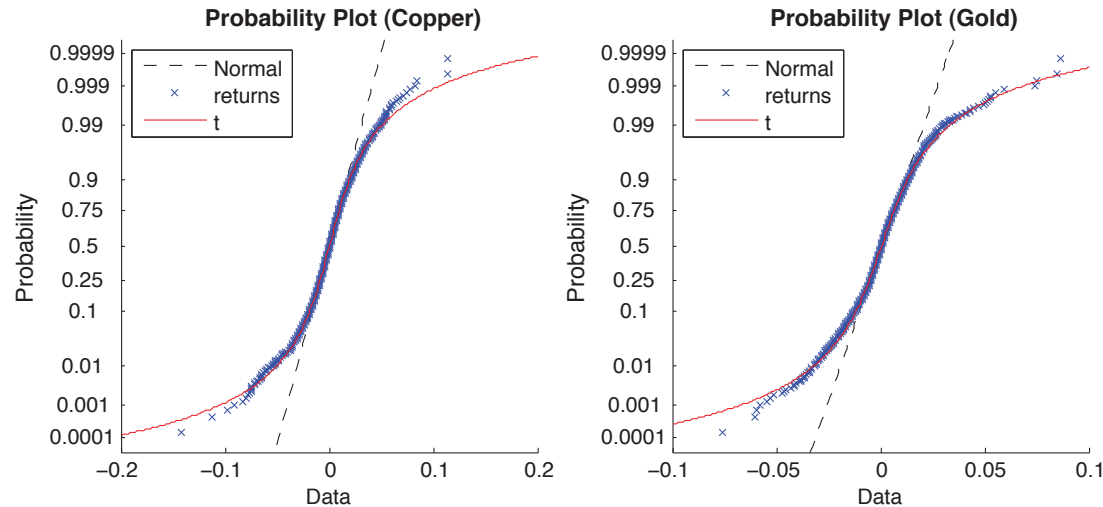
$$h_t^2 = a_0 + a_1 \varepsilon_{t-1}^2 + a_2 h_{t-1}^2 + b \psi(\varepsilon_{t-1}) \varepsilon_{t-1}^2 \quad (2)$$

- h_t^2 is the conditional variance of ε_t , $z_t = \varepsilon_t/h_t$, with $z_t \sim N(0, 1)$ or Student's t -distributed.

- An indicator function is introduced: $\psi(\varepsilon_{t-1}) = 1$ if ε_{t-1} (or z_{t-1}) is negative, or 0 if ε_{t-1} (or z_{t-1}) is positive.
- As there is no restriction on the sign of b , the model can be applied to describe both negatively or positively skewed data. We determine the model parameters by maximum likelihood estimation.

Model choice for the risk factors

- The model strongly underestimates extreme events:



- But exactly the extreme returns are the reason for stress testing
- Solution: we model the extremes with EVT (in accordance with Basel III).

