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**CYCLICAL LONG-TERM UNEMPLOYMENT, SKILL LOSS,
AND MONETARY POLICY**

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Abstract

Movements in long-term unemployment (LTU) exhibit a substantial cyclical component. I document a number of stylized facts and develop a business cycle model featuring labor market frictions and skill loss during unemployment to reproduce the findings in the data. I find that the skill loss mechanism helps to capture volatility patterns across macroeconomic variables, negative duration dependence, and the behavior of the LTU proportion among total unemployment around business cycle turning points. Optimal monetary policy in the presence of skill loss allows for a higher volatility of inflation after productivity shocks to reduce skill deterioration and mitigate consumption losses.

Keywords: Business cycles, labor market frictions, long-term unemployment, optimal monetary policy

JEL code: E24; E32; E52; J24

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1 Introduction

In this paper, I present a sticky price business cycle model which tracks durations in unemployment and introduces duration dependent skill loss during unemployment. I show that the model can reproduce several stylized facts about the cyclical component of long-term unemployment (LTU hereafter), defined as unemployment that lasts longer than four quarters. In particular, (i) hiring rates for long-term unemployed are lower than those for shorter-term unemployed (often called duration dependence); (ii) LTU is more volatile than total unemployment, which in turn is more volatile than the LTU proportion (the share of LTU among total unemployment), which in turn is more volatile than output; (iii) while the unconditional correlation between total unemployment and the LTU proportion is positive, the correlation around business cycle turning points is temporarily negative, meaning that the LTU proportion lags behind total unemployment. While stylized fact (i) is documented in the literature — for example, in Jackman and Layard (1991), Machin and Manning (1999), or Nickell (1979) — I use quarterly European data on LTU, total unemployment, and output to document the stylized facts (ii) and (iii).

In the model I use to reproduce those facts — which is based on the model in Lechthaler et al. (2010) — unemployed workers have different skill levels, depending on the duration of their unemployment spell. Unemployed workers are randomly assigned to firms at the beginning of a period. For their employment decisions, intermediate goods producers take into account skill differences between previously unemployed workers, a random operating cost shock hitting each firm-worker pair as well as hiring and firing costs. The influence of skill differences on the marginal revenue products of firms leads to different hiring thresholds for the operating cost shock depending on the previous duration of unemployment. This results in lower hiring rates for long-term unemployed workers and a lower volatility of the LTU proportion, in line with empirical data.

The unconditional correlation between total unemployment and the LTU proportion in the model is positive. As total unemployment increases during a recession, LTU increases stronger because hiring rates for all durations decrease. This implies that fewer people leave LTU and more and more shorter-term unemployed fall into LTU over time. As a consequence, the LTU proportion increases with total unemployment. However, at the beginning of a recession, an increasing firing rate creates a larger pool of shorter-term unemployed workers and hence a lower LTU proportion. Additionally, because of the effect of skill loss on the hiring thresholds, LTU lags total unemployment over the business cycle. Both effects imply that total un-

employment and the LTU proportion exhibit a negative correlation around business cycle turning points, as also pointed out by Machin and Manning (1999).

Based on the observation that skill loss helps to explain the stylized facts and the model bears empirical relevance, I examine optimal monetary policy in the presence of skill loss. Faia et al. (forthcoming) show that implementing the flexible price allocation is not optimal if firms face hiring and firing costs because of inefficient unemployment fluctuations. Hence, the monetary authority faces a trade-off between stabilizing inflation and stabilizing unemployment. I show that skill loss accentuates this trade-off. In response to a productivity shock, the monetary authority accepts more inflation variability to reduce skill deterioration in the workforce and losses in production and consumption possibilities.

To analyze these policy issues, I put the above described labor market framework in a general equilibrium context. The household consists of the different types of workers and decides on its optimal consumption and saving behavior. Households supply labor to intermediate firms which produce intermediate goods. Monopolistically competitive retailers buy these intermediate goods, transform them into final goods and sell these at a markup over marginal costs. Sticky prices are introduced through price adjustment costs. They allow for a meaningful role of monetary policy in the model. To reproduce the stylized facts, I assume that the central bank follows a Taylor rule. I calibrate the model to German data and solve it using perturbation methods.

The motivation to study the cyclical behavior of LTU and its policy implications comes from the recent developments in LTU. While high levels of LTU have long been an issue in European countries, the sharp increases in some countries in the course of the recent economic crisis suggest a substantial cyclical component in LTU. Figure 1 shows total unemployment rates in several European countries while figure 2 shows the corresponding LTU proportions. It is evident that a strong increase in total unemployment — as, for example, in Spain or Greece — is accompanied by a strong increase in LTU proportions, while a rather moderate development in total unemployment — as, for example, in Germany or France — is accompanied by a moderate development in the LTU proportion, illustrating the cyclicity of the latter.

This cyclicity in LTU has been recognized by researchers and policymakers. For example, Katz (2010, p. 4) remarks that "[...] cyclical problems swamp structural problems in terms of the source of unacceptably high overall and long-term unemployment [...]" and Bernanke (2012, p. 1-2) notes that "[...] while both cyclical and structural forces have doubtless contributed to the increase in long-term

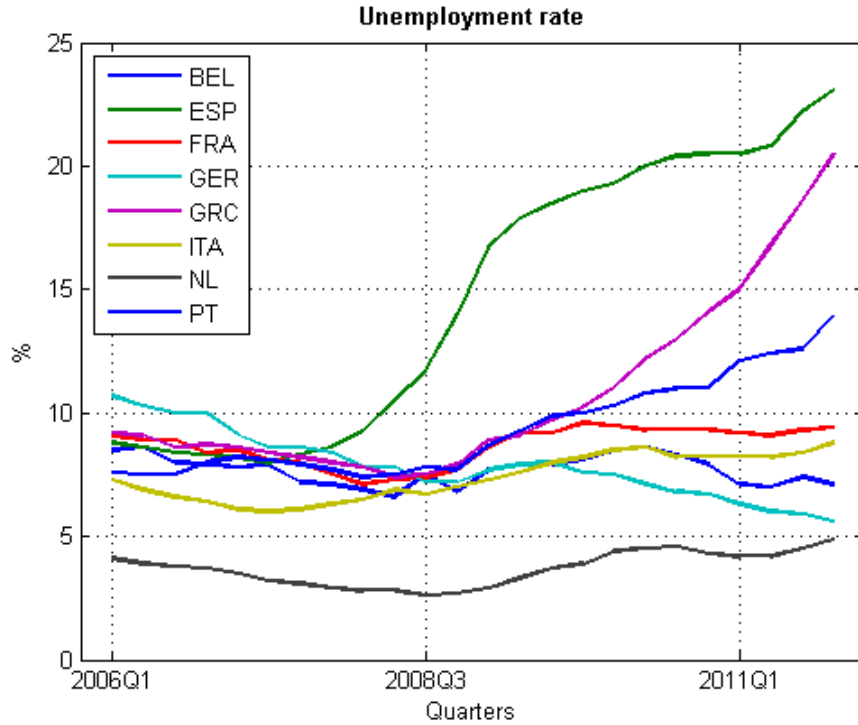


Figure 1: Total unemployment rates in Belgium (BEL), Spain (ESP), France (FRA), Germany (GER), Greece (GRC), Italy (ITA), Netherlands (NL) and Portugal (PT). Notes: Quarterly data are taken from the OECD and range from 2006Q1 to 2011Q4.

unemployment, the continued weakness in aggregate demand is likely the predominant factor.” If this is indeed the case, monetary policy can play a role in mitigating the negative impact of LTU on the economy.

The paper is related to several strands of the literature. Shimer (2005) compares the cyclical properties of the search and matching model in the tradition of Mortensen and Pissarides (1994) to the data and concludes that the model cannot reproduce the high volatility of labor market variables in the US (the “Shimer-puzzle”). Gartner et al. (2012) verify these findings for Germany. Hall (2005) proposes sticky wages as a mechanism to overcome this deficiency of the standard search and matching labor market model. Gertler and Trigari (2009) endogenize wage stickiness by assuming that not all firms and workers can renegotiate wages each period. An alternative solution to the Shimer-puzzle is presented by Brown et al. (forthcoming). These authors show that hiring and firing costs, in conjunction with operating cost shocks for firm-worker pairs, drive a wedge between the hiring and the retention rate, and in this way introduce realistic amplification patterns for macroeconomic shocks. Because of the good performance of their framework

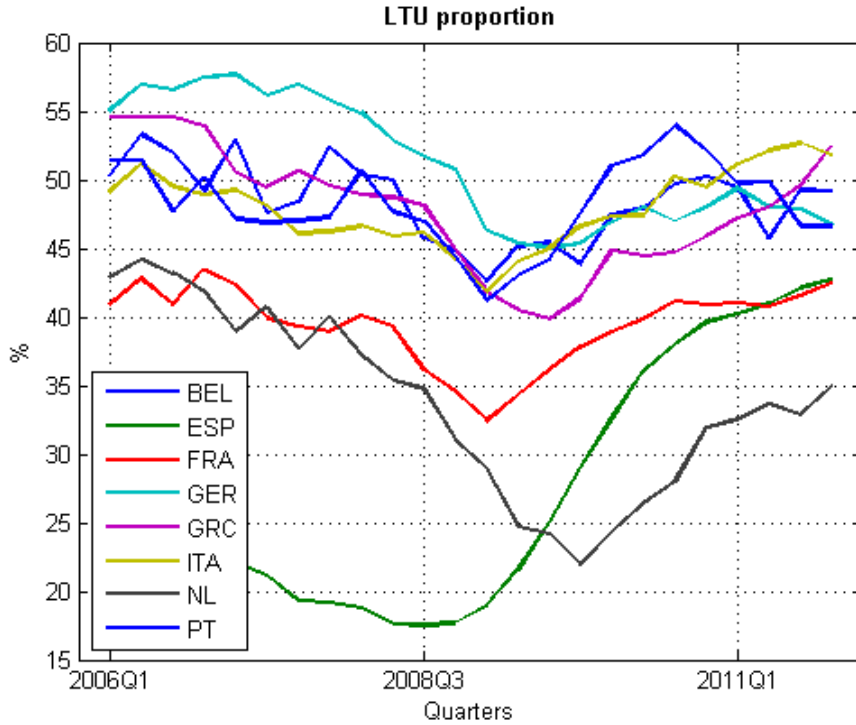


Figure 2: LTU proportion in Belgium (BEL), Spain (ESP), France (FRA), Germany (GER), Greece (GRC), Italy (ITA), Netherlands (NL) and Portugal (PT). Notes: Quarterly data are taken from the OECD and range from 2006Q1 to 2011Q4. The LTU proportion is the proportion of long-term unemployed workers among total unemployment.

regarding empirical relevance, I use the same labor market setup. My contribution is to analyze the cyclical behavior of LTU in this framework.

A number of recent papers, such as Krause and Lubik (2007), Thomas (2008), or Walsh (2003), have incorporated labor market frictions into general equilibrium monetary models in order to study labor market reactions to monetary shocks and policy. These papers adopt search and matching frictions as the basic labor market framework. Lechthaler et al. (2010) instead use the labor turnover cost approach of Brown et al. (forthcoming) described above. They show that, while the desirable properties for labor market variables generated by labor turnover costs are retained in general equilibrium, they additionally help to explain persistence and amplification properties of output in response to monetary and productivity shocks. Thus, the general equilibrium framework presented in section 3 will be based on Lechthaler et al. (2010). My paper adds the distinction between different unemployment durations and introduces skill loss during unemployment.

The idea that skill loss is a distinguishing feature between short- and long-term unemployment is not new. Ljungqvist and Sargent (1998) use skill loss to explain

how welfare states can slip into high unemployment regimes during turbulent times.¹ While their study is targeted at unemployment developments in the long run, I focus on cyclical movements in LTU. Pissarides (1992) shows in an overlapping generations model that skill loss during unemployment creates persistence in employment after macroeconomic shocks. A recent study by Laureys (2012) examines the implications of skill depreciation for optimal labor market policy. What sets my analysis apart from these studies is the nominal general equilibrium framework in which I can ultimately investigate monetary policy issues. Moreover, my research object is different from Laureys (2012) and Pissarides (1992). I seek to explain stylized business cycle facts about LTU with the help of skill loss. By contrast, Pissarides (1992) focuses on the persistence of employment and does not analyze the cyclical behavior of LTU. Similarly, Laureys (2012) does not relate the effects of skill loss to the behavior of LTU; in fact, her study does not distinguish between short- and long-term unemployment. Moreover, as described above, with the search and matching framework used in these papers, it is problematic to reproduce empirical observations about unemployment dynamics.

A recent study by Esteban-Pretel and Faraglia (2010) combines search and matching frictions and skill loss in a sticky price model and examines the role of skill loss for aggregate variables when monetary shocks hit the economy. Again, my analysis distinguishes itself from their paper regarding the labor market setting which allows for more realistic amplification patterns. I provide a systematic comparison of model outcomes to business cycle statistics. Moreover, I draw on Khan et al. (2003), Levin et al. (2006), and Schmitt-Grohe and Uribe (2004) to examine Ramsey optimal monetary policy in the presence of skill loss. To the best of my knowledge, this has not been done before.

