

Parallel Trade in General Equilibrium*

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Abstract

We introduce a simple two-country, monopolistic competition model of parallel trade policy with potentially imperfect enforcement of parallel trade prohibition. Consumers have 0/1 preferences such that large per capita income differences can induce some Northern firms to refrain from exporting to the South in order to avoid international arbitrage. We show that parallel trade restrictions have a pro-competitive effect on prices and change the terms of trade in favour of the poor country. Hence, the South prefers to ban parallel trade while the North prefers to allow it. The global welfare-maximising policy is to forbid parallel trade. Moreover, trade liberalisation increases the terms of trade of the South if there is a sufficiently strong parallel trade prohibition. Introducing income heterogeneity within countries may reveal diverging preferences on parallel trade policy between rich and poor consumers.

Keywords: non-homothetic preferences, parallel imports, arbitrage, extensive margin, North-South trade, income inequality

JEL classification: F10, F12, F13

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1 Introduction

In recent years there has been a significant shift in the approach to international patent exhaustion in many economies, including most major ones. This shift involves the adoption or confirmation of policies of international patent exhaustion, whereby patents are considered exhausted after the first sale of a good, regardless of where the sale takes place. This policy shift raises the question of whether it represents a universally optimal policy or whether it primarily reflects the preferences of a particular group of countries.

We present a two-country general equilibrium model of international trade with non-homothetic preferences, and discuss the role of parallel trade policy restrictions. The model builds on the framework introduced by [Föllmi, Hopenstrick and Zweimüller \(2018, hereafter FHZ\)](#). We extend their model along two different dimensions: (i) We include an explicit choice of parallel trade policy. Thus, a government's trade policy is a joint decision on the optimal level of parallel trade and trade costs. (ii) We abandon the assumption of a representative agent within countries and allow for income inequality by introducing heterogeneous labour endowments. To the best of our knowledge, we are the first to study the distributional aspects of parallel trade policy.

Our model reveals diverging preferences on parallel trade between the rich North and the poor South. The North prefers to allow parallel trade, because it can save resources by limiting exports to the South. Consequently, the South prefers to ban parallel trade in order to gain access to a wider range of goods. In case parallel trade is allowed, we also get diverging preferences regarding trade costs. The North has a standard preference for free trade, while the South wants to raise the barriers of trade. High trade costs reduce the arbitrage threat for Northern firms, thereby increasing exports to the benefit of Southern consumers. In contrast, if parallel trade is banned, both countries prefer free trade. The social planner would forbid parallel trade, because the welfare losses of the South caused by limited access to Northern goods strictly outweigh the gains for Northern consumers. Furthermore, we analyse the effects of a divergence or convergence in income between North and South. Unsurprisingly, an income convergence benefits both countries. The South can afford more goods from the North, while producing and exporting more varieties to the North, resulting in benefits for all consumers. However, only the North benefits from an income divergence. All additional resources of the North go into the production of new, non-traded varieties, which the South is unable to afford. Thus, our model predicts asymmetric welfare gains as countries grow richer.

In the presence of income inequality within countries, the threat of parallel imports is much less likely to give rise to export zeros. This is because the richest consumers in both countries can afford all products, unless they strongly differ in endowment or trade costs are prohibitively high. Consumer preferences for parallel trade policy within a country may no longer be uniform. Under FOSD, we can confirm the findings from the previous sections, that the rich country prefers to allow parallel trade, while the poor one prefers to forbid it. However, following a mean preserving spread in income in one country, diverging preferences emerge between wealthy consumers and most others. In addition, consumers within the same income group from different countries cannot agree either. Specifically, poor consumers in the less (more) equal country prefer to allow (forbid) parallel trade, while rich consumers prefer to forbid (allow) it.

1.1 Related Literature

Pricing-to-Market. [Krugman \(1986\)](#) was the first to study pricing-to-market.¹ He documented that European firms do not fully pass through movements in the exchange rate to US importers but rather maintain their export prices tied to local US prices. For more recent empirical evidence on pricing-to-market, see for example [Alessandria and Kaboski \(2011\)](#), [Gopinath, Itskhoki and Rigobon \(2010\)](#) or [Fitzgerald and Haller \(2014\)](#). On the theoretical side, [Atkeson and Burstein \(2008\)](#) can generate pricing-to-market behaviour of firms in a setting with Cournot competition, variable markups and marginal cost-push shocks. [Auer, Chaney and Sauré \(2018\)](#) explore how market-specific valuations for product quality affect relative prices. [Corsetti and Dedola \(2005\)](#) include nontradable inputs, such that the price elasticity of demand is country-specific and dependent on the exchange rate, causing international price discrimination. In our model, pricing-to-market is driven by differences in the willingness to pay across countries (and across consumers within countries). Moreover, poor consumers are worse off if they are a small minority in a predominantly rich country. We refer to this as the *Manhattan effect*. This is consistent with the empirical findings of [Handbury \(2021\)](#), who finds that poor households face higher food prices in rich relative to poor cities. [Alessandria and Kaboski \(2011\)](#), [Simonovska \(2015\)](#) or [Nord \(2022\)](#) find a similar pattern in pricing.

International Arbitrage. Pricing-to-market due to heterogeneous consumers within or

¹Pricing-to-Market refers to the situation where producers choose varying markups, and hence varying prices, for the same good in different markets.

across countries can create international arbitrage opportunities. In our representative agent case, firms face a trade-off between price and market size. If a firm charges a high price, it can only sell its products in the rich North and thus has a small market. This exclusion strategy leads to export zeros for this particular product. When a firm offers its product on the global market, it must consider the possibility of parallel imports.² Charging different prices in the two markets creates an opportunity for arbitrageurs to purchase the product cheaply in the South and ship it into the North where they can sell it at a slightly lower price than the official dealer and still make a profit. To avoid losing the Northern market to the arbitrageurs, the firm has to charge a reduced price in the North, constrained by the trade costs. [Malueg and Schwartz \(1994\)](#) present a partial equilibrium model with a single monopolistic producer per country. They find that banning parallel imports and allowing discriminatory pricing improves world welfare when demand dispersion across markets is large, because it allows to serve more markets. Our model predictions are consistent with their findings. [Grossman and Lai \(2008\)](#) discuss the effect of parallel imports on innovation in the context of government price controls applied to the pharmaceutical market. They show that incentives for product innovation in the North may be higher if the North allows parallel imports, but that the South will suffer welfare losses. [Li and Maskus \(2006\)](#) find that the distortions associated with parallel imports inhibit innovation. However, they point out that the global welfare implications depend on the exact interplay between parallel trade restrictions and trade costs. These two dynamic perspectives on the impact of parallel import policy can be seen as complementary to our static treatment focusing on inequality. [Roy and Saggi \(2012\)](#) model a North-South quality-differentiated duopoly competing in prices. They show that when between-country inequality is high, the North has an incentive to forbid parallel imports, since consumers in the North are harmed by the high price of the Northern duopolist, which refrains from exporting to the South due to the threat of arbitrage. In our model, the North always prefers to allow parallel imports as it benefits from an increased domestic variety when export exclusion occurs. The difference in our model's preferred parallel trade prediction is rooted in the possibility of firm entry. Allowing parallel trade induces some rich country firms to exclude the poor country market, which leaves more resources in the rich country and thus allows more firms to be established. We are the first to study

²Parallel trade occurs when a good protected by a patent, copyright, or trademark, which has been legally acquired in one country, is exported to another without the permission of the local intellectual property rights holder in the importing market (see [Maskus \(2000\)](#)). Hence, parallel imports are goods imported by unauthorized resellers.

parallel trade in general equilibrium and to point out this resource feedback mechanism.³

Inequality & Trade. Most of the literature focuses on the effect of trade on inequality.⁴ While there is a consensus that trade can be detrimental to employment and wages at the local level, the distributional consequences of trade from a consumer perspective (at the country level) are much more controversial, with findings ranging from pro-poor to distributionally neutral or even pro-rich. Our work goes in the opposite direction, i.e. the effect of inequality on trade. Under between-country inequality, our model can replicate the [Linder \(1961\)](#) hypothesis, namely that higher per capita income of the poor country unambiguously increases the intensity of trade. [Hallak \(2010\)](#) finds support for the Linder hypothesis, both empirically and theoretically, using GDP per capita to capture demand similarity across countries. [Fajgelbaum, Grossman and Helpman \(2015\)](#) can replicate the Linder hypothesis in an FDI context, such that FDI is greater between countries with more similar per capita income levels. [Bernasconi \(2013\)](#), [Bohman and Nilsson \(2007\)](#) and [Choi et al. \(2009\)](#) study the effect of within-country inequality on trade patterns, while [Kichko and Picard \(2023\)](#) show theoretically that lower inequality not only reduces the volume and value of trade, but also creates a general equilibrium effect that may negatively affect poor individuals.

Finally, our perspective of allowing demand heterogeneity to shape the patterns of trade should be seen as complementary to the firm heterogeneity approach initiated by [Melitz \(2003\)](#) and subsequently explored by [Arkolakis et al. \(2012\)](#); [Chaney \(2008\)](#) and empirically confirmed by [Bernard et al. \(2007, 2009\)](#); [Helpman et al. \(2008\)](#); [Hummels and Klenow \(2005\)](#). In our model, all firms have identical Dixit-Stiglitz production technology. Thus, trade patterns are entirely driven by consumer heterogeneity, in contrast to the Melitz framework, where productivity cutoffs explain product-level trade zeros.

The remainder of the paper is structured as follows: Section 2 provides an overview of the current parallel trade policies and reviews the empirical estimates of parallel trade volumes. Section 3 introduces the model and describes the autarky equilibrium. Section 4 presents the two trade equilibria, while section 5 discusses the arbitrage equilibrium and its welfare implications. Income inequality is introduced in section 6. Section 7 concludes.

³There exists a narrow literature on *compulsory licensing*, the second major flexibility that WTO members may use in their patent policy next to the exhaustion regime. See [Bond and Saggi \(2014, 2018, 2020\)](#) for theoretical work on this issue.

⁴See, for example, [Acemoglu et al. \(2016\)](#); [Adao et al. \(2022\)](#); [Autor et al. \(2013\)](#); [Behrens and Murata \(2012\)](#); [Borushy and Jaravel \(2021\)](#); [Fajgelbaum and Khandelwal \(2016\)](#); [Galle et al. \(2023\)](#); [Waugh \(2023\)](#).