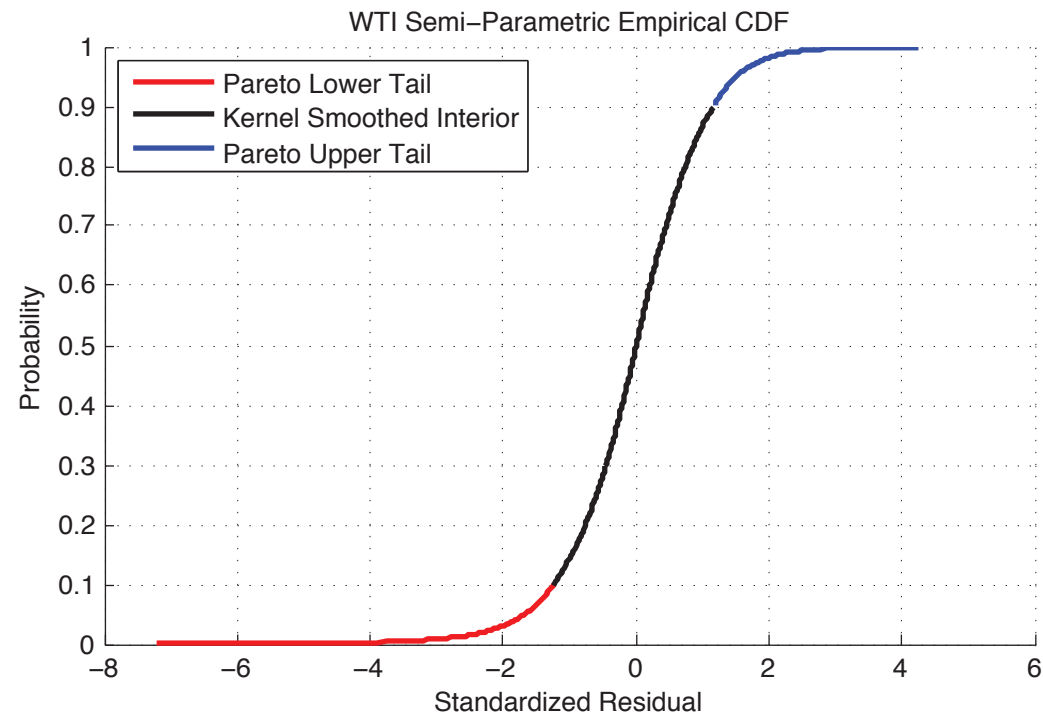
Modeling extreme events

- The standardized residuals z_t are modeled by a combined approach: kernel smooth interior (for the center of the distribution) and Pareto distribution for the tails (where data is scarce)
- We used the Hill's estimator to define the threshold where the extreme tail starts (for each commodity). On average, we looked at the upper and lower 10% tails
- The notation for a generalized Pareto (GP) distribution is introduced for any $\xi \in \mathbf{R}, \beta \in \mathbf{R}_+$:

$$GP_{\xi,\beta}(z) = 1 - \left(1 + \xi \frac{z}{\beta}\right)_+^{-\frac{1}{\xi}}, z \in \mathbf{R}$$

where item $1/\xi$ is known as the *tail index* and β a *scaling* parameter. We fit GP to the z_t standardized residual series that exceed the chosen threshold.

Semi-parametric CDF



Modeling joint extremes

- So far, we showed how we modeled the risk factors individually. However, for a realistic portfolio stress testing, **the evolution of dependency structures among the considered commodities** is of great importance.
- It is expected and empirically observed that in times of market stress, joint extreme returns occur in commodity markets. We therefore model joint positive or negative returns with a t -copula. In the case of t -distributions the d -dimensional t -copula with ν degrees of freedom is given by:

$$C_{\nu, \Sigma}^t(u) = t_{\nu, \Sigma}(t_{\nu}^{-1}(u_1), \dots, t_{\nu}^{-1}(u_d)) \quad (3)$$

where Σ is a correlation matrix, t_{ν} is the cumulative distribution function of the one dimensional t_{ν} distribution and $t_{\nu, \Sigma}$ is the cumulative distribution function of the multivariate $t_{\nu, \Sigma}$ distribution. We apply the semi-parametric maximum likelihood estimation to determine the parameters of the t -copula.

Stress scenarios

- **Historical scenario:**

- We construct the P&L of our portfolio for the next 22 day horizon based on the returns of the risk factors empirically observed during the financial crisis: 28 March 2008 – 31 March 2010
- Non-parametric method: no assumption for the dependence structure among the risk factors, no distributional assumption for the risk factors

- **Hybrid scenario:**

- The ARMA(1,1)-GARCH(1,1) GJR with EVT processes of the different commodity returns and the t -copula are calibrated on the financial crisis data ranging from 28 March 2008 to 31 March 2010
- Based on these parameters, the risk factors are simulated for the next 22 days, 10'000 scenarios, and the P&L is constructed

- **Baseline scenario (for comparison):**

- Aims at estimating the portfolio performance at the end of the 22 day period, without the impact of stress
- The ARMA(1,1)-GARCH(1,1) GJR with EVT processes of the different commodity returns and the t -copula are fitted to the entire data sample