The paper proceeds as follows: In the next section, I present stylized facts about the cyclical behavior of LTU. Section 3 presents the model and section 4 the calibration. Sections 5 and 6 compare the model predictions with the stylized facts from the data and examine the role of skill loss in explaining the stylized facts, respectively. Section 7 examines optimal monetary policy in the presence of skill loss. Section 8 concludes.

2 Stylized facts on long-term unemployment

Table 1 shows relative standard deviations, correlations and autocorrelations for output, total unemployment, the LTU proportion and LTU in several continental

¹This view is challenged by den Haan et al. (2005).

	BEL	ESP	FRA	GER	GRC	ITA	NL	PT
<i>Rel. std.</i>								
y	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
u^L/u	3.94	7.24	3.20	3.80	1.46	3.02	8.32	4.07
u^L	8.07	13.35	5.53	10.29	5.33	8.28	12.82	10.35
u	6.81	7.93	4.85	7.01	4.76	5.02	9.15	7.09
<i>Corr.</i>								
y, u	-0.52	-0.93	-0.76	-0.33	-0.83	-0.82	-0.56	-0.90
$y, (u^L/u)$	0.37	-0.91	0.27	0.38	0.36	-0.21	0.44	-0.34
y, u^L	-0.06	-0.94	-0.17	-0.14	-0.63	-0.67	-0.15	-0.39
$u, (u^L/u)$	0.13	0.83	0.24	0.49	0.25	0.49	0.16	0.48
u, u^L	0.78	0.86	0.75	0.92	0.97	0.87	0.73	0.57
<i>Autocorr.</i>								
y	0.92	0.98	0.97	0.89	0.95	0.95	0.95	0.96
u	0.73	0.98	0.83	0.89	0.83	0.91	0.93	0.91
u^L/u	0.81	0.98	0.91	0.98	0.97	0.97	0.95	0.91
u^L	0.91	0.98	0.96	0.96	0.97	0.98	0.94	0.96

Table 1: Unconditional moments of labor market variables and output in Belgium (BEL), Spain (ESP), France (FRA), Germany (GER), Greece (GRC), Italy (ITA), Netherlands (NL) and Portugal (PT). Notes: Quarterly data with different starting dates until 2011Q4 are taken from the OECD. u is the total unemployment rate and u^L/u is the LTU proportion among total unemployment. Unemployed workers are classified as LTU if their unemployment spell lasts more than four quarters. Output y is real GDP in USD with fixed PPPs. Variables are seasonally adjusted. All statistics are calculated using the cyclical component of the respective variable, defined as the deviation from the trend component of the HP filter. For relative standard deviations (rel. std.), percentage deviations from the trend component of the HP filter were used. Relative standard deviations are standard deviations divided by the standard deviation of output. The smoothing parameter for the HP filter is set to 100000 as in Shimer (2005).

European countries based on quarterly OECD data.² I present European data since my model in section 3 will feature characteristics often associated with European economies.

We can see several characteristic features of the data: Looking at relative standard deviations,³ we see a volatility pattern for all countries. The most volatile variable is LTU, followed by total unemployment. The LTU proportion, in turn is less volatile than total unemployment but still more volatile than output.

²For each country the end of the sample is 2011Q4. The beginning of the sample for output and total unemployment is 1995Q1; for LTU and the LTU proportion, it depends on data availability. Specifically, the first observations, respectively, are: Belgium (BEL): 1999Q1; Spain (ESP): 1992Q1; France (FRA): 2003Q1; Germany (GER): 1999Q1; Greece (GRC): 1998Q1; Italy (ITA): 1992Q4; Netherlands (NL): 2002Q2; Portugal (PT): 1992Q1. All variables were obtained seasonally adjusted, except LTU which I seasonally adjusted with the Census X-12 procedure. Following Shimer (2005), I apply the HP filter with smoothing parameter 10^5 to extract the cyclical component of the variables.

³A value bigger than 1 indicates higher a volatility in the respective variable than in output.

Turning to correlations, we see a less consistent picture. The correlations between output and total unemployment (negative), output and LTU (negative), total unemployment and the LTU proportion (positive) as well as total unemployment and LTU (positive) have the the same sign across all countries. In contrast, the signs of the correlation between output and the LTU proportion vary across countries. Regarding autocorrelations, all variables exhibit a similarly high degree of persistence across countries.

Moreover, there is an interesting feature in the relationship between total unemployment and the LTU proportion over the business cycle, pointed out by Machin and Manning (1999) for annual data. While the general relationship — as seen in row 4 of the correlations panel in table 1 — is positive, there seems to be a temporary negative relationships at the turning points of the business cycle. This means that when the economy starts to contract (expand), total unemployment rises (decreases) while the LTU proportion still declines (increases). In other words, the LTU proportion lags behind the total unemployment rate. Plotting and connecting the observations of these two variables results in a counter-clockwise movement when following the line through time. Figure 3 shows such a scatter plot for the most recent recession around 2008 and the following recovery.⁴ The described temporary negative correlation between total unemployment and the LTU proportion is most clearly visible for the turn into a recessionary period in or after the beginning of 2008. Note that the negative correlation is again visible for the turn into the recovery for countries which actually had one by the end of 2011 like France or Germany. For countries without a recovery like Greece or Spain, the slope stays positive after 2008.

Another characteristic of LTU is what the literature calls negative duration dependence. There are several studies which confirm that unemployed workers' hiring probability declines with higher duration in unemployment. Examples are Nickell (1979) and Jackman and Layard (1991) for the UK, Machin and Manning (1999) for several European countries and Aaronson et al. (2010) for the US. There has been a debate in this literature whether the lower hiring probabilities for longer-term unemployed reflect "true" duration dependence or merely unobserved heterogeneity. Machin and Manning (1999) make the argument that unobserved heterogeneity may itself be responsible for duration dependence by means of stigmatization of long-term unemployed on behalf of employers.

⁴The pattern is also visible for other turning points but for the sake of lucidity only the most recent period is shown.

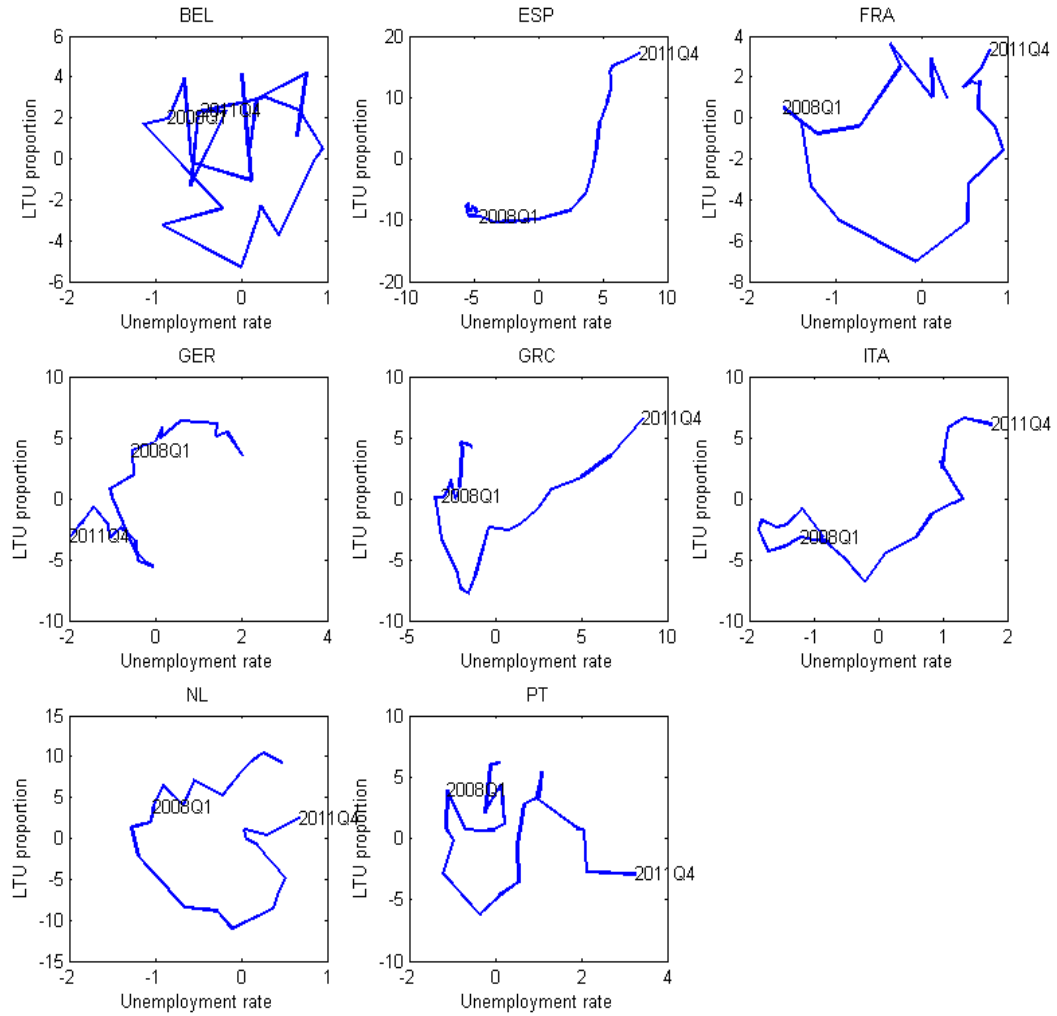


Figure 3: LTU proportion and total unemployment around business cycle turning points in Belgium (BEL), Spain (ESP), France (FRA), Germany (GER), Greece (GRC), Italy (ITA), Netherlands (NL) and Portugal (PT). Notes: Quarterly data are taken from the OECD. Shown are scatter plots of the cyclical components of the LTU proportion and the total unemployment rate from 2006Q1 to 2011Q4. This time period is meant to include the most recent recession and recovery, where applicable.

3 Model

In this section I present a model which explicitly takes into account LTU and duration dependent skill loss during unemployment. The model is then used to extract cyclical properties of LTU, total unemployment, and output, and compare them to the stylized facts presented above.

As a basic framework, I use a discrete time sticky price model with a frictional labor market. One time period in the model refers to one quarter. There are four types of agents: households, intermediate firms, retail firms and the monetary authority.

The labor market features frictions in the form of hiring and firing costs as in Brown et al. (forthcoming) and Lechthaler et al. (2010). These authors show that labor turnover costs can account better for labor market and business cycle stylized facts than the basic matching function approach in the tradition of Mortensen and Pissarides (1994). The main reason for the improved amplification and persistence effects is that hiring and firing costs drive a wedge between the job-finding rate and the job retention rate. In contrast, the search and matching framework implies that these two rates are equal, conditional on a match. In a partial equilibrium setting, Brown et al. (2011) show that the labor market framework with linear hiring and firing costs is particularly useful for considering heterogeneity in the duration of unemployment. I extend their work to a general equilibrium setting.

Households consume differentiated goods, save in bonds, and supply labor to intermediate firms. Members of the household are either employed or unemployed. If they are unemployed, they can be short-term unemployed (that is, they have been unemployed for up to four quarters) or long-term unemployed (that is, they have been unemployed for more than four quarters). If workers are employed, they can be incumbents (that is, they were employed during the previous period), hired out of short-term unemployment, or hired out of LTU. Employed workers are more productive than short-term unemployed workers, who in turn are more productive than long-term unemployed workers. These differences in productivities reflect skill loss processes during unemployment spells as emphasized in Pissarides (1992) or Ljungqvist and Sargent (1998).

Intermediate firms employ labor to produce intermediate goods. Unemployed workers are assigned randomly to intermediate firms at the beginning of a period. Worker-firm pairs are subject to idiosyncratic random operating costs each period which affect the profit generated by a worker. Intermediate firms make hiring and firing decisions according to the expected stream of profits taking into account hiring and firing costs and the skill level of workers.

Retail firms act under monopolistic competition and face price adjustment costs as in Rotemberg (1982). They sell their differentiated products to the households.

For the comparison of model outcomes to the data, the central bank is guided by a standard Taylor rule. Later, in section 7, the Taylor rule is replaced by Ramsey optimal policy.

3.1 Households

There is a continuum of agents in the economy represented by the unit interval. I follow Merz (1995) and assume that all agents belong to a large family-household. In this way, household members insure themselves against income risks arising from heterogeneous employment statuses across agents and time.