2 Parallel Trade in Practice

2.1 Institutional Context

In general, the Agreement on Trade-Related Aspects of Intellectual Property Rights (TRIPS), does not provide for a uniform approach to parallel trade policy, leaving it to each member state to decide on a preferred policy.⁵ At the same time, Article 6 of the Agreement states that nothing in the Agreement "shall be used to address the issue of the exhaustion of intellectual property rights" in dispute settlement, meaning that any position on parallel trade is valid in the eyes of the WTO and therefore cannot be challenged in court (WTO, 1994).⁶

Countries follow one of three approaches: national, regional, or international exhaustion of rights (Maskus, 2000). Under national exhaustion, the intellectual property right exhausts on first sale if the sale is made within the country. Under this rule, resales within a country are legal, but parallel trade is not. This allows firms to engage in third-degree price discrimination and thus segment markets.⁷ Consequently, this approach is the least friendly to international trade. Countries that follow national exhaustion are, for example, Brazil, Mexico, South Korea, Indonesia and several African economies, including Mozambique, Rwanda and Uganda (Calboli, 2022). Regional exhaustion means that patent protection is exhausted after an initial sale within a region, i.e. a particular group of countries such as the European Union. Therefore, parallel trade is allowed within that group of countries, but not if the sale was made outside of it. This system is applied in the EU⁸ and the OAPI (the African Intellectual Property Organisation), which includes, among others, countries such as Cameroon, Ivory Coast, Mali, and Senegal. Concerning the EU, the European Court of Justice (ECJ) ruled as early as 1974 that national exhaustion of rights violated the free movement of goods

⁵When talking about parallel trade, it is important to note that the term applies only to 'genuine' goods, i.e. goods that were manufactured and placed on the market either by the property rights holder or with his explicit consent. Goods manufactured without the consent of the rights holder are considered infringing goods and are not covered by exhaustion of rights regulations. While IPR infringements are an integral part of any FTA negotiation (especially for FTAs between rich and poor countries), they are not the focus of this paper. We assume that all goods are produced by the property rights holder. See for example Bond and Saggi (2020) for an analysis of IPR infringements and imitations.

⁶The General Provisions and Basic Principles (including Article 6) of the TRIPS Agreement can be found here: https://www.wto.org/english/docs_e/legal_e/trips_e.htm#part1.

⁷The assumption of national exhaustion is the standard assumption in international trade theory and allows firms to engage in pricing-to-market.

⁸More specifically, in the European Economic Area (EEA), including Liechtenstein, Norway and Island. Switzerland and the United Kingdom (after Brexit) have unilaterally tied themselves to that approach, which means that parallel imports from the EEA into Switzerland and the UK are allowed, while the EEA does not recognise parallel imports originating from these two countries.

postulated in Article 30 of the Treaty of Rome, meaning that the integrity of the single market trumped intellectual property rights.⁹ This extends to arbitrage opportunities created by national price controls, as regularly applied to pharmaceuticals (Maskus, 2000).¹⁰

Finally, international exhaustion means that the intellectual property right expires after the first sale of the good, regardless of where it is sold. This approach allows for parallel imports from all over the world and is therefore the most friendly to international trade. It is practised by Canada, China, India, Japan, and the US, among others (Calboli, 2022). Until recently, the US followed the approach of national exhaustion. However, following a series of Supreme Court decisions, the US has adopted international exhaustion. First, in the area of copyright, in *Kirtsaeng v. John Wiley & Sons, Inc.* (2013), the Supreme Court held that foreign sales of copyrighted products exhaust the US copyright. In 2017, the international exhaustion of rights was extended to patent rights in the decision *Lexmark v. Impression Products*, overturning several lower court decisions and decades of US patent policy (Ivus and Lai, 2017). However, notable exceptions remain for the pharmaceutical sector and for contractual restrictions that explicitly prohibit parallel imports. China shifted from national to international exhaustion with its new patent law in 2008, while India already did so in 2002 (Calboli, 2022). See table 1 for an overview.

Table 1: Countries by Patent Exhaustion Regime

National	Regional	International	Switched to International
Australia		Canada	
Brazil	EU	Chile	China (2008)
Indonesia	OAPI	Japan	India (2002)
Mexiko		Singapore	United States (2017)
South Korea		Vietnam	

Note: Own visualisation based on Calboli (2022); Maskus (2000). The list is non-exhaustive. For countries that switched to international exhaustion, the date of the switch is indicated in the bracket.

2.2 Parallel Trade Volume

Empirically, the extent of parallel trade is a rather understudied topic, partly because parallel trade is difficult to observe. Since parallel trade is legal in many parts of the world, there is no reason for customs offices to keep a record of whether a particular good is being imported

⁹See *Centrafarm B.V. and Adriaan de Peijper v. Sterling Drug Inc.*, 6 IIC 102 (1975) - *Negram III*.

¹⁰The ECJ rulings were codified by a European Council Directive in 2008 (Directive 2008/95/EC) and updated in 2015 (Directive 2015/2436/EC).

with or without the explicit consent of the patent holder. Nevertheless, there is convincing evidence that the possibility of parallel trade affects firms' pricing decisions in many markets.

The industries most affected are automobiles (Lutz, 2004; Yeung and Mok, 2013), consumer electronics (Burgess and Evans, 2005) and, most prominently, pharmaceuticals (Dubois and Sæthre, 2020; Dubois et al., 2022; Maskus, 2000; Goldberg, 2010). A report commissioned by the European Union estimated that parallel trade accounts for 10% - 20% of total trade volume within the EU (NERA, 1999), while Ganslandt and Maskus (2004) found that medicines subject to parallel imports experienced price reductions of between 12% - 19% compared to other medicines not subject to arbitrage.

Although these figures give the impression that parallel trade may be of secondary importance, this conclusion would be premature. The volume of parallel trade need not be large, or even exist, for it to have a significant impact on a firm's pricing and export decisions.¹¹ The mere possibility of parallel trade is enough for firms to adjust their pricing schedule or stop exporting altogether so that parallel trade does not occur.

Goldberg (2010) found that multinational pharmaceutical producers often refrain from exporting to India because the possibility of arbitrage and the practice of "global reference pricing" by several rich countries would lead to greater profit losses in high-price countries than profit gains from entering the Indian market. Lanjouw (2005) shows that pharmaceutical companies systematically delay entry into developing markets in order to enjoy monopoly profits unhindered by the threat of arbitrage. Danzon and Epstein (2008) find that a similar effect is at play between high- and low-price EU countries. Similarly, Ivus (2010) examines the effect of stronger intellectual property rights enforcement in response to the TRIPS agreement. She finds that stronger IPRs lead to a significant increase in export volumes to developing countries.

Regarding binding arbitrage price constraints, FHZ calibrate a multi-country trade model based on the framework of Simonovska (2015). They find that about 20% of all bilateral trade relations, representing 45% of total world trade, are subject to arbitrage threats. Furthermore, they find that arbitrage is particularly important for EU member states such as Germany, France and (at that time) the UK, due to the high degree of integration and low trade costs within Europe. This discussion suggests that the possibility of parallel trade has far-reaching implications for firms' pricing and export decisions, intending to effectively prevent parallel trade.

¹¹In our model, parallel trade will *never* occur in equilibrium because firms adjust their behaviour accordingly.

3 Autarky

3.1 Consumers and Labour Market

The economy is populated by $\mathcal{P}$ identical households. Each household is endowed with L units of labour, which it supplies inelastically to the labour market. We assume that the labour market is perfectly competitive within countries and that workers are paid their marginal product. Labour is immobile across countries. Households spend their income on a continuum of differentiated goods. Goods are indivisible and yield positive utility for the first unit and zero utility for each additional unit. Thus, the consumption decision for each good is a discrete choice between buying one unit of the good or not buying it at all.¹² Let $c(j)$ denote an indicator that takes the value 1 if good j is purchased and the value 0 if not. Then, preferences can be represented by the utility function:

$$U = \int_{\Omega} c(j) dj, \quad \text{with } c(j) \in \{0, 1\}$$

where Ω denotes the set of products available in a country. These preferences are additively separable, non-homothetic and have only an extensive consumption margin. The budget constraint is given by $\int_0^N p(j)c(j) \leq WL$, where W is the wage rate.¹³ Utility maximisation gives the binary purchase decision rule:

$$\begin{aligned} c(j) &= 1 & \text{if } 1 \geq \lambda p(j) \\ c(j) &= 0 & \text{if } 1 < \lambda p(j) \end{aligned}$$

where λ is the Lagrange multiplier and $1/\lambda$ can be interpreted as the willingness to pay. A consumer will purchase one unit of good j if the willingness to pay is at least equal to the good's price $p(j)$. Figure 1 illustrates the individual demand function that results from this utility function. Individual demand is a one-step function of the price, where the reservation

¹²Such preferences have been used, among others, by [Falkinger \(1994\)](#) and [Foellmi and Zweimüller \(2006\)](#) to analyse inequality and growth, by [Matsuyama \(2000\)](#) to explore non-homotheticities in a Ricardian trade model, by [Murphy et al. \(1989\)](#) to study the process of industrialisation, and by [Föllmi et al. \(2018\)](#) to explain trade zeros caused by demand-side heterogeneity. The preferences used in [Fajgelbaum, Grossman and Helpman \(2011, 2015\)](#); [Mongey and Waugh \(2024\)](#), which build on the theory of [McFadden \(1978\)](#), exhibit a similar discrete choice.

¹³Note that the integral in the utility function is defined over the whole set of goods available in the economy Ω , while the budget constraint is defined from zero to N , which denotes the set of goods actually purchased by the consumer.

price is the inverse of the consumer's marginal utility of income.¹⁴

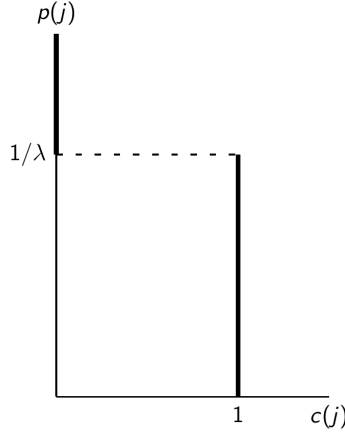


Figure 1: Individual demand function

3.2 Firms and Production

On the supply side, we assume a [Krugman \(1980\)](#) model of international trade with a mass of firms each producing a single, differentiated product, with fixed costs of production and labour as the only production input. Firms are homogeneous, with $1/a$ denoting the marginal costs of production and F the fixed costs (in units of labour). Producing a quantity q of good j thus requires a total labour input of $L(j) = F + q(j)/a$. Given our preferences, market size is given by the population size $\mathcal{P}$, which means that the firm's decision problem boils down to choosing the optimal price $p(j)$. There is free entry into the market. Therefore, the zero-profit conditions endogenously determine the number of active firms in equilibrium.

3.3 Equilibrium

The equilibrium is symmetric since both households and firms are homogeneous. The price of goods is determined by the consumer's willingness to pay, $p = 1/\lambda$, and firms sell one unit to each consumer, implying a total sales volume of $\mathcal{P}$. Without loss of generality, we use the wage rate as numéraire, $W = 1$. As the first equilibrium condition, the zero-profit condition

¹⁴Note that because of the symmetry between goods, there is ex-ante indeterminacy as to which products are consumed first. Introducing a "hierarchy of needs" would resolve this. However, this indeterminacy is unproblematic as long as we look only at equilibrium outcomes. A hierarchy may be operating in the background, but it does not need to be modelled explicitly.