Estimation results

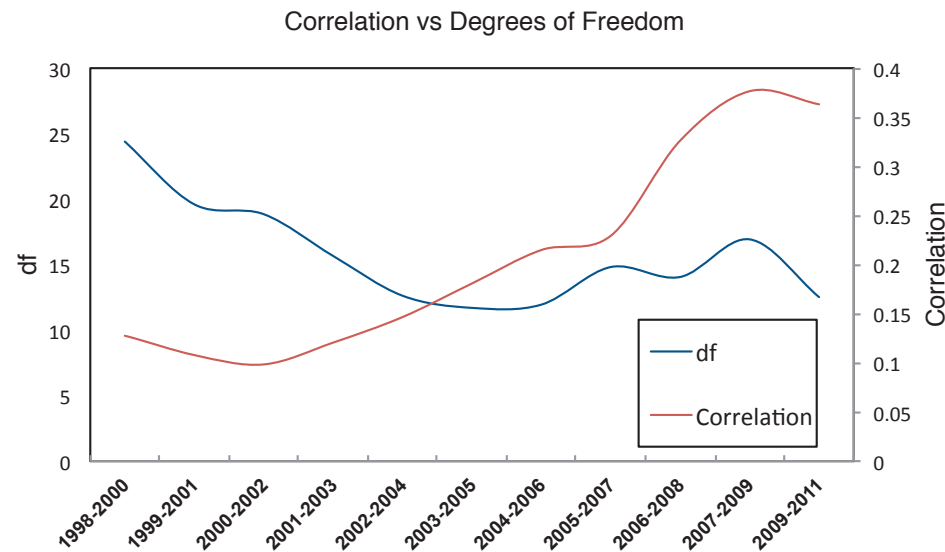
- **Drawbacks of the historical scenario:** limited time span, extreme quantiles are difficult to estimate; dependence structure among the risk factors is neglected and thus generally loss quantiles are underestimated
- **Hybrid scenario:** extrapolates beyond the historical data, "**main feature of a realistic stress testing technique**" (Basel III)
- **Joint extremes:** Surprisingly, higher degree of freedom for the t -copula with the hybrid scenario than in the case of the baseline scenario!! Joint extremes are less probable during market stress??

Metric	Baseline	Hybrid	Historical
Degrees of Freedom	12.79	15.25	N/A
Max. Simulated Loss	-37.86%	-64.08%	-32.93%
Max. Simulated Gain	30.57%	58.45%	14.49%
Simulated 90% VaR	-4.67%	-13.31%	-12.19%
Simulated 95% VaR	-6.35%	-17.96%	-16.56%
Simulated 99% VaR	-9.64%	-29.83%	-28.31%
Simulated 90% ES	-6.92%	-19.73%	-18.06%
Simulated 95% ES	-8.40%	-24.10%	-21.86%
Simulated 99% ES	-11.58%	-34.10%	-31.17%
Simulated 99.9% ES	-16.93%	-49.26%	N/A
Simulated 99.99% ES	-29.38%	-63.78%	N/A

Table 1: Metrics for hybrid and historical stress scenarios

Disentangling the contagion effect

- The tail dependence structures among commodities weakened during the financial crisis.
- Until 2006, we conclude that commodity returns became more correlated with each other, and joint extremes are more likely. However, during the boom and bust cycle of 2007–2009, and further during the 2008–2010 window, although correlations increased among commodities, we observe an increase in the degree of freedom as well
- Explanation: different dynamics among commodity prices during the financial crisis. Some underwent a relatively moderate growth and fall (agricultural commodities), while others, (oil, gas, copper) went through a massive boom and bust cycle



Conclusion

- Our results show that the reliance on standard assumptions like the increase in the probability of joint extremes among financial assets in times of market stress is not always realistic
- By contrary, we show that there have been structural breaks in commodity markets that temporarily led to a breakdown of expected statistical patterns, like tail dependence structures
- This fact should be explored by risk managers in performing hypothetical scenarios, by shocking arbitrary combinations of market factors, volatilities, and dependence structures
- The pure reliance on historical assumptions can have serious limitations for stress testing