There is a continuum of differentiated final goods in the interval $[0, 1]$ produced by retail firms. The quantity of the i th good consumed in period t is represented by $C_t(i)$. Agents consume a Dixit-Stiglitz aggregator C_t of these differentiated goods, where $C_t \equiv \int_0^1 [C_t(i)^{\frac{\epsilon-1}{\epsilon}} di]^{\frac{\epsilon}{\epsilon-1}}$. $\epsilon > 1$ is the elasticity of substitution between final goods. The household faces the following problem in each t :

$$\max_{C_t(i)} C_t \quad \text{s.t.} \quad Q_t = \int_0^1 P_t(i) C_t(i) di, \quad (1)$$

where $P_t(i)$ is the price of the differentiated good i . That is, the household maximizes consumption by deciding on the optimal expenditure on the differentiated goods given an expenditure level Q_t . Combining the first order conditions $\left(\frac{C_t}{C_t(i)}\right)^{\frac{1}{\epsilon}} = \lambda P_t(i)$ (λ is the Lagrange multiplier) with the expenditure constraint and defining the aggregate price level of the economy as $P_t = \left(\int_0^1 P_t(i)^{1-\epsilon} di\right)^{\frac{1}{1-\epsilon}}$ yields $C_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon} \frac{Q_t}{P_t}$. Using this expression and the definition of C_t , total expenditure of the household can be stated in terms of the aggregates:

$$\int_0^1 P_t(i) C_t(i) di = P_t C_t. \quad (2)$$

This implies the following demand functions for the differentiated goods that is relevant for retail firms:

$$C_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon} C_t. \quad (3)$$

In addition to the optimal consumption bundle, the infinitely-lived household with discount factor β has to decide on the optimal consumption/saving behavior. For this purpose, it maximizes the expected value of the sum of discounted period utility functions

$$U_t = E_t \left\{ \sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\sigma}}{1-\sigma} \right\}, \quad (4)$$

where σ is the coefficient of relative risk aversion, subject to the real budget constraint

$$C_t + \frac{B_t}{P_t} \leq N_t w_t + \frac{B_{t-1}}{P_t} (1 + i_{t-1}) + U_t b + \Theta_t. \quad (5)$$

Each period, the household can buy nominal bonds B_t which pay a gross interest rate $(1 + i_t)$ in the next period. N_t is the number of employed workers. Unemployed household members U_t receive real revenues from home production b . w_t represents the real wage. I assume that short- and long-term unemployed workers do not differ in terms of their home production. Since firms are owned by households, total firm profits Θ are declared to the households.

The first-order conditions of the household's problem are:

$$C_t^{-\sigma} = \lambda_t \quad (6)$$

and

$$E_t \left\{ \lambda_{t+1} \beta \frac{1}{P_{t+1}} \right\} (1 + i_t) = \lambda_t \frac{1}{P_t}. \quad (7)$$

Combining (6) and (7) gives the Euler equation

$$E_t \left\{ C_{t+1}^{-\sigma} \frac{1}{\pi_{t+1}} \right\} \beta (1 + i_t) = C_t^{-\sigma}, \quad (8)$$

where $\pi_{t+1} = \frac{P_{t+1}}{P_t}$ is the gross inflation rate in period $t + 1$. The Euler equation says that optimality requires the expected ratio of marginal utilities of consumption in the future and today to be equal to the price ratio of consumption today and in the future.

3.2 Firms

Intermediate firms employ labor and produce a homogeneous intermediate good. Retail firms take intermediate goods as input and produce differentiated final goods. The separation of intermediate goods producers and final goods producers is standard in the literature and avoids interactions between price setting and wage bargaining at the firm level, as Blanchard and Galí (2010) point out.

3.2.1 Intermediate goods firms and the labor market

I follow Faia et al. (forthcoming) and assume that there is a large number of workers and firms, and a lot more workers than firms. This ensures that intermediate firms face identical decision problems. The timing of the labor market is as follows: First, unemployed workers are randomly assigned to firms. Second, worker-firm pairs draw a random operating cost ϵ_t , where $E(\epsilon_t)$ is normalized to zero. This random costs

serves as a tool to endogenize both hiring and firing decisions. Third, wages w_t are negotiated. Fourth, hiring and firing decisions are made.

Intermediate firms operate under the production function

$$Z_t = A_t(a^I n_t^I + a^S n_t^S + a^L n_t^L), \quad (9)$$

where Z_t is the quantity of the intermediate good; n_t^I is the number of employed workers who were employed in the previous period and exhibit individual productivity a^I ; n_t^S is the number of employed workers who were short-term unemployed in the previous period and exhibit individual productivity a^S ; n_t^L is the number of employed workers who were long-term unemployed in the previous period and exhibit individual productivity a^L . A_t denotes the aggregate level of productivity in the economy. Intermediate firms sell their output to the retail firms in a perfectly competitive environment for the price P_z , so that real marginal costs mc_t must equal the real price: $mc_t = \frac{P_{z,t}}{P_t}$.

Firing rate for incumbent workers

The contemporaneous revenue of an incumbent worker for the intermediate firm is the marginal product of the worker times the additional revenue of one more unit of output. Additionally, with retention rate $1 - \phi_{t+1}$ the worker is not fired at the beginning of the next period and generates a future revenue stream. The costs for the firm are the wage and the random operating costs in the current and in future periods. If the worker is fired with rate ϕ_{t+1} in the next period, the firm has to pay firing costs f . In line with Krause and Lubik (2007), I assume that the firing rate consists of an external component ϕ^x and an internal component ϕ^n so that

$$\phi_t = \phi^x + (1 - \phi^x)\phi_t^n. \quad (10)$$

A recursive formulation for the profits generated by an incumbent worker, Θ_t^I , is

$$\Theta_t^I = A_t a^I mc_t - w_t - \epsilon_t + E_t \left\{ \beta_{t+1} \left((1 - \phi_{t+1}) \int_{-\infty}^{v_{f,t+1}} \frac{\Theta_{t+1}^I g(\epsilon_{t+1})}{G(v_{f,t+1})} d\epsilon_{t+1} - \phi_{t+1} f \right) \right\}. \quad (11)$$

β_{t+1} denotes the effective discount factor and is defined as $\beta_{t+1} = \beta \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma}$. It converts future profits — which eventually accrue to the households — in terms of current utility. $G(\cdot)$ and $g(\cdot)$ denote the cumulative distribution function and the probability density function, respectively, of the operating cost shock. Since incumbent workers are only employed in future periods if it is profitable for the firm to continue employment, future profits are conditional on the shock being below a certain threshold $v_{f,t+1}$, which will be determined below.

If future expected profits are expressed as

$$E_t\{\Theta_{t+1}^I\} = E_t\{(1 - \phi_{t+1})(A_{t+1}a^I mc_{t+1} - w_{t+1} - E_\epsilon\{\epsilon_{t+1} \mid \epsilon_{t+1} \leq v_{f,t+1}\} + E_{t+1}\{\beta_{t+2}\Theta_{t+2}^I\}) - \phi_{t+1}f\}, \quad (12)$$

the profits generated by an incumbent worker for an intermediate firm can be written as

$$\Theta_t^I = A_t a^I mc_t - w_t - \epsilon_t + E_t\{\beta_{t+1}\Theta_{t+1}^I\}. \quad (13)$$

Employing the incumbent worker only pays for the firm if these profits are higher than the profits from firing the worker, $-f$. Hence, the employment relationship is ended if the realization of the random cost shock ϵ_t implies $\Theta_t^I < -f$. Accordingly, the firing threshold is defined by

$$v_{f,t} = A_t a^I mc_t - w_t + f + E_t\{\beta_{t+1}\Theta_{t+1}^I\} \quad (14)$$

and the firing rate is the probability that the random cost shock is above the firing threshold:

$$\phi_t^n = 1 - G(v_{f,t}). \quad (15)$$

Hiring rate for short-term unemployed workers

An expression for the firm's profits Θ_t^S generated by a worker who was previously short-term unemployed is

$$\Theta_t^S = A_t a^S mc_t - w_t - \epsilon_t + E_t\left\{\beta_{t+1}\left((1 - \phi_{t+1}) \int_{-\infty}^{v_{f,t+1}} \frac{\Theta_{t+1}^I g(\epsilon_{t+1})}{G(v_{f,t+1})} d\epsilon_{t+1} - \phi_{t+1}f\right)\right\}. \quad (16)$$

The worker generates contemporaneous revenue but the firm has to pay a wage and operating costs. If the worker is not fired in the next period, she is an incumbent worker and accordingly generates the profits of an incumbent worker conditional on the operating costs being below the firing threshold. If the worker is fired in the next period, firing costs have to be paid by the firm. Expression (16) shows that the expected future profits generated by a previously short-term unemployed worker are equal to the expected future profits of a previously employed worker. Hence,

$$\Theta_t^S = A_t a^S mc_t - w_t - \epsilon_t + E_t\{\beta_{t+1}\Theta_{t+1}^I\}. \quad (17)$$

A previously short-term unemployed worker is hired if the random cost shock is low enough to generate positive profits taking into account the hiring costs h , that is, if $h < \Theta_t^S$. Thus, the hiring threshold for short-term unemployed workers, $v_{h,t}^S$ is defined by

$$v_{h,t}^S = A_t a^S mc_t - w_t - h + E_t\{\beta_{t+1}\Theta_{t+1}^I\} \quad (18)$$

and the hiring rate for short-term unemployed workers is

$$\eta_t^S = G(v_{h,t}^S). \quad (19)$$

Hiring rate for long-term unemployed workers

Profits Θ_t^L generated by a worker previously long-term unemployed can be written as

$$\Theta_t^L = A_t a^L m c_t - w_t - \epsilon_t + E_t \left\{ \beta_{t+1} \left((1 - \phi_{t+1}) \int_{-\infty}^{v_{f,t+1}^L} \frac{\Theta_{t+1}^I g(\epsilon_{t+1})}{G(v_{f,t+1}^L)} d\epsilon_{t+1} - \phi_{t+1} f \right) \right\}. \quad (20)$$

Similar explanations as for the profits generated by a previously short-term unemployed worker apply. Hence, profits can be written as

$$\Theta_t^L = A_t a^L m c_t - w_t - \epsilon_t + E_t \{ \beta_{t+1} \Theta_{t+1}^I \}, \quad (21)$$

where $E_t \{ \Theta_{t+1}^L \} = E_t \{ \Theta_{t+1}^I \}$. If the operating costs are low enough to generate positive profits despite hiring costs, that is, $h < \Theta_t^L$, a long-term unemployed worker is hired. The hiring threshold $v_{h,t}^L$ is defined as

$$v_{h,t}^L = A_t a^L m c_t - w_t - h + E_t \{ \beta_{t+1} \Theta_{t+1}^I \} \quad (22)$$

and the hiring rate for long-term unemployed workers is

$$\eta_t^L = G(v_{h,t}^L). \quad (23)$$

3.2.2 Employment and unemployment dynamics

Let n_t be the employment rate, that is, employment divided by the labor force, and u_t the total unemployment rate. The share of the labor force which is employed after being employed is

$$n_t^I = (1 - \phi_t) n_{t-1}. \quad (24)$$

The share of the labor force which is employed after being short-term unemployed is

$$n_t^S = \eta_t^S u_{t-1}^S, \quad (25)$$

where u_t^S is the short-term unemployment rate. Similarly, the share of the labor force which is employed after being long-term unemployed is

$$n_t^L = \eta_t^L u_{t-1}^L, \quad (26)$$

where u_t^L is the LTU rate. The short-term unemployment rate consists of the share of the labor force which is unemployed for less than or equal to four quarters. Hence, to determine the short-term unemployment rate I have to track the duration

of unemployment. Let ${}_d u_t$ denote the share of the labor force which has been unemployed for d periods. Then

$${}_1 u_t = \phi_t n_{t-1} \quad (27)$$

$${}_2 u_t = (1 - \eta_t^S) {}_1 u_{t-1} \quad (28)$$

$${}_3 u_t = (1 - \eta_t^S) {}_2 u_{t-1} \quad (29)$$

$${}_4 u_t = (1 - \eta_t^S) {}_3 u_{t-1}. \quad (30)$$

The short-term unemployment rate can then be determined by

$$u_t^S = {}_1 u_t + {}_2 u_t + {}_3 u_t + {}_4 u_t \quad (31)$$

and the LTU rate consists of those who are unemployed for more than four quarters:

$$u_t^L = (1 - \eta_t^L) u_{t-1}^L + (1 - \eta_t^S) {}_4 u_{t-1}. \quad (32)$$

Consequently, the employment and the total unemployment rate of the economy are

$$n_t = (1 - \phi_t) n_{t-1} + \eta_t^S u_{t-1}^S + \eta_t^L u_{t-1}^L \quad (33)$$

and

$$u_t = u_t^S + u_t^L, \quad (34)$$

respectively. Since the labor force is normalized to 1, it holds that

$$n_t + u_t = 1. \quad (35)$$

3.2.3 Wages

I follow Brown et al. (2011) and assume that the wage is the outcome of a Nash bargain between the intermediate firm and the median incumbent worker with operating costs $\bar{e}$ who faces no risk of dismissal at the negotiated wage.⁵ As Lechthaler et al. (2010) point out, this kind of wage determination is especially suited for European economies where unions play a crucial role in wage negotiations. In this setting, the fallback position is not unemployment and a vacancy for the workers and the firm, respectively, but disagreement. During disagreement, the worker receives a fallback income (e.g. support out of a union fund or from family members), which for simplicity is assumed to be equal to unemployment benefits, while the firm incurs costs s which can be seen as strike costs for example.⁶

Let V_t^N , V_t^U , and V_t^J be the value of a job for the worker, the value of unemployment for the worker, and the value of a job for an intermediate firm, respectively,

⁵This assumption implies that all workers receive the same wage.