$[p - W/a]\mathcal{P} = WF$ determines the equilibrium goods price at $p = (aF + \mathcal{P})/a\mathcal{P} = 1/\lambda$. The implied markup, the ratio of price over marginal costs, is then

$$\mu = \frac{aF + \mathcal{P}}{\mathcal{P}}$$

Note that markups are determined by technology parameters a and F and market size $\mathcal{P}$.¹⁵ The markup for products with a small market (luxury goods) is larger than the markup for products with a large market (necessary goods). This feature will be important when introducing income inequality in the later section.

The second equilibrium condition is the resource constraint in the economy, which ensures full employment $[F + \mathcal{P}/a]N = \mathcal{P}L$. The resource constraint determines the equilibrium product variety:

$$N = \frac{a\mathcal{P}}{aF + \mathcal{P}}L$$

4 Trade between Rich and Poor

Now consider two countries that trade with each other. We refer to them as North and South, where the South is assumed to be the poorer of the two and its variables are denoted by an asterisk. For now, we retain the assumption of the representative agent within economies. The population sizes are $\mathcal{P}$ and $\mathcal{P}^*$, where we make no ex-ante assumption about the relative sizes. The labour endowments are denoted by L and L^* , where we assume that $L > L^*$, which ensures that the North is the rich country and the South the poor one. We set the Northern wage as the numéraire and define the relative wage as $\omega = \frac{W^*}{W} = W^*$.

Trade is costly. We assume that trade costs are of the iceberg type: it costs τ unit of the good to get one unit of the good to arrive at its destination, which means that $\tau - 1$ units are lost during transport. We allow for the enforcement of parallel trade prohibition to be potentially imperfect. Before entering the market, a firm does not know whether parallel trade restrictions will be in place for its product. With probability π , price setting is unrestricted as international property rights are fully enforced, while with probability $1 - \pi$, parallel imports are allowed and firms' price setting is constrained by the threat of parallel imports. Thus, parameter π can be interpreted as a measure of the enforcement of

¹⁵Variable markups, especially in the context of international trade and pricing-to-market, have received considerable attention in recent years. See for example [Amiti, Itskhoki and Konings \(2019\)](#); [Demidova \(2017\)](#); [De Loecker and Eeckhout \(2020\)](#); [Edmond, Midrigan and Xu \(2015\)](#); [Feenstra \(2018\)](#); [Melitz and Ottaviano \(2008\)](#).

international property rights.¹⁶ We assume that parallel trade policy is symmetric, i.e. π is the same in the North and the South.¹⁷

An exporter from the North will charge a price p in the home market and a price p^* in the foreign market. The degree of price discrimination between markets reflects the difference in the willingness to pay between these markets. Naturally, we would expect that $p \geq p^*$, since consumers in the North have a higher willingness to pay. Optimally, producers would like to charge profit-maximising prices in both markets, unconstrained by any arbitrage threats.¹⁸ However, if arbitrage threats are present (the exact conditions for this will be presented later in the paper), the maximum price difference between North and South is equal to the trade costs τ .

When the income gap between North and South is sufficiently small, all goods are traded internationally and all firms can charge full price without the threat of arbitrage. We call this the *full trade equilibrium*. However, when the income gap is large, the price difference between markets is large enough to make arbitrage worthwhile. In such an *arbitrage equilibrium*, it will be the case that some firms in the North choose not to export their products to the South and sell only on the domestic market, at unconstrained prices. The remaining firms in the North will export their product to the South and charge a constrained price at home to prevent parallel imports into their home market.

¹⁶In reality, firms know perfectly well whether parallel trade is allowed or not. However, as discussed in the previous section, the actual extent of parallel trade is uncertain and is likely to vary from time to time. Therefore, a more realistic interpretation of the parameter π would be that parallel trade is legal, but firms are ex-ante uncertain about the extent of parallel trade.

¹⁷Note that the assumption of symmetry in parallel trade policy between North and South is not strictly necessary. We could alternatively set up the model with an asymmetric parallel trade policy. With a (potential) asymmetry, the North and the South would each set their own π , call it π_R and π_P . Strictly speaking, parallel trade restrictions are parallel *import* restrictions, countries allow or do not allow imports into their territory. Thus, π_R (π_P) denotes the probability that parallel imports *into the North (South)* are not allowed, thus protecting firms from arbitrage threats. Since the price level in the North is (weakly) higher than in the South, the prices in the South are effectively the lowest in the world. Products cannot be bought anywhere for less than the price in the South. Therefore, no one will want to conduct parallel imports into the South because the current prices cannot be undercut. Consequently, the parallel trade policy of the South is irrelevant in equilibrium. Hence, it does not matter whether we assume a symmetric or asymmetric parallel trade policy. The model in [Bond and Saggi \(2020\)](#) exhibits the same property.

¹⁸Consider the following example of arbitrage threats: A German pharmaceuticals producer wants to sell its product both in Germany and in Poland. Ideally, it would charge a price in Germany equal to the German consumer's willingness to pay $p^{DE} = 1/\lambda^{DE}$ and a price in Poland equal to the Polish consumer's willingness to pay $p^{PL} = 1/\lambda^{PL}$. If the difference between $1/\lambda^{DE}$ and $1/\lambda^{PL}$ is greater than the trade costs between Germany and Poland, arbitrage opportunities arise. Arbitrageurs can purchase the good cheaply in Poland, transport it back to Germany and undercut the producer on the German market. The producer would therefore charge a maximum price difference of τ in order to prevent arbitrage opportunities from arising in the first place.

4.1 Full Trade Equilibrium

In the full trade equilibrium, parallel trade will never take place due to small price differences between North and South. Free entry implies that firms will earn zero profits in equilibrium. For all firms, total revenue is given by $p\mathcal{P} + p^*\mathcal{P}^*$. Due to the assumption of 0-1 preferences, demand in each market is given by its respective population size. Total costs are $W^i[F + (\mathcal{P}^i + \tau\mathcal{P}^{-i})/a]$, which consist of the fixed costs for setting up the production facility, and the marginal costs of production, taking into account the iceberg trade costs for exported goods. To express costs in money terms, we multiply them with the respective wage rate. Combining the two zero-profit conditions delivers the relative wage rate:

$$\omega = \frac{W^*}{W} = \left(\frac{aF + \mathcal{P} + \tau\mathcal{P}^*}{aF + \mathcal{P}^* + \tau\mathcal{P}} \right)$$

Factor price equalisation can be achieved under one of two conditions: (1) there are no trade costs, $\tau = 1$, or (2) the two countries are equally populous ($\mathcal{P} = \mathcal{P}^*$).¹⁹ Within both economies, the total amount of resources required in production must equal the total amount of resources available. These aggregate resource constraints are given by:

$$\mathcal{P}L = N \left(F + \frac{\mathcal{P} + \tau\mathcal{P}^*}{a} \right) \quad \mathcal{P}^*L^* = N^* \left(F + \frac{\mathcal{P}^* + \tau\mathcal{P}}{a} \right)$$

where N is the number of varieties produced in the North, and N^* number of varieties produced in the South. Given that labour is the only factor of production, the aggregate resource constraints are effectively labour market clearing conditions. From the aggregate resource constraints, we get the equilibrium number of varieties produced in the North and South:

$$N = \frac{aL\mathcal{P}}{aF + \mathcal{P} + \tau\mathcal{P}^*} \quad N^* = \frac{aL^*\mathcal{P}^*}{aF + \mathcal{P}^* + \tau\mathcal{P}} \quad (1)$$

Higher trade costs τ imply that fewer varieties can be produced because more resources are required to cover the costs of shipment. Similarly, higher fixed costs imply that fewer varieties can be produced.

We require that countries have a balanced trade account, meaning that the value of imports must equal the value of exports. This balanced trade condition can be expressed as $p^*\mathcal{P}^*N = p\mathcal{P}N^*$. The left-hand side is the value of imports (exports) from the point of view

¹⁹Note that it is very much possible that the South has a higher wage rate than the North, however, relative nominal per capita income $\omega L^*/L$ would still be smaller than 1, ensuring that the South is the poor country of the two.

of the South (North). Plugging (1) into the balanced trade condition, we get an expression for relative prices:

$$\frac{p}{p^*} = \frac{L}{\omega L^*} \quad (2)$$

Relative prices are determined by relative labour income. For the full trade equilibrium to exist, we required that relative prices are smaller than the trade costs τ , such that arbitrage is never profitable. Together with the equation above, this implies that relative labour incomes must be smaller than the trade costs $\tau \geq L/\omega L^*$.

Finally, the zero-profit conditions together with the relative price, yield the absolute price level in the North and the South

$$p = \frac{L}{a} \left(\frac{aF + \mathcal{P} + \tau \mathcal{P}^*}{L\mathcal{P} + \omega L^* \mathcal{P}^*} \right) \quad p^* = p \frac{\omega L^*}{L} = \frac{\omega L^*}{a} \left(\frac{aF + \mathcal{P} + \tau \mathcal{P}^*}{L\mathcal{P} + \omega L^* \mathcal{P}^*} \right) \quad (3)$$

Having solved for the full trade equilibrium, we can work out the conditions under which this equilibrium persists. The first condition is $\tau \geq \frac{L}{\omega L^*}$. The second condition requires that Northern firms are willing to export to the South, given the price level in the South. Specifically, we require that prices in the South at least cover the marginal costs of exported products: $p^* \geq \tau/a$.²⁰ This constraint can be solved for the level of trade costs required $\tau \leq \frac{\omega L^*}{L} \left(\frac{aF + \mathcal{P}}{\mathcal{P}} \right)$. In this full trade equilibrium, consumers in both countries purchase all available goods. Hence, welfare is identical in both countries and given by:

$$U = U^* = \frac{aL\mathcal{P}}{aF + \mathcal{P} + \tau \mathcal{P}^*} + \frac{a\omega L^* \mathcal{P}^*}{aF + \mathcal{P} + \tau \mathcal{P}^*}$$

Consumers in the North pay higher prices for the same products, due to their larger willingness to pay. Consequently, they cover the majority of the total production costs. However, in sum, the higher prices and the higher income in the North offset each other such that welfare is identical in the South. We can state the following proposition

Proposition 1. *Assume a full trade equilibrium. Then, (a) all goods are traded. (b) Relative prices reflect relative purchasing power. (c) Welfare is equalised across countries. (d) A trade liberalisation increases welfare in both countries.*

Proof. In text. ■

²⁰Strictly speaking, we also require that the price level in the North covers the marginal costs of exported products from the South. However, given that prices in the North are strictly larger than in the South, this constraint will always be fulfilled if the other constraint is fulfilled as well.

4.2 Arbitrage Equilibrium

Full trade ceases to exist when per capita income differences exceed the trade costs. Then, price differences grow too large and arbitrage becomes possible. Firms respond by adjusting their pricing behaviour, or their exporter status, accordingly.