⁶Brown et al. (forthcoming) show that individual wage bargaining leads to similar results as the centralized procedure.

under agreement in period t . Let $\tilde{V}_t^N$ and $\tilde{V}_t^J$ be the value of a job for the median worker and the firm under disagreement in period t . Then, the expected present value of a job for the median worker under agreement is

$$V_t^N = w_t + E_t \left\{ \beta_{t+1} \left[(1 - \phi_{t+1}) V_{t+1}^N + \phi_{t+1} V_{t+1}^U \right] \right\}, \quad (36)$$

whereas under disagreement it is

$$\tilde{V}_t^N = b + E_t \left\{ \beta_{t+1} \left[(1 - \phi_{t+1}) V_{t+1}^N + \phi_{t+1} V_{t+1}^U \right] \right\}. \quad (37)$$

Under agreement, the worker receives the wage, under disagreement she receives fallback income. The continuation values are the same under agreement and disagreement since it is assumed that disagreement does not affect future returns. Either the worker is not fired in the next period and continues to receive the value of employment, or she is fired and receives the value of unemployment. The value of a job for the firm under agreement is

$$V_t^J = A_t a^I m c_t - w_t - \bar{\epsilon} + E_t \left\{ \beta_{t+1} \left[(1 - \phi_{t+1}) V_{t+1}^J - \phi_{t+1} f \right] \right\}, \quad (38)$$

whereas under disagreement it is

$$\tilde{V}_t^J = -s + E_t \left\{ \beta_{t+1} \left[(1 - \phi_{t+1}) V_{t+1}^J - \phi_{t+1} f \right] \right\}. \quad (39)$$

Under agreement, the firm receives the marginal profit generated by the median incumbent, while under disagreement it incurs the strike costs. In the next period the firm gets the value of a job or has to pay firing costs, depending on whether the worker is fired or not. It follows that the insiders bargaining surplus is $V_t^N - \tilde{V}_t^N$ and the firms bargaining surplus is $V_t^J - \tilde{V}_t^J$. Firms and workers bargain over the wage to maximize the Nash product

$$\begin{aligned} \Upsilon &= (V_t^N - \tilde{V}_t^N)^\gamma (V_t^J - \tilde{V}_t^J)^{(1-\gamma)} \\ &\Leftrightarrow \Upsilon = (w_t - b)^\gamma (A_t a^I m c_t - w_t - \bar{\epsilon} + s)^{(1-\gamma)}. \end{aligned} \quad (40)$$

Rearranging the first-order condition of this maximization problem gives the wage

$$w_t = (1 - \gamma)b + \gamma(A_t a^I m c_t - \bar{\epsilon} + s). \quad (41)$$

3.2.4 Retail sector

There is a continuum of monopolistic competitive firms in the retail sector indexed by $i \in (0; 1)$. Retailers buy inputs from intermediate firms and transform them one-to-one to differentiated final consumption goods $Y_t(i)$. They face quadratic price adjustment costs as in Rotemberg (1982). Firm i in the retail sector chooses the price in order to maximize its expected discounted profit stream subject to the

household's demand function (3). Hence, the optimization problem for retailers reads

$$\begin{aligned} \max_{\{P_t(i)\}} \Theta_t^R &= E_t \left\{ \sum_{t=0}^{\infty} \beta_t \left[\frac{P_t(i)}{P_t} Y_t(i) - mc_t Y_t(i) - \frac{\psi}{2} \left(\frac{P_t(i)}{P_{t-1}(i)} - \pi \right)^2 Y_t \right] \right\} \\ \text{s.t. } Y_t(i) &= \left(\frac{P_t(i)}{P_t} \right)^{-\epsilon} Y_t \end{aligned}$$

where π denotes steady state inflation, $Y_t \equiv \int_0^1 [Y_t(i)]^{\frac{\epsilon-1}{\epsilon}} di]^{\frac{\epsilon}{\epsilon-1}}$ and $\frac{\psi}{2} \left(\frac{P_t(i)}{P_{t-1}(i)} - \pi \right)^2 Y_t$ are the price adjustment cost. The first order condition reads

$$\begin{aligned} Y_t P_t^{\epsilon-1} (1 - \epsilon) P_t(i)^{-\epsilon} + \epsilon mc_t Y_t P_t^{\epsilon} P_t(i)^{-\epsilon-1} - \psi Y_t \left(\frac{P_t(i)}{P_{t-1}(i)} - \pi \right) \frac{1}{P_{t-1}} + \psi E_t \left\{ Y_{t+1} \left(\frac{P_{t+1}(i)}{P_t(i)} - \pi \right) \frac{P_{t+1}(i)}{P_t(i)^2} \right\} &= 0. \end{aligned} \quad (42)$$

By symmetry, the i 's can be neglected. Multiplying by P_t and dividing by Y_t yields the supply equation

$$(1 - \epsilon) + mc_t \epsilon - \psi(\pi_t - \pi)\pi_t + \psi E_t \left\{ (\pi_{t+1} - \pi)\pi_{t+1} \frac{Y_{t+1}}{Y_t} \right\} = 0. \quad (43)$$

3.3 Monetary authority and aggregate productivity process

The central bank follows a Taylor-type rule, that is, it reacts to deviations of output from steady state output and to deviations of inflation from steady state inflation. Additionally, and as is common in the literature, it is assumed that the monetary authority engages in interest rate smoothing, that is, it also reacts to the past interest rate level. The monetary policy rule is thus described by

$$\frac{1 + i_t}{1 + i} = \left(\frac{1 + i_{t-1}}{1 + i} \right)^{\gamma_i} \left[\left(\frac{\pi_t}{\pi} \right)^{\gamma_\pi} \left(\frac{y_t}{y} \right)^{\gamma_y} \right]^{1-\gamma_i} e^{\mu_t^i}. \quad (44)$$

γ_i , γ_π and γ_y are the central banks interest rate, inflation and output reaction parameters, respectively, and variables without time subscript are steady state values. μ_t^i is a monetary shock term.

Aggregate productivity is assumed to follow the AR(1) process

$$A_t = A^{1-\rho} A_{t-1}^\rho e^{\mu_t^A}, \quad (45)$$

where μ_t^A is an aggregate productivity shock.

3.4 Aggregation and equilibrium

Real profits for intermediate firms Θ_t^F are real revenues minus total costs, where the latter comprise wage payments, operating costs and hiring and firing costs:

$$\begin{aligned} \Theta_t^F = & mc_t A_t (a^I n_t^I + a^S n_t^S + a^L n_t^L) - w_t n_t - n_t^I \left(\frac{\int_{-\infty}^{v_{f,t}} \epsilon_t g(\epsilon_t) d\epsilon_t}{1 - \phi_t} \right) \\ & - n_t^S \left(\frac{\int_{-\infty}^{v_{h,t}^S} \epsilon_t g(\epsilon_t) d\epsilon_t}{\eta_t^S} \right) - n_t^L \left(\frac{\int_{-\infty}^{v_{h,t}^L} \epsilon_t g(\epsilon_t) d\epsilon_t}{\eta_t^L} \right) - u_{1,t} f - n_t^S h - n_t^L h. \end{aligned} \quad (46)$$

Retailers make real revenues Y_t , have to pay for their inputs and incur price adjustment costs. Hence profits for the retailers are

$$\Theta_t^R = Y_t - mc_t A_t (a^I n_t^I + a^S n_t^S + a^L n_t^L) - \frac{\psi}{2} (\pi_t - \pi)^2 Y_t. \quad (47)$$

Aggregate profits in the economy are intermediate firm profits plus retailer profits:

$$\begin{aligned} \Theta_t = & Y_t - w_t n_t - n_t^I \left(\frac{\int_{-\infty}^{v_{f,t}} \epsilon_t g(\epsilon_t) d\epsilon_t}{1 - \phi_t} \right) - n_t^S \left(\frac{\int_{-\infty}^{v_{h,t}^S} \epsilon_t g(\epsilon_t) d\epsilon_t}{\eta_t^S} \right) \\ & - n_t^L \left(\frac{\int_{-\infty}^{v_{h,t}^L} \epsilon_t g(\epsilon_t) d\epsilon_t}{\eta_t^L} \right) - u_{1,t} f - n_t^S h - n_t^L h - \frac{\psi}{2} (\pi_t - \pi)^2 Y_t. \end{aligned} \quad (48)$$

Combining (48) with the budget constraint (5) yields

$$\begin{aligned} C_t = & Y_t - n_t^I \left(\frac{\int_{-\infty}^{v_{f,t}} \epsilon_t g(\epsilon_t) d\epsilon_t}{1 - \phi_t} \right) - n_t^S \left(\frac{\int_{-\infty}^{v_{h,t}^S} \epsilon_t g(\epsilon_t) d\epsilon_t}{\eta_t^S} \right) \\ & - n_t^L \left(\frac{\int_{-\infty}^{v_{h,t}^L} \epsilon_t g(\epsilon_t) d\epsilon_t}{\eta_t^L} \right) - u_{1,t} f - n_t^S h - n_t^L h - \frac{\psi}{2} (\pi_t - \pi)^2 Y_t. \end{aligned} \quad (49)$$

For given interest rate and aggregate productivity processes $\{A_t, i_t\}_{t=0}^{\infty}$, a competitive equilibrium in this economy is defined as a sequence of variables

$$\begin{aligned} & \{C_t, mc_t, w_t, \Theta_t^I, \phi_t, \phi_t^n v_{f,t}, v_{h,t}^S, v_{h,t}^L, \\ & \eta_t^S, \eta_t^L, n_t^I, n_t^S, n_t^L, {}_1u_t, {}_2u_t, {}_3u_t, {}_4u_t, u_t^S, u_t^L, Y_t, \pi_t\}_{t=0}^{\infty} \end{aligned}$$

which satisfy the household optimality condition (8), the profit process (12), the hiring and firing thresholds and rates (10), (14), (15), (18), (19), (22), and (23), the labor market flow processes (24), (25), (26), (27), (28), (29), (30), (31), and (32), the wage equation (41), the supply equation (43), the economy budget constraint (49), and the production function (9). A collection of the equilibrium conditions is provided in appendix A.

4 Calibration and simulation

I calibrate the model to German data and simulate second moments and dynamic responses to shocks hitting the economy. I focus on Germany because Lechthaler et al. (2010) provide a good reference point for calibrating hiring and firing costs in Germany, and because certain characteristics of the model, like the type of labor market frictions or centralized wage bargaining, are often associated with European economies.

Table 2 gives an overview of the calibration. The household's discount rate β is assumed to be 0.99, implying an annual real interest rate of 4%. Following Lechthaler et al. (2010), the elasticity of substitution between final consumption goods, ϵ , is set to 10, the coefficient of risk aversion σ to 2, and the parameter of price adjustment, ψ , to 104.85 to match an average price duration of four quarters (see e.g. Nakamura and Steinsson (2008)). As no definitive evidence on the bargaining power of workers is available in the literature, I follow Krause and Lubik (2007) and set $\gamma = 0.5$.

Bentolila and Bertola (1990) report that firing costs in Germany amount to approximately 75% of the annual wage, which implies $f = 2.6$. Mortensen and Pissarides (1999) use hiring costs amounting to 15% of annual output leading to $h = 0.54$. As in Lechthaler et al. (2010), unemployment benefits are chosen to be 65% of the level of productivity. Keane and Wolpin (1997) estimate the rate of skill loss to be approximately 30% after one year of unemployment. I thus normalize a^I to 1 and set a^L to 0.7 and a^S to 0.92. These numbers will be changed later on to show how the results depend on the relative productivities of the heterogeneous labor force.

Following Brown et al. (forthcoming, 2011) and Lechthaler et al. (2010), I assume that the random operating cost is logistically distributed with cumulative distribution function $G(\epsilon; sd) = \frac{1}{1+e^{-\epsilon/sd}}$ and probability density function $g(\epsilon; sd) = \frac{e^{-\epsilon/s}}{s(1+e^{-\epsilon/s})^2}$ (recall that $E(\epsilon)$ is normalized to 0). The distributional parameter sd and the strike cost parameter s are calibrated so that the steady state values of the total unemployment rate and the LTU proportion match the average of the respective series over the period 1998Q1-2011Q4 in Germany. Thus, to obtain $u = 0.085$ and $\frac{u^L}{u} = 0.51$, s is set to 0.1 and sd to 0.6.

Steady state aggregate productivity is assumed to be 1 and the inflation and output reaction coefficients are set to $\gamma_\pi = 1.5$ and $\gamma_y = 0.125$. Söderlind et al. (2005) and Belke and Polleit (2007) report an estimated smoothing degree of approximately 0.7, so $\gamma_i = 0.7$. The autocorrelation coefficient of productivity, ρ , is 0.94. This value is in line with estimates from Smets and Wouters (2005). The monetary shock is a one-off shock, that is, it does not exhibit autocorrelation.

Parameter	Description	Value	Source
β	Discount rate	0.99	Lechthaler et al. (2010), standard value
ϵ	Elasticity of substitution	10	Lechthaler et al. (2010), standard value
σ	Coefficient of risk aversion	2	Lechthaler et al. (2010), standard value
ψ	Parameter of price adjustment	104.85	Equiv. to avg. price dur. of 1 year
γ	Bargaining power of workers	0.5	Krause and Lubik (2007)
f	Firing costs	2.6	Bentolila and Bertola (1990)
h	Hiring costs	0.54	Mortensen and Pissarides (1999)
a^I	Prod. of incumbent worker	1	Normalized
a^L	Prod. of worker prev. in LTU	0.7	Keane and Wolpin (1997)
a^S	Prod. of worker prev. in STU	0.92	Assuming linear skill loss
sd	Distributional parameter.	0.6	To match avg. $u, u^L/u$ in data
s	Disagreement costs for firm	0.1	To match avg. $u, u^L/u$ in data
γ_π	Inflation reaction coefficient	1.5	Standard value
γ_y	Output reaction coefficient	0.125	Standard value
γ_i	Degree of interest rate smoothing	0.7	Söderlind et al. (2005)
ρ	Autocorr. coefficient of productivity	0.94	Smets and Wouters (2005)

Table 2: Calibration and source of calibrated value. Notes: STU abbreviates short-term unemployment.