In equilibrium, all firms from the South choose to export. However, in the North, some firms choose not to export their product and only sell in the domestic market, at unconstrained prices. Adopting this strategy implies a smaller market but allows the firm to charge higher prices due to the Northern consumers' higher willingness to pay. The other firms from the North will export their product to the South while charging a reduced price at home. To see why this has to be an equilibrium, consider the case in which all firms export. If all Northern firms were to export as well, then the consumers in the North would not fully exhaust their budget constraint and have an infinitely large willingness to pay for additional products, which cannot be an equilibrium. This is because the reduced price p_T lies below the willingness to pay of the consumers in the North. Consequently, there exist two different prices in the North, one for domestic products and one for traded products. The zero-profit conditions can be written as:

$$p\mathcal{P} = W\left(F + \frac{\mathcal{P}}{a}\right) \quad (4)$$

$$\underbrace{\pi p\mathcal{P} + (1 - \pi)\tau p^*\mathcal{P}}_{\text{Expected Revenue from North}} + \underbrace{p^*\mathcal{P}^*}_{\text{Revenue from South}} = W^i\left(F + \frac{\mathcal{P}^i + \tau\mathcal{P}^{-i}}{a}\right) \quad (5)$$

(where $-i$ = South if i = North and vice versa). The ZPC of a domestic Northern producer is (4), while (5) is for a Northern exporter and all Southern firms. Since parallel trade enforcement is uncertain, exporters can charge unconstrained prices with probability π , and with probability $(1 - \pi)$, they have to charge the constrained price, to avoid arbitrage by third parties. The left-hand side is (expected) revenue, which is given by the number of consumers in each market multiplied by the sales price (remember that every consumer only ever buys one unit, thus sales volume is given by the population size). The right-hand side is total costs. From (4), the price of non-traded goods in the North is given by:

$$p = \left(\frac{aF + \mathcal{P}}{a\mathcal{P}}\right) \quad (6)$$

Note that the price of non-traded varieties is identical to the price under autarky.

Within both economies, the total number of resources must equal the total number of resources available. These aggregate resource constraints are given by:

$$\mathcal{P}L = N_T \left(F + \frac{\mathcal{P} + \tau\mathcal{P}^*}{a} \right) + N_N \left(F + \frac{\mathcal{P}}{a} \right) \quad \mathcal{P}^*L^* = N^* \left(F + \frac{\mathcal{P}^* + \tau\mathcal{P}}{a} \right)$$

where N_N N_T are the number of non-traded and traded varieties from the North and N^* is the total number of products from the South.

From the resource constraint in the South, we can determine the number of varieties in the South

$$N^* = \frac{a\mathcal{P}^*L^*}{aF + \mathcal{P}^* + \tau\mathcal{P}} \quad (7)$$

The price level in the South is calculated using (5) and (6):

$$p^* = \frac{(1 - \pi)(aF + \mathcal{P}) + \tau\mathcal{P}^*}{a(\mathcal{P}^* + (1 - \pi)\tau\mathcal{P})} \quad (8)$$

Notice that the prices in the South depend on the enforcement of parallel trade restrictions π . Consequently, also the price of traded varieties in the North, given by $p_T = \tau p^*$, depends on π . Finally, we require a balanced trade condition, such that the value of exports equals the value of imports:

$$\mathcal{P}^*p^*N_T = [(1 - \pi)\tau p^* + \pi p]\mathcal{P}N^* \quad (9)$$

The bracket on the right-hand side of the equation denotes the price level in the North, which is a weighted average of the non-traded goods price and the traded goods price.

Given the price levels in the North and South, we can determine the number of traded varieties from the North, N_T , using the balanced trade condition (9):

$$N_T = \frac{\mathcal{P}}{\mathcal{P}^*} \underbrace{\left[(1 - \pi)\tau + \pi \frac{(1 + a\frac{F}{\mathcal{P}})(\mathcal{P}^* + (1 - \pi)\tau\mathcal{P})}{(1 - \pi)(aF + \mathcal{P}) + \tau\mathcal{P}^*} \right]}_{\text{Terms of Trade: } \mathbb{E}p/p^* \equiv \xi(\tau, \pi)} \underbrace{\frac{a\mathcal{P}^*L^*}{aF + \mathcal{P}^* + \tau\mathcal{P}}}_{N^*} \quad (10)$$

Finally, from the resource constraint in the North, we get the number of non-traded varieties:

$$N_N = \underbrace{\left(\frac{a\mathcal{P}}{aF + \mathcal{P}} \right)}_{1/p} \left[L - L^* \underbrace{\left(\frac{aF + \mathcal{P} + \tau\mathcal{P}^*}{aF + \mathcal{P}^* + \tau\mathcal{P}} \right)}_{\omega} \underbrace{\left((1 - \pi)\tau + \pi \frac{(1 + a\frac{F}{\mathcal{P}})(\mathcal{P}^* + (1 - \pi)\tau\mathcal{P})}{(1 - \pi)(aF + \mathcal{P}) + \tau\mathcal{P}^*} \right)}_{\text{Terms of Trade}} \right] \quad (11)$$

Note that the number of exported varieties, N_T , and the number of varieties not exported, N_N , both depend on the enforcement of parallel trade restrictions π . For non-exported varieties N_N to exist, we require that the relative labour endowment between North and South is sufficiently large. Specifically, we need that $L/\omega L^* > \xi(\tau, \pi)$. In the special case of $\pi = 0$, when parallel trade is fully allowed, this condition simplifies to $L/\omega L^* > \tau$. Additionally, we have the trade condition, i.e. a condition on the trade costs τ , such that arbitrage trade actually happens. For a firm to be willing to export, the export price must at least cover the marginal costs $p^* \geq \tau/a$. Using (8), the trade condition is determined by:

$$\tau \leq \sqrt{\frac{aF + \mathcal{P}}{\mathcal{P}}} \quad (12)$$

5 Welfare Analysis of Arbitrage Equilibrium

5.1 Parallel Trade Policy

Prices. The prices of non-traded goods in the North, are independent of parallel trade restrictions. Due to the existence of arbitrage constraints, traded goods prices no longer depend on labour endowments L and L^* as was the case in the full trade equilibrium, but rather on the enforcement of parallel trade restrictions π . Specifically, stronger IPR enforcement lowers prices in the South and traded goods prices in the North. Hence, there is a pro-competitive effect of parallel trade enforcement on p^* . This is because stricter parallel trade restrictions make exports to the South more attractive for Northern firms. There is increased competition in the Southern goods market, which drives down prices. This is consistent with [Dubois, Gandhi and Vasserman \(2022\)](#), who find that allowing parallel trade mostly leads to price convergence *to the top*, implying that firms will raise prices in the poor country rather than lower prices in the rich country.

Terms of Trade. Let us define the terms of trade as $\mathbf{Ep}/p^* \equiv \xi(\tau, \pi) = \frac{\pi p + (1-\pi)\tau p^*}{p^*}$ (see (10)). The terms of trade, as they are defined here, are from the point of view of the South. Thus, an increase in ξ captures a shift in the terms of trade in favour of the South.

Non-trade goods prices in the North are independent of π , and all traded goods prices fall in π , hence the terms of trade $\xi(\tau, \pi)$ are increasing in π because the price level in the South falls faster. Northern exports to the South are worth relatively less than Southern exports to the South, which benefits the South. Moreover, relative (and absolute) population sizes do

not affect parallel trade policy considerations: a rich (poor) country does not have a different parallel trade policy when trading with a large or small poor (rich) country. More parallel trade restrictions always shift the terms of trade in favour of the South. The North would therefore prefer fewer restrictions on parallel trade.

Export Probability. In the South, the export probability is always one, as every producer is also an exporter. In the North, however, some firms do not export, so the export probability is less than one and defined as $\frac{N_T}{N_T + N_N}$. Intuitively, the number of traded goods increases in π . As the threat of parallel imports diminishes, more firms are willing to export to the South, thereby increasing product variety in the South. This effect is the driving force behind the pro-competitive effect of parallel trade restrictions on prices in the South, as established previously. At the same time, the number of non-traded varieties in the North decreases with stronger parallel trade restrictions. In sum, the reduction in non-traded varieties strictly dominates the increase in traded varieties. Due to the iceberg trade costs τ , a disproportionate amount of resources is required to produce these export goods, which in turn means that fewer resources are available to produce non-traded varieties. Therefore, with a strict parallel trade ban, the North produces fewer goods. Together with the fact that more goods are exported to the South, this implies that the export probability is strictly increasing in π . This is consistent with empirical findings on exports and export probabilities such as [Goldberg \(2010\)](#) or [Ivus \(2010\)](#).

Trade Intensity. Let us define the "trade intensity" ϕ as the ratio between the total value of world trade and world GDP. The total value of trade is given by the sum of all exports $p^*P^*N_T + [(1 - \pi)\tau p^* + \pi p]P N^*$, while world GDP is $LP + \omega L^*P^*$. Using equations (6), (7) and (8), trade intensity is given by

$$\phi = \frac{2aPP^*L^*}{aF + P^* + \tau P} \frac{1}{LP + \omega L^*P^*} \left[\pi \frac{(aF + P)}{aP} + (1 - \pi)\tau \frac{(1 - \pi)(aF + P) + \tau P^*}{a(P^* + (1 - \pi)\tau P)} \right]$$

Naturally, trade intensity depends on the two trade policy choices, τ and π , and on per capita income in the South L^* . Specifically, higher per capita income in the South increases the intensity of trade. This result goes back to [Linder \(1961\)](#). He postulated that more similar countries should trade more with each other.²¹ Finally, tighter parallel trade restrictions

²¹Similarity should be understood as similarity in per capita income or more generally as similarity in demand, which does not necessarily coincide due to non-homothetic demand.

increase the intensity of trade. Countries trade the most, both absolutely and in proportion to GDP, when parallel trade is prohibited, ie. when $\pi = 1$. Intuitively, this result follows from the fact that the export probability of the North increases with π .

Proposition 2. *Assume that per capita income differences are large enough to maintain an arbitrage equilibrium. Then, some firms in the North do not export. Stricter IPR enforcement leads to (a) lower prices in the South, (b) an improvement in the terms of trade of the South ξ , (c) increases the export probability from the North and (d) raises trade intensity ϕ .*

Proof. In Appendix A. ■

Welfare. Given that consumers only ever purchase one unit of every good, utility is given by the sum of varieties consumed. Thus, we have:

$$U = N_N + N^* \left(1 + \xi(\tau, \pi) \frac{\mathcal{P}}{\mathcal{P}^*} \right) \quad U^* = N^* + N_T = N^* \left(1 + \xi(\tau, \pi) \frac{\mathcal{P}}{\mathcal{P}^*} \right)$$

The effects of parallel trade restrictions on welfare in North and South are given by:

$$\frac{\partial U}{\partial \pi} = \frac{\partial \xi(\tau, \pi)}{\partial \pi} \left(\frac{\partial N_N}{\partial \xi(\tau, \pi)} + \frac{\mathcal{P} N^*}{\mathcal{P}^*} \right) < 0 \quad \frac{\partial U^*}{\partial \pi} = \frac{\mathcal{P} N^*}{\mathcal{P}^*} \frac{\partial \xi(\tau, \pi)}{\partial \pi} > 0$$

The shift in the terms of trade has a direct impact on utility: although consumers in the North benefit from an increased variety of traded Northern products (N_T increases), the decrease in non-traded varieties is strictly greater. Therefore, consumers in the North are harmed by more parallel trade restrictions. Southern consumers strictly benefit from more parallel trade restrictions due to more imports from the North. The intuition behind this result is simple: the shift in the terms of trade requires the North to export more varieties in order to meet the balanced trade condition, since its export goods have become comparatively less valuable. Due to the iceberg trade costs τ , a disproportionate amount of resources are required to produce these export goods, resulting in the North producing fewer varieties overall.

As a result, there are divergent preferences between North and South on the optimal parallel trade policy. The rich North prefers to allow parallel trade, while the poor South prefers to ban it in order to gain access to more Northern products. This result contrasts with Roy and Saggi (2012), who find that the North also prefers to ban parallel trade when income differences become too large. However, their model does not allow for firm entry and thus misses a key benefit for Northern consumers.

Proposition 3. *In an arbitrage equilibrium, stronger IPR enforcement increases welfare in the South, but decreases it in the North.*

Proof. In Appendix B. ■

5.2 Trade Costs

The effect of the trade costs τ on prices, trade intensity, and ultimately welfare depend strongly on the level of parallel trade restrictions π . To build intuition and to simplify notation, the discussion focuses on the two corner solutions $\pi = 0$ and $\pi = 1$.

Prices. The price of non-traded goods in the North is independent of trade costs. If there are no restrictions on parallel trade $\pi = 0$, prices in the South are given by $p^* = (aF + \mathcal{P} + \tau\mathcal{P}^*)/(a\mathcal{P}^* + a\tau\mathcal{P})$. In this case, a trade liberalisation *increases* prices in the South.²² Lower trade costs increase the risk of arbitrage for Northern exporters. As a result, fewer of them are willing to export to the South in the first place, which reduces competition in the Southern goods market and thus raises prices. The possibility of parallel trade thus causes trade liberalisation to generate *less* competition instead of more.

If parallel trade is prohibited, $\pi = 1$, the price level in the South will be driven down to just cover the marginal costs of exporters $p = \tau/a$. This is due to the increased competition from Northern firms when the arbitrage threat is absent. Trade liberalisation then produces the standard result of lower prices. In fact, there is a complete pass-through of the cost reduction to the benefit of the consumer. These two results imply that there exists some intermediate level of parallel trade restriction $\hat{\pi}$, such that trade liberalisation does not affect p^* .²³

Terms of Trade. If there are no restrictions on parallel trade, $\pi = 0$, then a trade liberalisation decreases the terms of trade ξ . This is because trade liberalisation raises prices in the South, as previously established. Thus, exports from the North become more valuable relative to their Southern counterparts. Consequently, the South would prefer to restrict trade by imposing higher trade costs τ , while the North would prefer free trade.

²²This is conditional on the two countries being similarly populous. If the South is significantly larger than the North, the reverse may be true and we have the standard result of a pro-competitive effect of trade liberalisation.

²³Specifically, this intermediate level of π is determined by $\hat{\pi} = 1 - \frac{\mathcal{P}^*}{\sqrt{\mathcal{P}(aF + \mathcal{P})}}$.

In the opposite case of $\pi = 1$, trade liberalisation benefits the South. Since there is no arbitrage threat in this case, and many Northern firms are willing to export, trade liberalisation lowers prices in the South and shifts the terms of trade in their favour.

Trade Intensity. Given the results just established, it is clear that a trade liberalisation reduces the export probability of the North and consequently also the intensity of trade ϕ if parallel imports are allowed. Lower trade barriers increase the possibility of parallel trade, which discourages some Northern firms from exporting their goods to the South, thus avoiding the threat of arbitrage altogether. This result was established in Föllmi et al. (2018), and we can confirm their finding in our generalised setting.

Only when parallel trade restrictions are sufficiently strong does this turn into the more familiar finding that trade liberalisation increases both the probability of exporting and the intensity of trade. Thus, one of the core results of trade theory rests on the assumption that parallel imports do not play an outsized role in trade policy.

Proposition 4. *Assume an arbitrage equilibrium. If IPR enforcement is weak, a trade liberalisation (a) raises prices in the South, (b) lower terms of trade of South and (c) decreases the trade intensity. The opposite holds in case of strong IPR enforcement.*

Proof. In Appendix C. ■

Welfare. First, consider the case with parallel trade, $\pi = 0$, which can be considered the more important case from a legal point of view, since most countries allow it. Welfare in both countries is again given by the sum of all varieties consumed:

$$U(\pi = 0, \tau) = \frac{a\mathcal{P}L}{aF + \mathcal{P}} + \frac{a\mathcal{P}^*L^*(aF + \mathcal{P} - \tau^2\mathcal{P})}{(aF + \mathcal{P})(aF + \mathcal{P}^* + \tau\mathcal{P})} \quad U^*(\pi = 0, \tau) = \frac{aL^*(\mathcal{P}^* + \tau\mathcal{P})}{aF + \mathcal{P}^* + \tau\mathcal{P}}$$

It is straightforward to verify that $\partial U / \partial \tau < 0$ and $\partial U^* / \partial \tau > 0$. Consumers in the South prefer high trade costs because they get access to more products from the North. The high trade barriers reduce the threat of arbitrage, allowing Northern firms to charge higher prices in the South, making exports more lucrative, so more firms become exporters. This increase in the trade intensity benefits consumers in the South but is detrimental to welfare in the North. The additional exports require many resources for production, leaving fewer for non-traded goods production. In the left part of figures 2a and 2b, when parallel trade restrictions are low, higher trade costs strictly reduce welfare in the North while increasing it in the South.

The other extreme case of no parallel trade, $\pi = 1$, can be considered more relevant from a practical point of view, since the volume of parallel trade is far from being the majority of the total volume of trade and, as it will turn out, is the preferable policy from a global welfare point of view. Then, welfare is given by:

$$U(\pi = 1, \tau) = \frac{a\mathcal{P}L}{aF + \mathcal{P}} \quad U^*(\pi = 1, \tau) = \frac{aL^*}{\tau} \frac{aF + \mathcal{P} + \tau\mathcal{P}^*}{aF + \mathcal{P}^* + \tau\mathcal{P}}$$

Clearly, the North is indifferent to the level of trade costs in this scenario, the welfare effects of more non-traded goods and fewer traded goods exactly cancel out. In the absence of parallel trade, all firms wishing to export their product already do so,²⁴ so any change in the trade costs does not induce any further reallocation in the export status of Northern firms. See in figure 2a, when parallel trade is ruled out, different levels of trade costs τ have no further effect on welfare. We conclude that the North (weakly) prefers free trade. The South is now also a free trader. When parallel trade is prohibited, prices in the South are determined by marginal costs of the exporters $p^* = \tau/a$ due to the high degree of competition in the market. A reduction in trade costs lowers marginal costs of exporting and thus product prices, which benefits the consumers. In contrast to the case of $\pi = 0$, Northern firms do not reduce exports to the South when trade costs are lower. In figure 2b on the right, the welfare gains of trade liberalisation are shown to be significant. As the South moves from being contra-free trade to pro-free trade as parallel trade restrictions increase, there must exist some intermediate level of parallel trade restriction at which the South is indifferent. In the figure, this turning point is found at about $\pi = 0.3$.

Proposition 5. *Assume an arbitrage equilibrium. If IPR enforcement is weak, a trade liberalisation raises welfare in the North, but lowers it in the South. If IPR enforcement is strong, a trade liberalisation leaves welfare in the North unaffected, while raising it in the South.*

Proof. In Appendix D. ■

²⁴Even in this case, not all Northern firms export their goods, because the relative income difference is still too large to generate a full trade equilibrium.

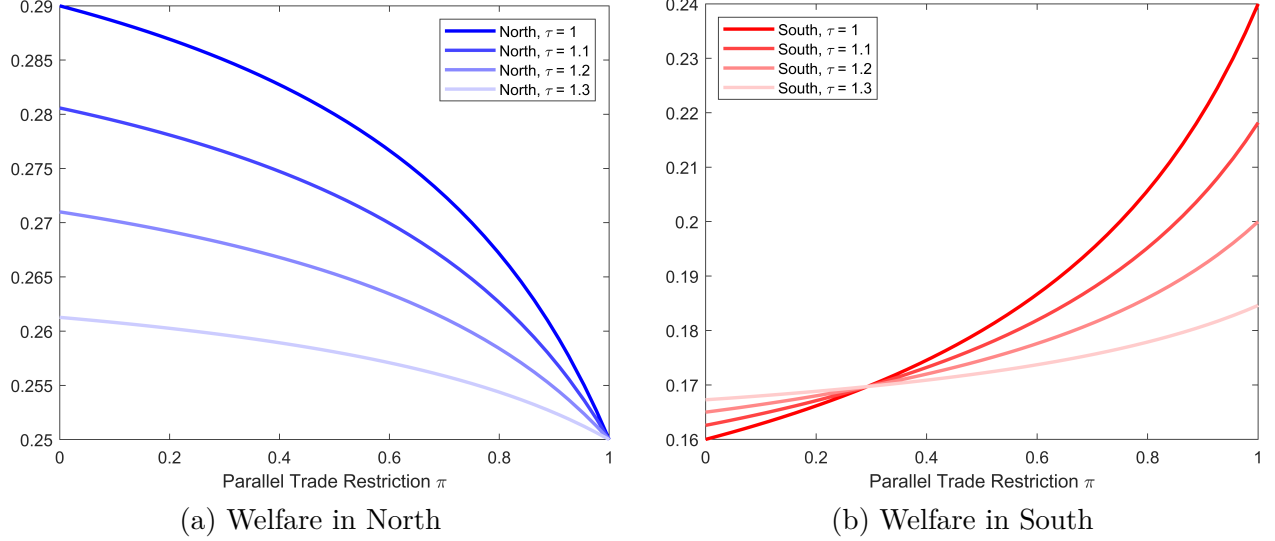


Figure 2: Welfare Effects of Parallel Trade Restrictions

5.3 Optimal Trade Policy

Having established the welfare implications of the two trade policy choices π and τ separately, the question naturally arises as to which mix is optimal from both a country and a global perspective. Figure 3 plots the welfare of the North and the South, given a joint choice of π and τ (note that the axes are inverted in the two figures). Welfare in the North is a well-behaved function of trade policy, with the maximum being at free trade ($\tau = 1$) and allowing parallel trade ($\pi = 0$). Welfare in the South shows the previously established conflictive preference for free trade, depending on the level of parallel trade restrictions. When parallel trade is allowed, raising trade barriers can mitigate the welfare losses to consumers. However, they are best off when parallel trade is banned and free trade prevails.