Finally, I follow Lechthaler et al. (2010) and set the standard deviation of the productivity and the monetary shock to 0.5% and 0.15%, respectively.⁷

The model is solved using perturbation methods. A solution of the model is defined as a set of decision rules for the endogenous variables expressed as policy functions of the lagged state variables and aggregate shocks of the economy. Technically, the deterministic steady state of the model economy is computed to obtain linear approximations of the policy functions about the steady state.

The model moments reported below are produced as follows: In each simulation run, 250 time series observations are generated by drawing monetary and productivity shocks from a normal distribution with means zero and standard deviations as reported above. The first 100 observations are discarded and the remaining 150 are used to calculate relative standard deviations, correlations and autocorrelations for each variable. This procedure is repeated 1000 times. The mean of the resulting sample with size 1000 is the statistic of interest.

5 Comparing model outcomes to the data

I now compare the model-generated moments with the stylized facts presented in section 2. Note that in this section, I look at moments for the model with skill loss, while the next section compares these outcomes with those of the model without skill loss.

⁷See also Smets and Wouters (2005, 2003)

The model is in line with the evidence of negative duration dependence. That is, in the model the hiring rate for long-term unemployed workers, which is 14.23%, is lower than the hiring rate for short-term unemployed workers, which is 25.66%. The resulting average job finding rate of around 20% is in line with the value found in Lechthaler et al. (2010).

Second moments are presented in table 3. For ease of comparison, the first data column repeats the stylized facts for Germany (see table 1). The second, fourth and sixth data columns in table 3 contain moments obtained by simulating the model with skill loss ("sl") for joint, productivity and monetary shocks, respectively. It is apparent that productivity shocks exert the main influence on the overall performance of the model as the results for joint shocks are very close to those for productivity shocks alone.

Looking at the panel with relative standard deviations in table 3, the model can reproduce the volatility pattern of labor market variables and output. LTU is the most volatile variable, followed by total unemployment. The LTU proportion is less volatile than total unemployment but still more volatile than output. This is true for joint, productivity, and monetary shocks.

	GER	Joint shock		Productivity shock		Monetary shock	
		sl	nsL	sl	nsL	sl	nsL
Rel. std.							
y	1.000	1.000	1.000	1.000	1.000	1.000	1.000
u^L/u	3.800	4.067	9.044	4.038	8.986	7.199	21.043
u^L	10.294	8.343	14.084	8.294	13.989	13.312	32.402
u	7.014	4.282	4.906	4.257	4.874	8.823	13.023
Corr.							
y,u	-0.333	-0.963	-0.966	-0.963	-0.967	-0.994	-1.000
$y,(u^L/u)$	0.381	-0.857	-0.882	-0.857	-0.881	-0.452	-0.798
y,u^L	-0.138	-0.921	-0.918	-0.921	-0.918	-0.903	-0.921
$u,(u^L/u)$	0.495	0.955	0.968	0.959	0.969	0.378	0.798
u,u^L	0.920	0.989	0.986	0.990	0.987	0.866	0.921
Autocorr.							
y	0.885	0.964	0.948	0.963	0.947	0.779	0.821
u	0.892	0.987	0.982	0.989	0.983	0.707	0.821
u^L/u	0.976	0.991	0.989	0.992	0.989	0.832	0.941
u^L	0.962	0.992	0.990	0.992	0.989	0.928	0.924

Table 3: Data and model moments. Notes: The first data column shows empirical moments for Germany (see table 1). The rest of the data columns show simulated moments generated using the model in section 3 in the case of joint, productivity, and monetary shocks. The model is simulated without skill loss (nsL) and with skill loss (sl).

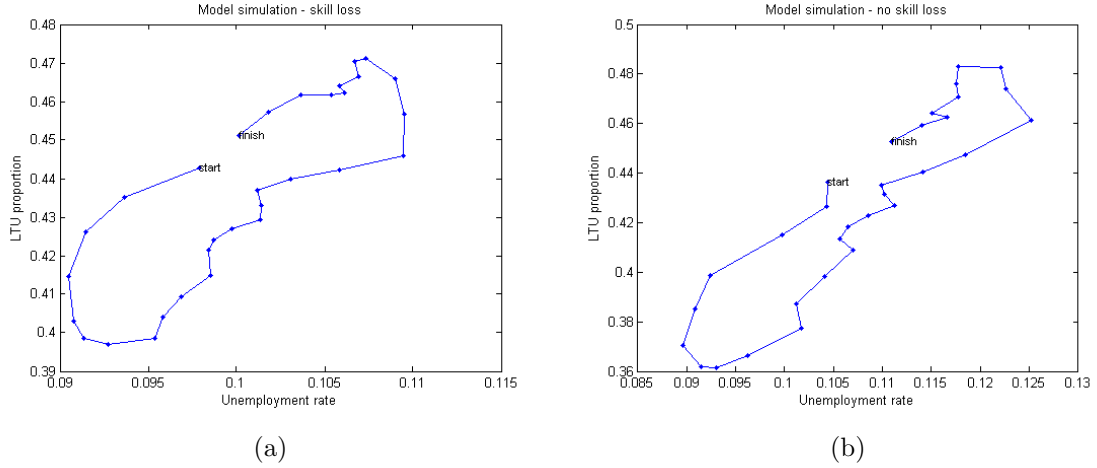


Figure 4: Model-generated plot of LTU proportion and total unemployment over the business cycle. Notes: The graphs show scatter plots for simulated data (joint shocks) with the LTU proportion and the total unemployment rate on the axes. Panel (a) is the result of the model including skill loss and panel (b) is the result of the model without skill loss for the same sequence of shocks.

When it comes to correlations, the model captures the correct sign in all but one case, namely the correlation between output and the LTU proportion. This correlation seems hard to capture as it switches signs across countries as shown in section 2. In the other cases where the signs are correct, the correlations produced by the model tend to be too high in absolute value.

While the fourth row in the correlations panel in table 3 shows the positive correlation between total unemployment and the LTU proportion, figure 4(a) inspects the model-generated correlation between these two variables more closely. It is the simulated counterpart for figure 3.⁸ While the overall positive correlation is obvious, the two variables are negatively correlated around business cycle turning points. The model can thus reproduce the counterclockwise movement of the line connecting the single observations through time.

There are several other findings which I do not seek to explain explicitly in this paper, but can still be connected to the existing literature. First, as shown in data row four of the relative standard deviation panel in table 3, the model is able to produce a higher volatility for total unemployment than for output, which reproduces one of the basic findings in Lechthaler et al. (2010). However, the magnitude of total unemployment volatility for joint shocks is still not large enough to reproduce that of the empirical observations. The very high volatility of total unemployment

⁸The figure shows the case of joint shocks. For the sake of lucidity, the scatter plot shows only 30 observations over approximately one cycle.

relative to output in Germany is also documented in Gartner et al. (2012). As we see in column six and row four of the relative standard deviations panel in table 3, monetary shocks are in principle capable of producing such high volatilities. This indicates that models with several demand shocks might be able to produce a higher overall volatility of total unemployment. Second, the model is able to generate large degrees of persistence for all types of shocks. For joint and productivity shocks, these are even a bit bigger than those found in the data.

6 The role of skill loss

The skill loss mechanism is the main distinguishing feature of the model in this paper compared to most existing models in the literature. This section aims at illustrating the importance of skill loss in reproducing stylized facts. To this end, I switch off the skill loss mechanism in the model presented in section 3, that is I set $a^I = a^S = a^L$. The resulting model is one with a homogeneous work force. I can now compare moments of the model with and without skill loss.

As a first result, and contrary to the model with skill loss, the model without the skill loss mechanism is not able to reproduce a lower hiring rate for workers in LTU than for workers with shorter durations in unemployment. The hiring rate for both types of unemployed is around 19%. This result is a direct consequence of the influence of skill loss on the hiring thresholds of short- and long-term unemployed workers, as seen in equations (19) and (23).

Turning to second moments, data columns three, five and seven in table 3 show the outcomes for the model without skill loss ("nsl"). The most striking result is that the skill loss mechanism is crucial for reproducing the volatility pattern in the data. With a homogeneous work force, LTU and the LTU proportion are far too volatile. In addition, the model without skill loss predicts, counterfactually, that the LTU proportion is more volatile than the total unemployment rate. Both of these results are true for joint, productivity as well as monetary shocks. The skill loss mechanism mutes the impact of the shock on the hiring threshold for long-term unemployed workers more than on the hiring threshold for short-term unemployed workers, leading to a volatility ordering which is in line with the empirical evidence.

In terms of unconditional correlations, there is little difference between the model with heterogeneous and homogeneous skills. If anything, skill loss improves the model's performance by reducing the absolute value of the correlation between output and the LTU proportion as well as between total unemployment and the LTU proportion, moving the model moments in the direction of the empirical moments.

These improvements are especially noticeable when it comes to the specific correlation pattern between total unemployment and the LTU proportion. Figure 4(b) shows a scatter plot of simulated values of those variables for the model without skill loss. The shock sequence is the same as in 4(a) with skill loss present. While the positive correlation is again clearly visible, the model without skill loss does not exhibit the temporary negative correlation around business cycle turning points (bottom left and upper right corner of the graph) as clearly as the model with skill loss. In this sense, LTU lags the development in total unemployment over the business cycle more when workers lose skills during unemployment.

When it comes to autocorrelations, we see that for joint shocks, skill loss adds persistence to all variables compared to the model without skill loss. However, this is due to the dominating nature of the productivity shock. For monetary shocks, the autocorrelation even declines somewhat for the skill loss model vis-a-vis the model excluding skill loss. This result corroborates findings in related studies: Using real labor market models, Merkl and Snower (2008) and Pissarides (1992) show that labor market persistence increases for heterogeneous productivities in the labor force, while the monetary model described in Esteban-Pretel and Faraglia (2010) yields a decrease in persistence when skill loss is present.

To gain further insight into the dynamics of the model, figures 5 and 6 show the reaction of several variables to a monetary and productivity shock, respectively. The horizontal axes show quarters, while the vertical axes show percentage deviations of the respective variable from the steady state.⁹ The solid lines represent impulse response functions for the model with skill loss, while the dashed lines are impulse responses for the model without skill loss.

As shown in figure 5, output decreases in response to a contractive monetary shock. This is due to a decrease in demand since consumers postpone consumption into the future. As can be seen in equations (14), (18) and (22), lower marginal costs decrease the hiring and firing thresholds due to lower unitary profits, causing the hiring rates for short- and long-term unemployment durations to drop and the firing rate to increase.¹⁰ This translates into higher short- and long-term unemployment and consequently into higher total unemployment. Note that the drop in the hiring rate for long-term unemployed is smaller than that for the short-term unemployed.

With skill loss present in the economy, LTU and the LTU proportion show a more muted response, in line with the data. Note that with skill loss, the LTU proportion

⁹For inflation and the nominal interest rate, percentage point deviations from steady state are shown.

¹⁰Note that wages also decrease when a contractive monetary shock hits, but this decrease is weaker than the decrease in marginal costs.

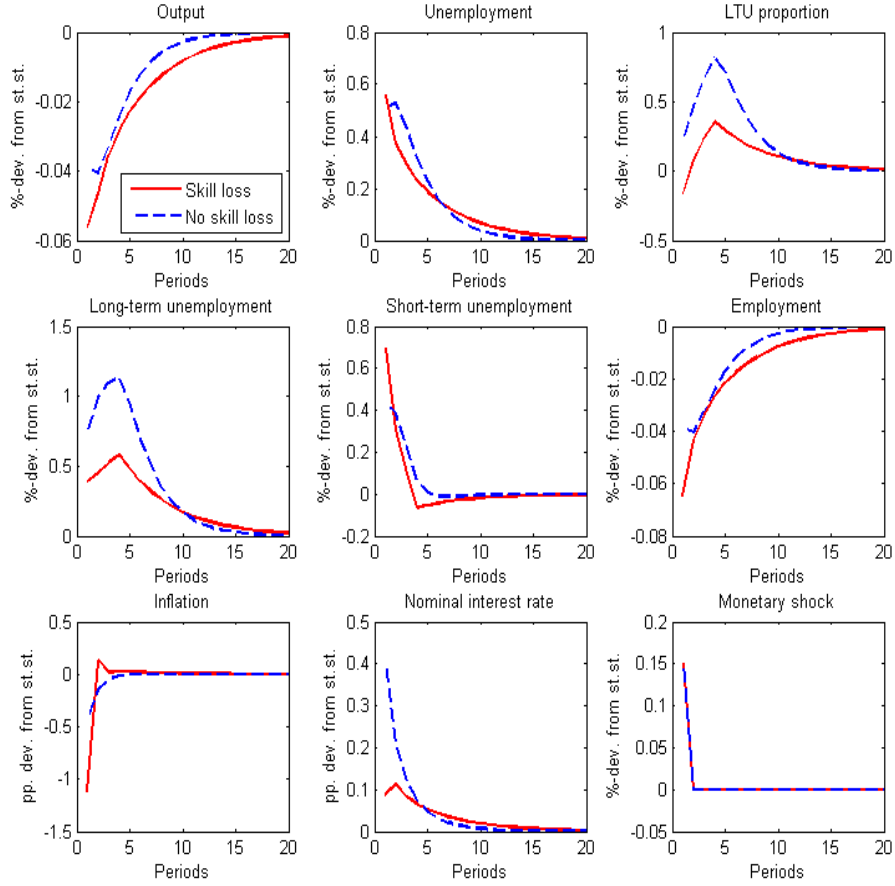


Figure 5: Impulse response functions for a contractive monetary shock. Notes: The graphs show a comparison of IRFs generated by the model with (solid lines) and without (dashed lines) skill loss, respectively, where the central bank follows a Taylor rule. Time periods correspond to quarters.

even decreases initially as the increase in total unemployment is stronger than the increase in LTU. Only in the subsequent quarters the LTU proportion increases before converging back to the steady state in accordance with total unemployment. This pattern illustrates again the overall positive but temporary negative correlation between total unemployment and the LTU proportion seen in figures 3 and 4(a).