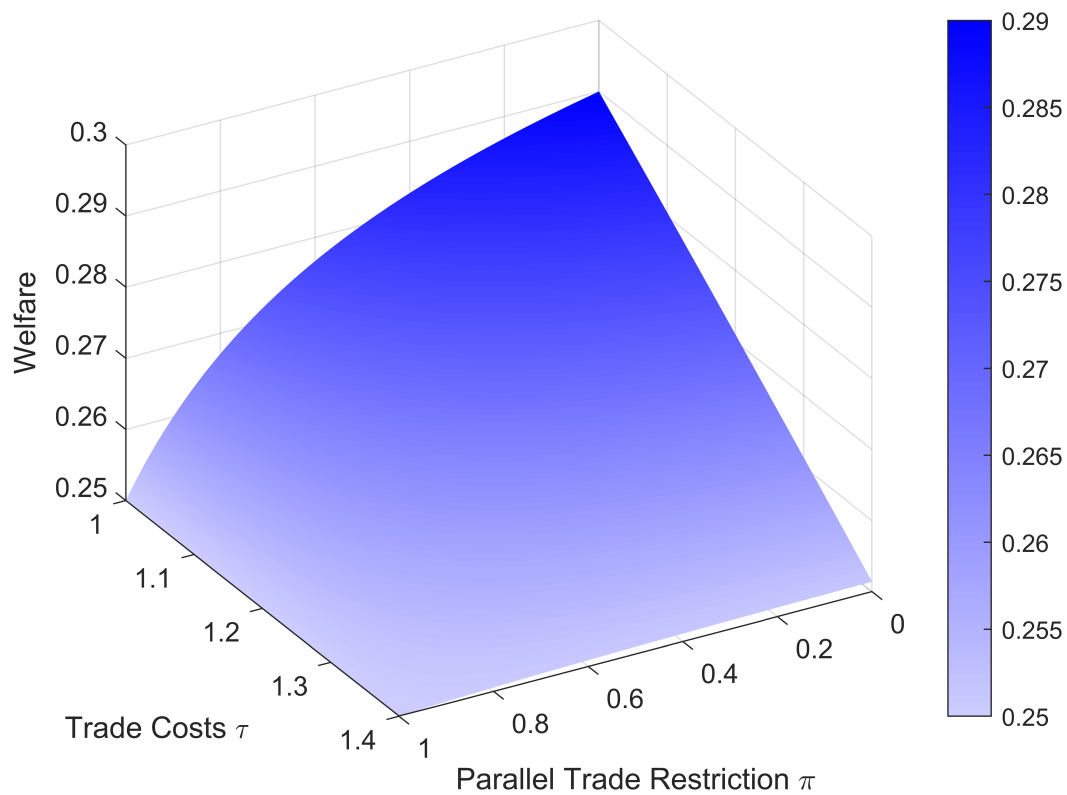
From a global welfare perspective, therefore, the key question concerns parallel trade policy. Suppose there exists a world government (or a world trade regulator), that wanted to maximise aggregate welfare of the North and the South. What level of parallel trade restriction would it choose? The world government chooses π to maximise aggregate welfare:

$$\frac{\partial}{\partial \pi} V(\pi) = \frac{\partial}{\partial \pi} \left[\mathcal{P}U(\pi) + \mathcal{P}^*U^*(\pi) \right] = \underbrace{\frac{\partial \xi}{\partial \pi}}_{>0} \left[\underbrace{\mathcal{P} \frac{\partial N_N}{\partial \xi}}_{<0} + \underbrace{\frac{\mathcal{P}(\mathcal{P} + \mathcal{P}^*)}{\mathcal{P}^*} N^*}_{>0} \right]$$

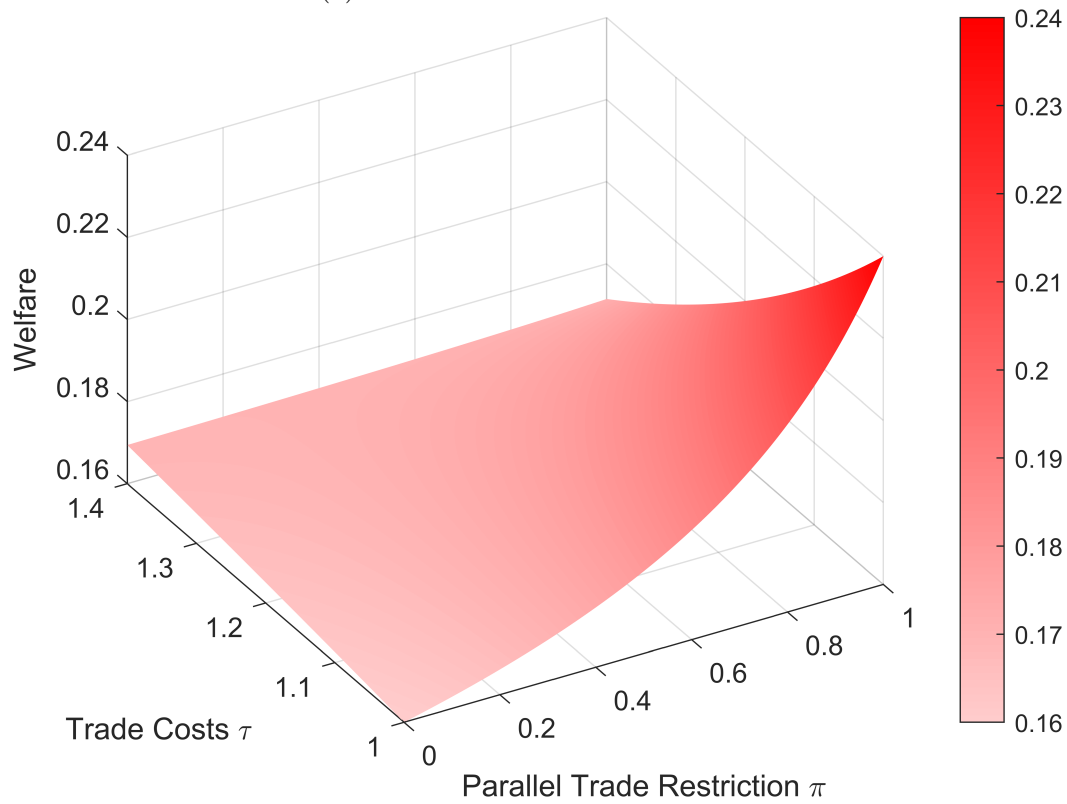
It can be shown that the world government would choose to completely restrict parallel trade, and consequently, to allow free trade (both countries have a preference for free trade

when parallel trade is banned). Although the North would strictly prefer a different parallel trade policy, the welfare gains in the South outweigh the losses in the North. Note that the result is independent of the relative population sizes of North and South.

Thus, from a global welfare perspective, price discrimination by limiting parallel trade is preferable to a globally uniform price, because it allows more markets and consumers to be served. This result is similar to the findings of the early literature on third-degree price discrimination and social welfare, as found in [Schmalensee \(1981\)](#), [Varian \(1985\)](#) and [Schwartz \(1990\)](#). They found that third-degree price discrimination can be welfare-enhancing as long as total output increases in response to the price discrimination. We can confirm and generalise the result of [Malueg and Schwartz \(1994\)](#), who proved this result in a monopoly setting. The proof is in [Appendix E](#).



(a) Welfare in North



(b) Welfare in South

Figure 3: Welfare Effects of Trade Policy between North and South.

5.4 Further Analysis

Big or Small Trade Partners? In case the South becomes relatively more populous, its terms of trade are better compared to a less populous poor country. This is due to the price effect. A larger population means that the fixed costs of production can be spread over more people, so goods can be sold at a lower price. Northern exporters have to match this price, so they also lower their prices. Since Southern exporters can still sell their goods in the North at the old price, this means that the terms of trade have shifted in favour of the South. Note that, if $\pi = 1$, this effect disappears. This is because, if parallel trade is fully restricted, the price in the South p^* has already been driven down to the marginal costs of exporting $p^* = \tau/a$, due to the effect of increased competition from Northern firms in the South. At this point, no further reduction in p^* is possible, given Northern firms exporting incentives.

Note that the income per capita difference has not changed. The comparison is between a large and a small, equally poor, Southern country. We have already shown that $\frac{\partial(N_T+N_N)}{\partial\xi} \leq 0$, i.e., the number of varieties produced in the North decreases in the terms of trade. Consequently, the North would strictly prefer many, less populous trading partners to a few, but very populous ones.

Income Convergence. What happens if we observe convergence between rich and poor, implying that inequality between North and South is decreasing? It is straightforward to verify that the terms of trade are independent of labour endowment $\frac{\partial\xi}{\partial L^*} = 0$. Poorer countries do not *per se* have worse terms of trade than richer ones. This is because prices, both in the North and South, are independent of labour endowments in an arbitrage equilibrium. So there is no change in the relative price, i.e. the terms of trade.

However, as the South becomes richer, the more varieties it produces. The absence of an intensive margin implies that all extra income is spent on additional varieties, thereby increasing N^* . Both the North and the South benefit from this, *ceteris paribus*. A richer South also implies that the North exports more varieties. This follows from the balanced trade condition: If the South exports more varieties, then the North must also export more varieties (holding prices fixed), so that the value of imports equals the value of exports. These two facts together imply that welfare in the South U^* is strictly increasing in its labour endowment. As for the North, we have already established that an increase in traded varieties necessarily implies a decrease in non-traded varieties. The conversion rate is greater than one, meaning that the decrease in non-traded varieties is greater than the increase in

traded varieties, implying that the total number of varieties produced in the North is now smaller. However, taking into account the increased variety from the South, the North also benefits from having a richer trading partner.²⁵ This result is independent of the degree of parallel trade restriction π .

The Divergence. As a complement to the previous section, we examine the welfare implications of the North’s labour endowment L . Under CES preferences, we would expect the welfare implications to be symmetric, both countries should benefit if one of the two becomes richer (in terms of higher labour endowment). This is, after all, what neoclassical trade theory is all about. But it turns out that this is not the case in our model.

Clearly, the number of varieties produced in the South, the number of traded varieties from the North, and the terms of trade are independent of L . Thus, welfare in the South does not change in response to a richer North. All additional resources from the North go into the production of new, non-traded varieties N_N . Note that the increase in non-traded varieties is inversely related to the price of these goods. Intuitively, if goods are more expensive, the same amount of resources will buy fewer goods.

Consequently, welfare in the North rises as a result of an increase in labour endowment through an increased variety of non-traded goods. Consumers in the South are equally well off as before because the number of traded varieties has not changed. Our model therefore predicts asymmetric welfare gains: If the South gets richer (increase in L^*), both countries benefit. However, if the North gets richer (increase in L), only the North benefits. This is reminiscent of the Prebisch-Singer hypothesis, according to which the South faces ever-worsening terms of trade because the relative price of commodities to manufactured goods falls over time due to different price elasticities of demand.²⁶

Proposition 6. *Assume an arbitrage equilibrium. (a) The rich North prefers to have many, but small trading partners. (b) Income convergence (i.e. richer South) benefits both countries, but the South more so. (c) Income divergence (i.e. richer North) benefits only the North.*

Proof. In Appendix F. ■

²⁵This result is consistent with empirical estimates of the welfare effects of the China shock. See for example Arkolakis, Costinot and Rodríguez-Clare (2012), Caliendo, Dvorkin and Parro (2019) or Galle, Rodríguez-Clare and Yi (2023).