In response to an adverse productivity shock, as shown in figure 6, output decreases on impact, reaching its strongest reaction with a lag. Total unemployment and LTU also respond with a lag and reach their peak reaction after eight quarters. The lagged reaction of the unemployment variables can be explained by offsetting effects of productivity and wages (negative) on the one hand and marginal costs (positive) on the other hand on the hiring and firing thresholds. It is noteworthy that the output reaction to a productivity shock peaks later when skill loss is present. Since the responses of short-term unemployment are similar in magnitude

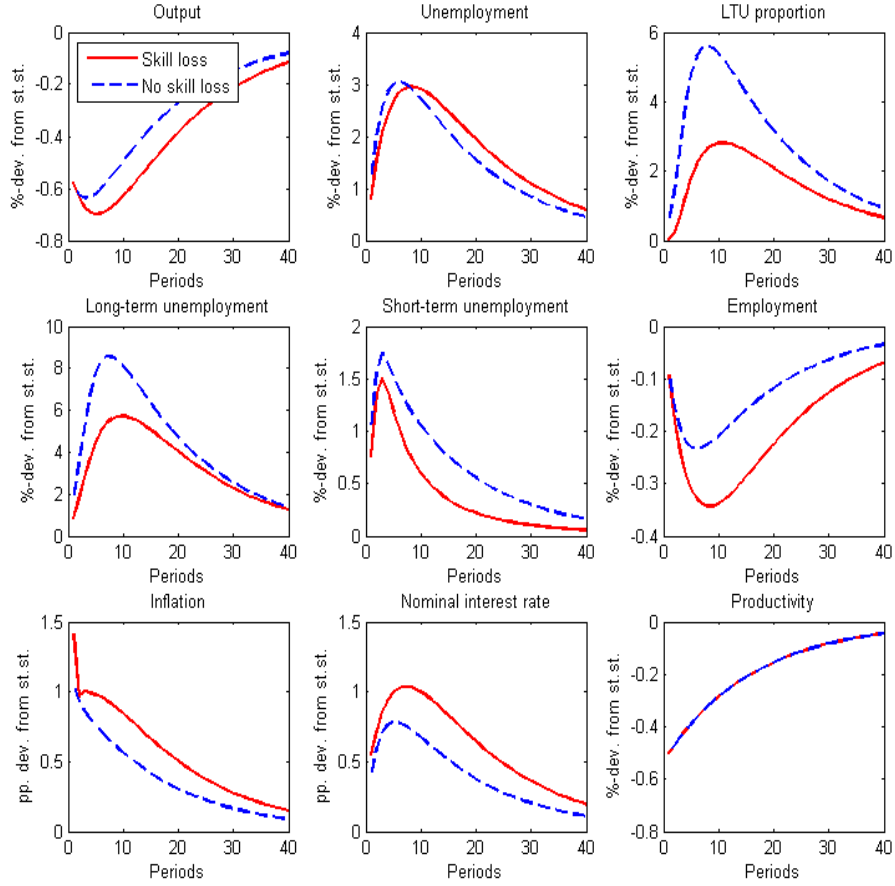


Figure 6: Impulse response functions for an adverse productivity shock. Notes: The graphs show a comparison of IRFs generated by the model with (solid lines) and without (dashed lines) skill loss, respectively, where the central bank follows a Taylor rule. Time periods correspond to quarters.

and timing for both cases this is due to the delayed peak reaction in LTU in the case of skill loss.

Moreover, the lower volatilities for LTU and the LTU proportion are evident in the impulse-response functions. The adverse productivity shock has a direct impact on the marginal revenue product of workers as shown in equations (18) and (22). Basically, skill loss has a dampening effect on the marginal revenue product of labor in the sense that movements in productivity and marginal costs in response to shocks are not fully transmitted into unitary profits and hence hiring and firing thresholds.

7 Optimal monetary policy in the presence of skill loss

As shown above, modeling the skill loss mechanism is important for capturing stylized facts from the data. Consequently, the implications of skill loss for monetary policy are examined. In a recent study, Faia et al. (forthcoming) show that labor turnover costs induce a trade-off for the central bank. Hiring and firing costs imply a waste of resources. Hence, implementing the flexible price allocation is not optimal anymore because of inefficient unemployment fluctuations. Consequently, the monetary authority has to strike a balance between stabilizing inflation and smoothing out unemployment fluctuations. This trade-off is also at play here. However, an additional factor relevant for welfare considerations enters the picture as firms do not account for skill loss processes during unemployment.

The optimal policy plan is determined by a Ramsey planner who maximizes the family household's utility function (4) subject to the equilibrium conditions of the competitive economy. The Taylor rule is replaced by the Ramsey planner's first-order conditions, shown in appendix B. Similar as in section 4, the deterministic steady state of the Ramsey problem's first-order conditions is computed to obtain linear approximations of the policy functions around that steady state.

It is instructive to first look at the difference between the Ramsey policy and the Taylor rule. Figure 7 looks at impulse response functions after an adverse productivity shock for the Ramsey monetary policy and the Taylor rule policy.¹¹ The solid line shows the Ramsey policy, the dashed line the Taylor rule policy. As expected, the Ramsey policy tolerates more short-term, long-term, and hence total unemployment, compared to the policy of a Taylor rule. This comes at the benefit of a much lower inflation volatility.

However, skill loss during unemployment accentuates the unemployment-inflation trade-off. This becomes clear by comparing the Ramsey policies for the case with and without skill loss. Figure 8 shows the optimal path of model variables in response to an adverse productivity shock. Here, the solid lines represent impulse responses for the model with skill loss, while the dashed lines show impulse responses without skill loss.

In the case of an adverse productivity shock, output declines while short-term, long-term and total unemployment as well as inflation increase. The rise in the unemployment variables is driven by declining hiring rates and an increasing firing

¹¹Note that monetary shocks cannot be examined since the interest rate rule is replaced by the interest rate path supporting the optimal allocation.

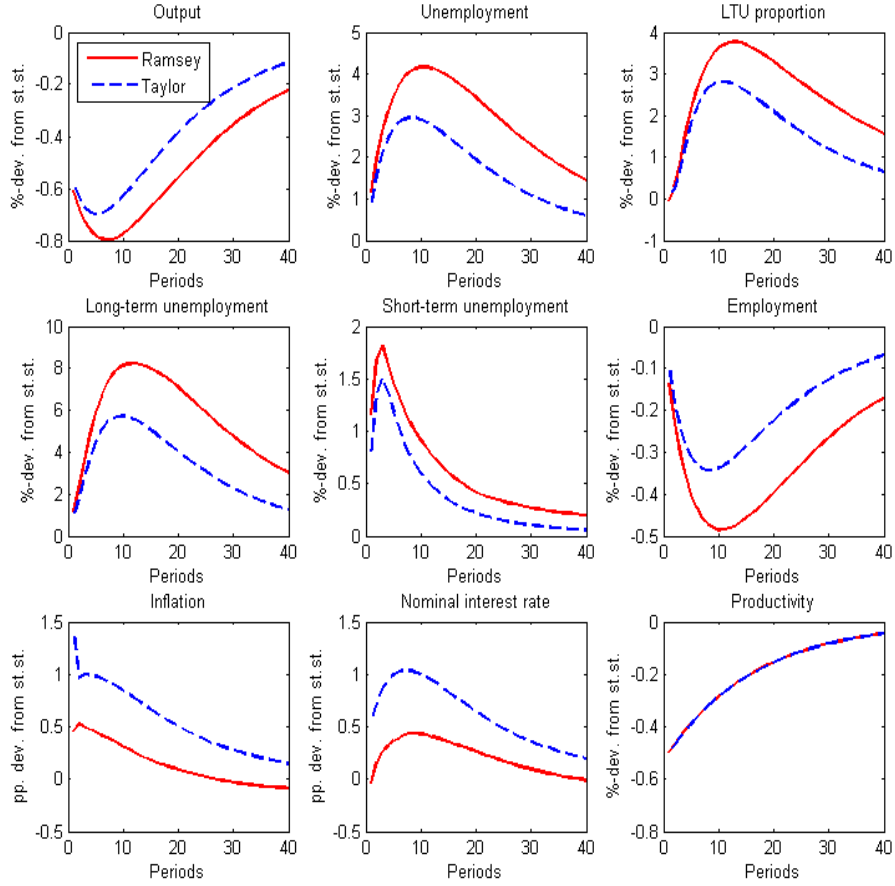


Figure 7: Ramsey monetary policy vs. Taylor rule for an adverse productivity shock. Notes: The graphs show impulse response functions generated by the model with skill loss. The dashed and solid lines correspond to the model with Taylor rule and Ramsey policy, respectively. Inflation and the interest rate are annualized percentage point deviations from the steady state, the rest is shown in percentage deviations from the steady state. Time periods correspond to quarters.

rate. As in Faia et al. (forthcoming), the Ramsey planner can use inflation to reduce inefficient unemployment fluctuations. However, in the setting with skill loss, this trade-off is more accentuated than in the setting without skill loss. The lower left graph in figure 8 shows that the Ramsey planner allows for a bigger increase in inflation in the presence of skill loss. Note that the unemployment variables increase less in this case. In particular, the response of LTU is more muted and peaks later. This is also reflected in the smoothed response of the LTU proportion. In this way, the Ramsey planner reduces skill loss processes during unemployment at the cost of higher inflation volatility and hence mitigates production and consumption losses due to a lower average productivity. Consequently, the implied interest rate processes that support the optimal adjustment paths of inflation and the unemployment

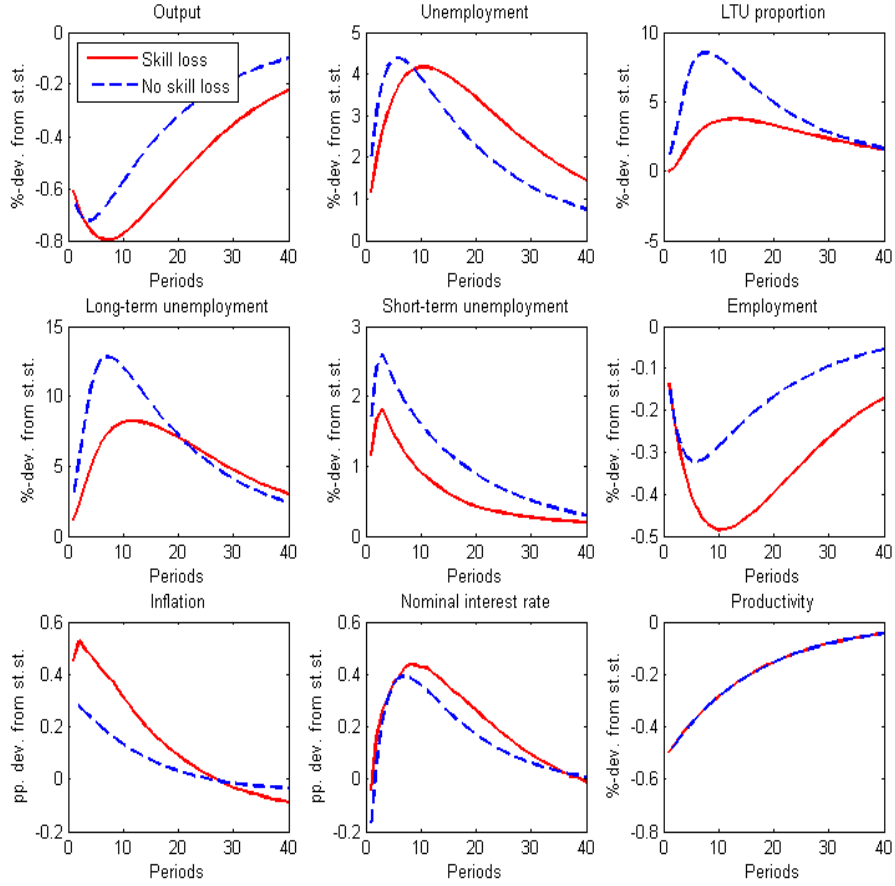


Figure 8: Ramsey monetary policy with and without skill loss for an adverse productivity shock. Notes: The graphs compare impulse response functions for the Ramsey optimal monetary policy with (solid line) and without (dashed line) skill loss. Inflation and the interest rate are annualized percentage point deviations from the steady state, the rest is shown in percentage deviations from the steady state. Time periods correspond to quarters.

variables in the two scenarios proceed lower than if the monetary authority employs a Taylor rule but the shift is relatively more pronounced in the scenario with skill loss.

A look at the model-generated moments shows that the optimal inflation volatility approximately doubles when skill loss is present. The relative standard deviation of inflation, measured as described in section 2, is 0.14 in the model with heterogeneous skills compared to 0.08 in the model with homogeneous skills.

8 Conclusion

LTU exhibits substantial variations over the business cycle. The cyclical components of LTU and the LTU proportion are several times more volatile than the cyclical component of output. The same is true for total unemployment, but its volatility is lower than that of LTU and higher than that of the LTU proportion. Total unemployment and the LTU proportion are generally positively correlated but exhibit a temporary negative correlation around the turning points of the business cycle. Furthermore, the literature documents that unemployment is subject to negative duration dependence, that is, the unemployed workers' hiring rate declines with higher duration in unemployment.

A New Keynesian business cycle model with a frictional labor market and skill loss during unemployment is able to match these stylized facts. The skill loss mechanism makes it relatively less attractive for firms to hire workers from the LTU pool, which explains negative duration dependence. The impact of skill loss on the marginal revenue product of firms and hiring and firing thresholds helps to reproduce the empirical evidence on the volatility pattern of output and labor market variables, as well as the behavior of the LTU proportion around business cycle turning points.

Due to hiring and firing costs, the monetary authority faces a trade-off between stabilizing inflation and stabilizing unemployment. Skill loss accentuates this trade-off. Optimal monetary policy in the presence of skill loss therefore accepts more inflation after adverse productivity shocks to reduce skill deterioration and mitigate production and consumption losses.

Appendix A

Define Ξ^I as the expected operating costs for incumbent workers conditional on not being fired and Ξ^S and Ξ^L as the expected operating costs for short- and long-term unemployed workers, respectively, conditional on being hired. Then, there are 28 endogenous variables: $c, y, i, v_h^S, v_h^L, \eta^S, \eta^L, v_f, \phi, \phi^n, n^S, n^L, n^I, {}_1u, {}_2u, {}_3u, {}_4u, u^L, u^S, w, mc, \Theta^I, \Theta^S, \Theta^L, \pi, \Xi^I, \Xi^S$, and Ξ^L . The competitive economy can be characterized by the following 27 equilibrium conditions plus the interest rate rule:

$$c_t^{-\sigma} = E_t \left\{ c_{t+1}^{-\sigma} \beta \frac{i_t}{\pi_{t+1}} \right\} \quad (\text{A.1})$$

$$0 = 1 - \epsilon + \epsilon mc_t - \pi_t \psi (\pi_t - 1) + \psi \beta E_t \left\{ \left(\frac{c_{t+1}}{c_t} \right)^{-\sigma} (\pi_{t+1} - 1) \pi_{t+1} \frac{y_{t+1}}{y_t} \right\} \quad (\text{A.2})$$

$$w_t = (1 - \gamma) b + \gamma (mc_t A_t a^I + s - \bar{e}) \quad (\text{A.3})$$

$$\phi_t^n = 1 - \frac{1}{1 + \exp \left(\frac{-v_{f,t}}{sd} \right)} \quad (\text{A.4})$$

$$\phi_t = \phi^x + \phi_t^n (1 - \phi^x) \quad (\text{A.5})$$

$$v_{f,t} = mc_t A_t a^I - w_t + f + \beta E_t \left\{ \left(\frac{c_{t+1}}{c_t} \right)^{-\sigma} \Theta_{t+1}^I \right\} \quad (\text{A.6})$$

$$\begin{aligned} \Theta_t^I &= (1 - \phi_t) \left(mc_t A_t a^I - w_t - \Xi_t^I \right. \\ &\quad \left. + \beta E_t \left\{ \Theta_{t+1}^I \left(\frac{c_{t+1}}{c_t} \right)^{-\sigma} \right\} \right) - \phi_t f \end{aligned} \quad (\text{A.7})$$

$$\Xi_t^I = \frac{v_{f,t} - \frac{v_{f,t}}{1 + \exp \left(\frac{v_{f,t}}{sd} \right)} - sd \log \left(1 + \exp \left(\frac{v_{f,t}}{sd} \right) \right)}{1 - \phi_t} \quad (\text{A.8})$$

$$\Theta_t^S = (1 - \phi_t) \left(mc_t A_t a^I - w_t - \Xi_t^I + \beta E_t \left\{ \Theta_{t+1}^I \left(\frac{c_{t+1}}{c_t} \right)^{-\sigma} \right\} \right) - \phi_t f \quad (\text{A.9})$$

$$\Theta_t^L = (1 - \phi_t) \left(mc_t A_t a^I - w_t - \Xi_t^I + \beta E_t \left\{ \Theta_{t+1}^I \left(\frac{c_{t+1}}{c_t} \right)^{-\sigma} \right\} \right) - \phi_t f \quad (\text{A.10})$$

$$v_{h,t}^S = mc_t A_t a^S - w_t - h + \beta E_t \left\{ \left(\frac{c_{t+1}}{c_t} \right)^{-\sigma} \Theta_{t+1}^S \right\} \quad (\text{A.11})$$

$$\eta_t^S = \frac{1}{1 + \exp \left(\frac{-v_{h,t}^S}{sd} \right)} \quad (\text{A.12})$$

$$v_{h,t}^L = mc_t A_t a^L - w_t - h + \beta E_t \left\{ \left(\frac{c_{t+1}}{c_t} \right)^{-\sigma} \Theta_{t+1}^L \right\} \quad (\text{A.13})$$

$$\eta_t^L = \frac{1}{1 + \exp \left(\frac{-v_{h,t}^L}{sd} \right)} \quad (\text{A.14})$$

$$\Xi_t^S = \frac{v_{h,t}^S - \frac{v_{h,t}^S}{1 + \exp \left(\frac{v_{h,t}^S}{sd} \right)} - sd \log \left(1 + \exp \left(\frac{v_{h,t}^S}{sd} \right) \right)}{\eta_t^S} \quad (\text{A.15})$$

$$\Xi_t^L = \frac{v_{h,t}^L - \frac{v_{h,t}^L}{1 + \exp \left(\frac{v_{h,t}^L}{sd} \right)} - sd \log \left(1 + \exp \left(\frac{v_{h,t}^L}{sd} \right) \right)}{\eta_t^L} \quad (\text{A.16})$$

$$n_t^L = \eta_t^L u_{t-1}^L \quad (\text{A.17})$$

$$n_t^S = \eta_t^S u_{t-1}^S \quad (\text{A.18})$$

$$n_t^I = (1 - \phi_t) (n_{t-1}^L + n_{t-1}^S + n_{t-1}^I) \quad (\text{A.19})$$

$${}_1u_t = \phi_t (n_{t-1}^L + n_{t-1}^S + n_{t-1}^I) \quad (\text{A.20})$$

$${}_2u_t = (1 - \eta_t^S) {}_1u_{t-1} \quad (\text{A.21})$$

$${}_3u_t = (1 - \eta_t^S) {}_2u_{t-1} \quad (\text{A.22})$$

$${}_4u_t = (1 - \eta_t^S) {}_3u_{t-1} \quad (\text{A.23})$$

$$u_t^L = (1 - \eta_t^S) {}_4u_{t-1} + u_{t-1}^L (1 - \eta_t^L) \quad (\text{A.24})$$

$$u_t^S = 1 - u_t^L - n_t^I - n_t^S - n_t^L \quad (\text{A.25})$$

$$y_t = A_t (a^I n_t^I t + a^S n_t^S + a^L n_t^L) \quad (\text{A.26})$$

$$c_t = y_t - \Xi_t^I n_t^I t - \Xi_t^L n_t^L - \Xi_t^S n_t^S - f_1 u_t - h n_t^S - h n_t^L - y_t \frac{\psi}{2} (\pi_t - 1)^2 \quad (\text{A.27})$$

$$\frac{1 + i_t}{1 + i} = \left(\frac{1 + i_{t-1}}{1 + i} \right)^{\gamma_i} \left[\left(\frac{\pi_t}{\pi} \right)^{\gamma_\pi} \left(\frac{y_t}{y} \right)^{\gamma_y} \right]^{1-\gamma_i} \exp(\mu_t^i). \quad (\text{A.28})$$

Total unemployment is defined as $u_t = u_t^L + u_t^S$ and aggregate employment as $n_t = n_t^L + n_t^S + n_t^I$. The exogenously driven process for technology is given by $A_t = A^{1-\rho} A_{t-1}^\rho e^{\mu_t^A}$.

Appendix B

The Ramsey planner maximizes the expected discounted sum of period utility functions conditional on information at time 0 subject to the equilibrium conditions of the competitive economy. For a given stochastic process for productivity, the Ramsey planner chooses $c_t, y_t, i_t, v_{h,t}^S, v_{h,t}^L, \eta_t^S, \eta_t^L, v_{f,t} \phi_t, \phi_t^n, n_t^S, n_t^L, n_t^I, {}_1u_t, {}_2u_t, {}_3u_t, {}_4u_t, u_t^L, u_t^S, w_t, mc_t, \Theta_t^I, \Theta_t^S, \Theta_t^L, \pi_t, \Xi_t^I, \Xi_t^S, \text{ and } \Xi_t^L$ to maximize

$$\begin{aligned} & E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\sigma}}{1-\sigma} \right\} \\ & \text{s.t. (A.1)-(A.27).} \end{aligned}$$

Let $\lambda_i, i = 1, 2, \dots, 27$ be the Lagrange multipliers on constraints (A.1)-(A.27) of the Lagrangian $\mathcal{L}$. The first order conditions of the maximization problem are (A.1)-(A.27) and

$$\begin{aligned}
\frac{\partial \mathbf{L}}{\partial c_t} = & \lambda 27_t + \frac{1}{c_t^\sigma} + \frac{\sigma c_t^{\sigma-1} \lambda 1_{t-1}}{\beta} - \mathbf{E}_t \left\{ \frac{\Theta_{t+1}^I \sigma \beta c_t^{\sigma-1} \lambda 6_t}{c_{t+1}^\sigma} \right\} \\
& - \mathbf{E}_t \left\{ \frac{\Theta_{t+1}^S \sigma \beta c_t^{\sigma-1} \lambda 11_t}{c_{t+1}^\sigma} \right\} - \mathbf{E}_t \left\{ \frac{\Theta_{t+1}^L \sigma \beta c_t^{\sigma-1} \lambda 13_t}{c_{t+1}^\sigma} \right\} \\
& - \frac{\sigma i_t \beta c_t^{\sigma-1} \lambda 1_t}{\pi_{t+1}} + \frac{\Theta_t^I \sigma c_{t-1}^\sigma \lambda 6_{t-1}}{c_t^{1+\sigma}} + \frac{\Theta_t^S \sigma c_{t-1}^\sigma \lambda 11_{t-1}}{c_t^{1+\sigma}} + \frac{\Theta_t^L \sigma c_{t-1}^\sigma \lambda 13_{t-1}}{c_t^{1+\sigma}} \\
& + \mathbf{E}_t \left\{ \frac{\Theta_{t+1}^I \sigma c_t^{\sigma-1} \beta \lambda 7_t (\phi_t - 1)}{c_{t+1}^\sigma} \right\} + \mathbf{E}_t \left\{ \frac{(\phi_t - 1) \Theta_{t+1}^I \sigma c_t^{\sigma-1} \beta \lambda 9_t}{c_{t+1}^\sigma} \right\} \\
& + \mathbf{E}_t \left\{ \frac{(\phi_t - 1) \Theta_{t+1}^I \sigma c_t^{\sigma-1} \beta \lambda 10_t}{c_{t+1}^\sigma} \right\} - \frac{\Theta_t^I \sigma c_{t-1}^\sigma \lambda 7_{t-1} (\phi_{t-1} - 1)}{c_t^{1+\sigma}} \\
& - \frac{(\phi_{t-1} - 1) \Theta_t^I \sigma c_{t-1}^\sigma \lambda 9_{t-1}}{c_t^{1+\sigma}} - \frac{(\phi_{t-1} - 1) \Theta_t^I \sigma c_{t-1}^\sigma \lambda 10_{t-1}}{c_t^{1+\sigma}} \\
& - \mathbf{E}_t \left\{ \frac{(\pi_{t+1} - 1) y_{t+1} \sigma \psi \pi_{t+1} \beta c_t^{\sigma-1} \lambda 2_t}{c_{t+1}^\sigma y_t} \right\} + \frac{(\pi_t - 1) y_t \sigma \psi \pi_t c_{t-1}^\sigma \lambda 2_{t-1}}{c_t^{1+\sigma} y_{t-1}} = 0
\end{aligned} \tag{B.1}$$

$$\begin{aligned}
\frac{\partial \mathbf{L}}{\partial \eta_t^L} = & \lambda 14_t - u_{t-1}^L \lambda 17_t + u_{t-1}^L \lambda 24_t \\
& - \frac{\lambda 16_t \left(sd \log \left(1 + \exp \left(\frac{v_{h,t}^L}{sd} \right) \right) + \frac{v_{h,t}^L}{1 + \exp \left(\frac{v_{h,t}^L}{sd} \right)} - v_{h,t}^L \right)}{(\eta_t^L)^2} = 0
\end{aligned} \tag{B.2}$$