²⁶See Harvey et al. (2010) or Singer (1950). Of course, in our simple framework, the mechanism is not the same as in the Prebisch-Singer hypothesis. The terms of trade are unaffected by economic divergence, the effect works through the “new gains from trade”, namely increasing product variety in the North.

6 Trade with Income Inequality

We now relax the assumption of a representative agent within economies. We introduce income inequality as a result of heterogeneous labour endowments. Each consumer is endowed with θ units of labour, where $\theta \in [\underline{\theta}, \bar{\theta}]$ has a probability density function $g(\theta)$ and a cumulative density function $G(\theta)$.

Individuals with a lower labour endowment have a lower income and therefore a lower willingness to pay for products. Each firm charges a unique price for the good it sells, which means that depending on the price it charges, it serves a certain proportion of the population. The higher the price, the smaller the market size. If $\tilde{\theta}$ is the labour endowment of a firm's marginal consumer, the price set by the firm will equal the marginal consumer's willingness to pay, so that all richer consumers who buy that good get a surplus equal to the difference between their willingness to pay and the price of the good.

6.1 Autarky

For any given θ , firms must make zero profits, otherwise there would be deviation incentives. Thus, we have:

$$\mathcal{P}(1 - G(\theta)) \left(p(\theta) - \frac{W}{a} \right) = WF, \quad \forall \theta$$

where $(1 - G(\theta))$ is the share of consumers willing to buy the good at the given price. Solving for $p(\theta)$ delivers the pricing schedule in the economy:

$$p(\theta) = \frac{W}{a} + \frac{WF}{(1 - G(\theta))\mathcal{P}} \quad (13)$$

Prices are given by marginal costs W/a plus a markup, such that firms can recover the fixed costs of production. The markup, and hence prices, increase in θ and converge to infinity as θ approaches its upper bound $\bar{\theta}$ (since $G(\bar{\theta}) = 1$). This is intuitive: the smaller the market size, the larger the share of the fixed costs that each consumer has to bear, which raises the price. Therefore, necessities sold to everyone are cheapest, while luxury goods will be more expensive. This pricing schedule differs from the one in models with an intensive margin of consumption featuring income inequality, e.g. in [Bertoletti and Etro \(2017\)](#); [Fajgelbaum et al. \(2011\)](#); [Kichko and Picard \(2023\)](#). In these models, prices of all goods are identical and a function of the average willingness to pay of all consumers. Thus, rich and poor consumers pay the same prices, poor consumers simply buy less of each good, rather than abstaining

from certain varieties altogether, as in our model.

The budget constraint of a consumer with income $\tilde{\theta}$ is:

$$\tilde{\theta}W = \int_{\underline{\theta}}^{\tilde{\theta}} p(\theta) dN(\theta) + p(\underline{\theta})N(\underline{\theta}) \quad (14)$$

which simplifies to $\underline{\theta}W = p(\underline{\theta})N(\underline{\theta})$ for the poorest consumer. Using (13), we obtain the number of mass products consumed by the poorest individual $N(\underline{\theta}) = \frac{a\mathcal{P}}{aF+\mathcal{P}}\underline{\theta}$. Differentiating the budget constraint (14) with respect to $\tilde{\theta}$ and then integrating again over θ gives the consumption profile of consumer $\tilde{\theta}$:

$$\begin{aligned} N(\tilde{\theta}) &= \int_{\underline{\theta}}^{\tilde{\theta}} \frac{W}{p(\theta)} d\theta + N(\underline{\theta}) \\ &= \int_{\underline{\theta}}^{\tilde{\theta}} \frac{a\mathcal{P}(1-G(\theta))}{aF+(1-G(\theta))\mathcal{P}} d\theta + \frac{a\mathcal{P}}{aF+\mathcal{P}}\underline{\theta} \end{aligned} \quad (15)$$

Clearly, the number of varieties consumed increases with income θ , so richer consumers buy a greater variety of products than poorer consumers do. Total product variety in the economy is given by $N(\bar{\theta})$ and depends on the shape of the income distribution $G(\theta)$. Following a mean preserving spread in income, prices of mass products rise while prices of luxury goods fall. Thus, poor consumers are worse off after an increase in income inequality, while rich consumers benefit from it. In [Bertoletti and Etro \(2017\)](#) and [Kichko and Picard \(2023\)](#), prices fall uniformly following a mean preserving spread.

6.2 Trade

Let $G(\cdot)$ and $G^*(\cdot)$ denote the distributions of labour endowments in the North and South, respectively. The distributions are defined over the same support $\theta \in [\underline{\theta}, \bar{\theta}]$. For the sake of clarity, we assume that $\tau = 1$, such that trade is costless. This implies that Northern and Southern firms will choose the same marginal consumer within each market. Consequently, the pricing schedules will be the same for each market, regardless of the location of production. Factor price equalisation prevails, i.e. the relative wage is $\omega = 1$.

When we allow for parallel trade to occur, firms have to apply global uniform pricing, any other pricing rule would be exploited by arbitrageurs making use of costless trade. Again,

for any given income level, firms must make zero profits in equilibrium:

$$\begin{aligned}
WF &= (1 - G(\theta))\mathcal{P}\left(p(\theta) - \frac{W}{a}\right) + (1 - G^*(\theta))\mathcal{P}^*\left(p(\theta) - \frac{W}{a}\right), \quad \forall \theta \\
\Leftrightarrow \quad p(\theta) &= p^*(\theta) = \frac{W}{a} + \frac{WF}{(1 - G(\theta))\mathcal{P} + (1 - G^*(\theta))\mathcal{P}^*}
\end{aligned} \tag{16}$$

where $p(\theta)$ and $p^*(\theta)$ are the prices chosen by the home and foreign firms, respectively. As in the case of the representative agent, prices are lower with trade than under autarky because firms can spread their fixed costs over more consumers. The market size for each firm is given by the sum of consumers with income above $\tilde{\theta}$, i.e. the income threshold chosen by the firm.²⁷

In case parallel trade is forbidden, firms can arbitrarily set different prices and markups in the markets they serve, depending on the local distribution of income. In equilibrium, the fixed costs of production may be split unevenly between both markets. Denote profits in the North and South by Π_N and Π_S , respectively. The sum of profits must equal the fixed costs:

$$WF = \Pi_N + \Pi_S. \tag{17}$$

The pricing schedules in the North and South, as a function of Π_N and Π_S , are given by:

$$\begin{aligned}
p(\theta) = p^*(\theta) &= \frac{W}{a} + \frac{\Pi_N}{\mathcal{P}(1 - G(\theta))} \\
p(\theta^*) = p^*(\theta^*) &= \frac{W}{a} + \frac{\Pi_S}{\mathcal{P}^*(1 - G^*(\theta^*))}
\end{aligned} \tag{18}$$

Similar to the case in autarky, we may calculate the number of products bought by consumers

²⁷Let us briefly consider the case of costly trade $\tau > 1$ and highlight the differences. If $\tau > 1$, domestic and foreign firms would not choose the same pricing schedule in a given market. Foreign firms have to price in the trade costs, which they would do both by raising prices *and* by reducing markups in order to minimise losses in market size. The imported goods are therefore luxury goods, consumed only by richer consumers. The zero-profit condition in this general case is:

$$\left(p(\theta) - \frac{W}{a}\right)(1 - G(\theta))\mathcal{P} + \left(\tau p(\theta) - \frac{\tau W}{a}\right)(1 - G^*(\theta^*))\mathcal{P}^* = WF$$

where θ^* is implicitly defined by the arbitrage condition $p(\theta^*) = \tau p(\theta)$. Depending on the slope of the income distribution at the point θ , the choice between price increase (with loss of market size) and markup reduction tends to one side or the other. The equilibrium pricing schedule is a weighted average of the price that home firms would charge and the price that foreign firms would charge in the export market.

with income $\tilde{\theta}$:

$$\begin{aligned} N(\tilde{\theta}) &= \int_{\underline{\theta}}^{\tilde{\theta}} \frac{W}{p(\theta)} d\theta + N(\underline{\theta}) \\ &= \int_{\underline{\theta}}^{\tilde{\theta}} \frac{a\mathcal{P}(1 - G(\theta))}{a\Pi_N + (1 - G(\theta))\mathcal{P}} d\theta + \frac{a\mathcal{P}}{a\Pi_N + \mathcal{P}} \underline{\theta} \end{aligned} \quad (19)$$

Since all firms serve both markets, the number of products bought by the richest consumer $\bar{\theta}$ must be the same in the North and South. Hence, it must hold that:

$$\int_{\underline{\theta}}^{\bar{\theta}} \frac{a\mathcal{P}(1 - G(\theta))}{a\Pi_N + (1 - G(\theta))\mathcal{P}} d\theta + \frac{a\mathcal{P}}{a\Pi_N + \mathcal{P}} \underline{\theta} = \int_{\underline{\theta}}^{\bar{\theta}} \frac{a\mathcal{P}(1 - G^*(\theta^*))}{a\Pi_S + (1 - G^*(\theta^*))\mathcal{P}} d\theta^* + \frac{a\mathcal{P}}{a\Pi_S + \mathcal{P}} \underline{\theta} \quad (20)$$

Combining (17) and (20) allows us to solve for Π_N and Π_S , which fully determines the pricing schedules (18) as functions of $G(\theta)$ and $G^*(\theta^*)$ and model parameters.²⁸

Having established the pricing schedules in the North and South, both with and without parallel trade, we can determine the parallel trade policy preferences for rich and poor consumers. For this part, we assume that North and South are equally populous $\mathcal{P} = \mathcal{P}^*$.

First, let us consider the case in which the North is on average the richer country. Specifically, we assume that the income distribution in the North first-order stochastically dominates (FOSD) the income distribution in the South: $G(\theta) \leq G^*(\theta)$, $\forall \theta$. The integrand in (20) is monotonically decreasing in $G(\theta)$ or $G^*(\theta^*)$. Thus, if $G(\theta)$ FOSD $G^*(\theta^*)$, we must have $\Pi_N > \Pi_S$, implying that consumers in the North carry the bulk of the fixed costs. Therefore, basic goods are more expensive in the North. In other words, poor consumers in the rich country are worse off than their counterparts in the poor country, because the presence of many rich compatriots induces firms to charge higher prices. We refer to this as the *Manhattan effect*. Such a positive relation between average income and prices is well-established, both theoretically (Bertoletti and Etro, 2017; Fajgelbaum et al., 2011; Kichko and Picard, 2023) and empirically (Anderson et al., 2018; Alessandria and Kaboski, 2011; Handbury, 2021; Simonovska, 2015). However, this result no longer holds at the upper end of the distribution as the market size effect starts to dominate the pricing-to-market effect: although consumers in the South carry a smaller share of the fixed costs, they have to be dis-

²⁸Note that an equal mean in income is not a sufficient condition for $\Pi_N = \Pi_S = 1/2$. The shape of the entire income distribution matters to determine the profit shares.

tributed across an ever smaller number of consumers as we move up the income distribution. The North, with more rich consumers, is much less affected by this.