$$\begin{aligned}
\frac{\partial \mathbf{L}}{\partial \eta_t^S} = & \lambda 12_t + {}_1 u_{t-1} \lambda 21_t + {}_2 u_{t-1} \lambda 22_t + {}_3 u_{t-1} \lambda 23_t + {}_4 u_{t-1} \lambda 24_t - u_{t-1}^S \lambda 18_t \\
& - \frac{\lambda 15_t \left(sd \log \left(1 + \exp \left(\frac{v_{h,t}^S}{sd} \right) \right) + \frac{v_{h,t}^S}{1 + \exp \left(\frac{v_{h,t}^S}{sd} \right)} - v_{h,t}^S \right)}{(\eta_t^S)^2} = 0
\end{aligned} \tag{B.3}$$

$$\frac{\partial \mathbf{L}}{\partial i_t} = \mathbf{E}_t \left\{ \frac{c_t^\sigma \beta \lambda 1_t}{\pi_{t+1}} \right\} = 0 \tag{B.4}$$

$$\begin{aligned}
\frac{\partial \mathbf{L}}{\partial m c_t} = & (\phi_t - 1) A_t a^I \lambda 7_t - A_t a^I \lambda 6_t - A_t a^L \lambda 13_t - A_t a^S \lambda 11_t - \epsilon \lambda 2_t \\
& + (\phi_t - 1) A_t a^I \lambda 9_t + (\phi_t - 1) A_t a^I \lambda 10_t - \gamma A_t a^I \lambda 3_t = 0
\end{aligned} \tag{B.5}$$

$$\begin{aligned}
\frac{\partial \mathbf{L}}{\partial n_t^I} = & \lambda 19_t + \lambda 25_t + \Xi_t^I \lambda 27_t + \beta \mathbf{E}_t \{ \lambda 19_{t+1} (\phi_{t+1} \\
& - 1) \} - A_t a^I \lambda 26_t - \beta \mathbf{E}_t \{ \phi_{t+1} \lambda 20_{t+1} \} = 0
\end{aligned} \tag{B.6}$$

$$\begin{aligned} \frac{\partial \mathbf{L}}{\partial n_t^L} &= \beta \mathbf{E}_t \{ \lambda 19_{t+1} (\phi_{t+1} - 1) \} + \lambda 17_t + \lambda 25_t + \lambda 27_t (h \\ &\quad + \Xi_t^L) - A_t a^L \lambda 26_t - \beta \mathbf{E}_t \{ \phi_{t+1} \lambda 20_{t+1} \} = 0 \end{aligned} \quad (\text{B.7})$$

$$\begin{aligned} \frac{\partial \mathbf{L}}{\partial n_t^S} &= \beta \mathbf{E}_t \{ \lambda 19_{t+1} (\phi_{t+1} - 1) \} + \lambda 18_t + \lambda 25_t + \lambda 27_t (h \\ &\quad + \Xi_t^S) - A_t a^S \lambda 26_t - \beta \mathbf{E}_t \{ \phi_{t+1} \lambda 20_{t+1} \} = 0 \end{aligned} \quad (\text{B.8})$$

$$\begin{aligned} \frac{\partial \mathbf{L}}{\partial \phi_t} &= \lambda 5_t + \lambda 19_t (n_{t-1}^S + n_{t-1}^L + n^I t - 1) - (n_{t-1}^S + n_{t-1}^L + n^I t - 1) \lambda 20_t + \lambda 7_t \left(m c_t \right. \\ &\quad \left. A_t a^I + f - w_t - \Xi_t^I + \mathbf{E}_t \left\{ \frac{\beta c_t^\sigma \Theta_{t+1}^I}{c_{t+1}^\sigma} \right\} \right) + \lambda 9_t \left(m c_t A_t a^I + f - w_t - \Xi_t^I + \mathbf{E}_t \right. \\ &\quad \left. \left\{ \frac{\beta c_t^\sigma \Theta_{t+1}^I}{c_{t+1}^\sigma} \right\} \right) + \lambda 10_t \left(m c_t A_t a^I + f - w_t - \Xi_t^I \right. \\ &\quad \left. + \mathbf{E}_t \left\{ \frac{\beta c_t^\sigma \Theta_{t+1}^I}{c_{t+1}^\sigma} \right\} \right) + \frac{\lambda 8_t \left(s d \log (1 + \exp (\frac{v_{f,t}}{s d})) + \frac{v_{f,t}}{1 + \exp (\frac{v_{f,t}}{s d})} - v_{f,t} \right)}{(\phi_t - 1)^2} = 0 \end{aligned} \quad (\text{B.9})$$

$$\frac{\partial \mathbf{L}}{\partial \phi_t^n} = \lambda 4_t + \lambda 5_t (\phi^x - 1) = 0 \quad (\text{B.10})$$

$$\begin{aligned} \frac{\partial \mathbf{L}}{\partial \pi_t} &= \lambda 2_t (\psi (\pi_t - 1) + \psi \pi_t) - \lambda 2_{t-1} \left(\frac{(\pi_t - 1) y_t \psi c_{t-1}^\sigma}{c_t^\sigma y_{t-1}} \right. \\ &\quad \left. + \frac{y_t \psi \pi_t c_{t-1}^\sigma}{c_t^\sigma y_{t-1}} \right) + y_t \psi \lambda 27_t (\pi_t - 1) + \frac{\lambda 1_{t-1} c_{t-1}^\sigma i_{t-1}}{\pi_t^2} = 0 \end{aligned} \quad (\text{B.11})$$

$$\begin{aligned} \frac{\partial \mathbf{L}}{\partial \Theta_t^I} &= \lambda 7_t + \frac{c_{t-1}^\sigma \lambda 7_{t-1} (\phi_{t-1} - 1)}{c_t^\sigma} + \frac{c_{t-1}^\sigma (\phi_{t-1} - 1) \lambda 9_{t-1}}{c_t^\sigma} \\ &\quad + \frac{c_{t-1}^\sigma (\phi_{t-1} - 1) \lambda 10_{t-1}}{c_t^\sigma} - \frac{c_{t-1}^\sigma \lambda 6_{t-1}}{c_t^\sigma} = 0 \end{aligned} \quad (\text{B.12})$$

$$\frac{\partial \mathbf{L}}{\partial \Theta_t^L} = \lambda 10_t - \frac{c_{t-1}^\sigma \lambda 13_{t-1}}{c_t^\sigma} = 0 \quad (\text{B.13})$$

$$\frac{\partial \mathbf{L}}{\partial \Theta_t^S} = \lambda 9_t - \frac{c_{t-1}^\sigma \lambda 11_{t-1}}{c_t^\sigma} = 0 \quad (\text{B.14})$$

$$\frac{\partial \mathbf{L}}{\partial u_t} = \lambda 20_t + f \lambda 27_t + \beta \mathbf{E}_t \{ \lambda 21_{t+1} (\eta_{t+1}^S - 1) \} = 0 \quad (\text{B.15})$$

$$\frac{\partial \mathbb{L}}{\partial_2 u_t} = \lambda 21_t + \beta \mathbb{E}_t \{ (\eta_{t+1}^S - 1) \lambda 22_{t+1} \} = 0 \quad (\text{B.16})$$

$$\frac{\partial \mathbb{L}}{\partial_3 u_t} = \lambda 22_t + \beta \mathbb{E}_t \{ (\eta_{t+1}^S - 1) \lambda 23_{t+1} \} = 0 \quad (\text{B.17})$$

$$\frac{\partial \mathbb{L}}{\partial_4 u_t} = \lambda 23_t + \beta \mathbb{E}_t \{ (\eta_{t+1}^S - 1) \lambda 24_{t+1} \} = 0 \quad (\text{B.18})$$

$$\frac{\partial \mathbb{L}}{\partial u_t^L} = \lambda 24_t + \lambda 25_t - \beta \mathbb{E}_t \{ \eta_{t+1}^L \lambda 17_{t+1} \} + \beta \mathbb{E}_t \{ \lambda 24_{t+1} (\eta_{t+1}^L - 1) \} = 0 \quad (\text{B.19})$$

$$\frac{\partial \mathbb{L}}{\partial u_t^S} = \lambda 25_t - \beta \mathbb{E}_t \{ \eta_{t+1}^S \lambda 18_{t+1} \} = 0 \quad (\text{B.20})$$

$$\begin{aligned} \frac{\partial \mathbb{L}}{\partial v_{f,t}} = & \lambda 6_t - \frac{\lambda 8_t \left(\frac{\exp\left(\frac{v_{f,t}}{sd}\right)}{1+\exp\left(\frac{v_{f,t}}{sd}\right)} + \frac{1}{1+\exp\left(\frac{v_{f,t}}{sd}\right)} - \frac{v_{f,t} \exp\left(\frac{v_{f,t}}{sd}\right)}{sd \left(1+\exp\left(\frac{v_{f,t}}{sd}\right)\right)^2} - 1 \right)}{\phi_t - 1} \\ & + \frac{\exp\left(\frac{(-v_{f,t})}{sd}\right) \lambda 4_t}{sd \left(1 + \exp\left(\frac{(-v_{f,t})}{sd}\right)\right)^2} = 0 \end{aligned} \quad (\text{B.21})$$

$$\begin{aligned} \frac{\partial \mathbb{L}}{\partial v_{h,t}^L} = & \lambda 13_t + \frac{\lambda 16_t \left(\frac{\exp\left(\frac{v_{h,t}^L}{sd}\right)}{1+\exp\left(\frac{v_{h,t}^L}{sd}\right)} + \frac{1}{1+\exp\left(\frac{v_{h,t}^L}{sd}\right)} - \frac{v_{h,t}^L \exp\left(\frac{v_{h,t}^L}{sd}\right)}{sd \left(1+\exp\left(\frac{v_{h,t}^L}{sd}\right)\right)^2} - 1 \right)}{\eta_t^L} \\ & - \frac{\exp\left(\frac{(-v_{h,t}^L)}{sd}\right) \lambda 14_t}{sd \left(1 + \exp\left(\frac{(-v_{h,t}^L)}{sd}\right)\right)^2} = 0 \end{aligned} \quad (\text{B.22})$$

$$\begin{aligned} \frac{\partial \mathbb{L}}{\partial v_{h,t}^S} = & \lambda 11_t + \frac{\lambda 15_t \left(\frac{\exp\left(\frac{v_{h,t}^S}{sd}\right)}{1+\exp\left(\frac{v_{h,t}^S}{sd}\right)} + \frac{1}{1+\exp\left(\frac{v_{h,t}^S}{sd}\right)} - \frac{v_{h,t}^S \exp\left(\frac{v_{h,t}^S}{sd}\right)}{sd \left(1+\exp\left(\frac{v_{h,t}^S}{sd}\right)\right)^2} - 1 \right)}{\eta_t^S} \\ & - \frac{\exp\left(\frac{(-v_{h,t}^S)}{sd}\right) \lambda 12_t}{sd \left(1 + \exp\left(\frac{(-v_{h,t}^S)}{sd}\right)\right)^2} = 0 \end{aligned} \quad (\text{B.23})$$

$$\begin{aligned} \frac{\partial \mathbf{L}}{\partial w_t} = & \lambda 13_t + \lambda 11_t + \lambda 6_t + \lambda 3_t - \lambda 7_t (\phi_t - 1) - (\phi_t - 1) \lambda 9_t \\ & - (\phi_t - 1) \lambda 10_t = 0 \end{aligned} \quad (\text{B.24})$$

$$\begin{aligned} \frac{\partial \mathbf{L}}{\partial \Xi_t^I} = & \lambda 8_t + n_t^I \lambda 27_t - \lambda 7_t (\phi_t - 1) - (\phi_t - 1) \lambda 9_t - (\phi_t \\ & - 1) \lambda 10_t = 0 \end{aligned} \quad (\text{B.25})$$

$$\frac{\partial \mathbf{L}}{\partial \Xi_t^L} = \lambda 16_t + n_t^L \lambda 27_t = 0 \quad (\text{B.26})$$

$$\frac{\partial \mathbf{L}}{\partial \Xi_t^S} = \lambda 15_t + n_t^S \lambda 27_t = 0 \quad (\text{B.27})$$

$$\begin{aligned} \frac{\partial \mathbf{L}}{\partial y_t} = & \lambda 26_t + \lambda 27_t \left(\frac{\psi (\pi_t - 1)^2}{2} - 1 \right) \\ & - \frac{(\pi_t - 1) \psi \pi_t c_{t-1}^\sigma \lambda 2_{t-1}}{y_{t-1} c_t^\sigma} + \mathbf{E}_t \left\{ \frac{(\pi_{t+1} - 1) y_{t+1} \psi \pi_{t+1} c_t^\sigma \beta \lambda 2_t}{c_{t+1}^\sigma y_t^2} \right\} = 0. \end{aligned} \quad (\text{B.28})$$

Note that λj , $j = \{1, 2, 6, 7, 9, 10, 11, 13\}$ are state variables. From a timeless perspective, the λj_{-1} take on their steady state values.

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