In terms of welfare, however, this is merely a catch-up, as (20) establishes that the richest consumers are equally well off irrespective of location. Everyone else is worse off in the North than in the South, given the same nominal income $\tilde{\theta}W$. Consequently, Northern consumers prefer to allow for parallel trade, such that global uniform pricing prevails and the Manhattan effect disappears. On the other hand, consumers in the South would prefer to forbid parallel trade. Figure 4a shows the gains/losses from parallel trade policy on consumers along the entire income distribution. Introducing income inequality reveals *heterogeneous gains* from parallel trade policy within countries. However, it does not reveal *heterogeneous preferences* for it: rich and poor consumers within a given country share the same preference. Hence, proposition 3 can be generalised to the heterogeneous agent setting: the rich country prefers to allow parallel trade, the poor one prefers not to.²⁹ The welfare losses of the South appear to be slightly larger than the gains of the North, which would replicate our previous finding on the optimal policy from section 5.3.

Second, we consider the case where both countries are equally rich on average, but income is not equally distributed across countries. Suppose that $\int_{\underline{\theta}}^x G(\theta)d\theta \leq \int_{\underline{\theta}}^x G^*(\theta)d\theta$, $\forall x \in (\underline{\theta}, \bar{\theta})$, such that the income distribution in the South $G^*(\theta)$ is a mean preserving spread of the income distribution in the North $G(\theta)$, i.e. the South is the less equal country.³⁰ As shown in the autarky case, a mean preserving spread leads to higher prices for poor consumers and lower prices for rich consumers. Therefore, basic goods are more expensive in the South than in the North, which means that Southern consumers are worse off in comparison. They would therefore prefer to *allow* parallel trade, in which case prices are averaged between the higher prices in the South and the lower prices in the North. Conversely, poor consumers in the North would prefer a ban on parallel trade, which would allow them to maintain low prices without being lumped together with Southern consumers. For rich consumers, higher inequality is good news because luxury goods become cheaper.

So, rich consumers from the South prefer no parallel trade, while those from the North want to allow parallel trade. In Figure 4b, the difference in welfare is plotted. Clearly,

²⁹Note however that the underlying reason is different. Under homogeneity, allowing for parallel trade caused some Northern firms to abstain from exporting, benefiting Northern consumers. Under heterogeneity, firms always export. Here, the effect is driven by the fact that Northern consumers would be charged higher prices if firms were allowed to discriminate in prices.

³⁰We assume that the mean preserving spread is such that the profit shares are constant and equal, i.e. $\Pi_N = \Pi_S = 1/2$. However, depending on the distribution functions, this assumption may not always hold.

poor consumers in the North would prefer to forbid parallel trade, while only the very rich consumers prefer to allow it. Consumers in the middle part of the income distribution, where prices change only little (or not at all), still have the same preference for parallel trade policy as their poorer compatriots. This is because they also have to bear the higher costs of basic products. Only when one is sufficiently rich do the gains from cheaper luxury goods outweigh the losses from more expensive basic goods. Hence, most people (from the poorest person up to the middle class) have the same preference for parallel trade, while only the very rich have a different one.

We summarise this discussion in the following proposition:

Proposition 7. *Under 0/1 preferences with costless trade and within-country inequality:*

1. *Every consumer in the rich country prefers to allow parallel trade, while consumers in the poor country prefer not to.*
2. *Following a mean-preserving spread, poor consumers in the less (more) equal country prefer to allow (forbid) parallel trade, while rich consumers prefer to forbid (allow) it.*

Proof. In text. ■

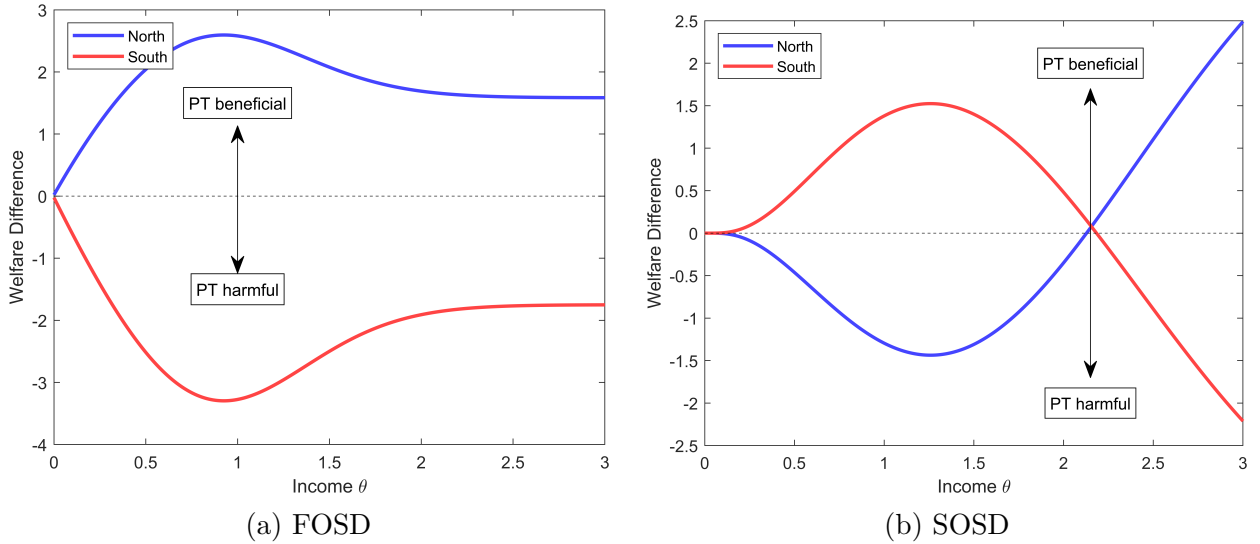


Figure 4: Parallel Trade Policy and Welfare

Notes: Welfare differentials are calculated as the difference between welfare in case parallel trade is allowed and in case parallel trade is forbidden. Hence, a value above the zero intercept indicates that welfare is higher in case parallel trade is allowed, and vice versa. The income distributions are generated using a truncated lognormal distribution. In Panel A, the distributions are determined by $\begin{bmatrix} \theta \\ \theta^* \end{bmatrix} \sim LN \left(\begin{bmatrix} 0.3 \\ 0.0 \end{bmatrix}, \begin{bmatrix} 0.7 & 0 \\ 0 & 0.7 \end{bmatrix} \right)$ and in Panel B by $\sim LN \left(\begin{bmatrix} 0.0 \\ 0.0 \end{bmatrix}, \begin{bmatrix} 0.7 & 0 \\ 0 & 1 \end{bmatrix} \right)$.

7 Conclusion

This article develops a model of international trade model in which an importer's per capita income plays a crucial role in determining both export zeros and the prices of exported goods. This phenomenon arises from a demand-side effect: consumers in lower-income countries are less willing to pay for differentiated products than consumers in higher-income countries. As a result, firms in wealthier regions have less incentive to export to destinations in lower-income regions. Our model attributes the occurrence of export zeros to non-homothetic preferences, which differs from the conventional approach that emphasises firm heterogeneity.

The presence of arbitrage threats reduces the probability of exports. Firms from the rich North face a trade-off between market size and pricing power: (1) refrain from exporting to the South and charge higher prices in the North, or (2) sell in all markets but with limited price discrimination. In this case, the South prefers high barriers to trade. High trade costs reduce the arbitrage threat for Northern firms (because making reimports more expensive), thereby increasing exports to the benefit of Southern consumers. This creates a paradoxical situation where higher trade costs actually lead to higher trade volumes.

The preferences of the North and South regarding parallel trade policy diverge. The South favours restrictions on parallel trade in order to gain access to more Northern goods. Conversely, the North prefers to allow parallel trade because it lowers prices and expands the range of available products. From a global welfare perspective, banning parallel trade and eliminating trade costs emerges as the optimal trade policy mix.

Introducing income inequality within countries confirms the main findings: all consumers in the rich country prefer to allow parallel trade, while consumers in the poor one prefer to forbid it, although to varying degrees. However, following a mean preserving spread in income, we reveal diverging preferences for parallel trade policy between the very rich and all other consumers. Interestingly, these preferences also diverge between countries within the same income group. If the rich consumers in one country prefer parallel trade, the rich consumers in the other country prefer to ban it.

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APPENDIX

A Proof of Proposition 2

Part (a): This follows from differentiating p^* with respect to π

$$\frac{\partial p^*}{\partial \pi} = \frac{\partial}{\partial \pi} \left[\frac{(1-\pi)(aF + \mathcal{P}) + \tau \mathcal{P}^*}{a(\mathcal{P}^* + (1-\pi)\tau \mathcal{P})} \right] = \frac{-\mathcal{P}^*(aF + \mathcal{P} - \tau^2 \mathcal{P})}{a(\mathcal{P}^* + (1-\pi)\tau \mathcal{P})^2} < 0$$

Part (b):

$$\frac{\partial \xi(\tau, \pi)}{\partial \pi} = \frac{\mathcal{P}^*}{\mathcal{P}} \frac{(aF + \mathcal{P} + \tau \mathcal{P}^*)(aF + \mathcal{P} - \tau^2 \mathcal{P})}{\left((1-\pi)(aF + \mathcal{P}) + \tau \mathcal{P}^* \right)^2} > 0$$

Part (c):

$$\frac{\partial(N_T + N_N)}{\partial \pi} = \frac{a\omega \mathcal{P}^* L^*(aF + \mathcal{P} - \tau^2 \mathcal{P})(1 - aF - \mathcal{P} - \tau \mathcal{P}^*)}{(aF + \mathcal{P}) \left((1-\pi)(aF + \mathcal{P}) + \tau \mathcal{P}^* \right)^2} < 0$$

Part (d): Note that $\text{sign}(\partial \phi / \partial \pi) = \text{sign}(\partial \log \phi / \partial \pi)$. Then, we can write:

$$\frac{\partial \log \phi}{\partial \pi} = \frac{\mathcal{P}^{*2}(aF + \mathcal{P} - \tau^2 \mathcal{P})}{a\mathcal{P}(\mathcal{P}^* + (1-\pi)\tau \mathcal{P})} > 0$$

where the inequality sign follows from the trade condition (12).

B Proof of Proposition 3

By appendix A, we already know that $\frac{\partial(N_T + N_N)}{\partial \pi} < 0$. Hence, what remains to be shown is that:

$$\begin{aligned} \frac{\partial N_T}{\partial \pi} &= \frac{\mathcal{P} N^*}{\mathcal{P}^*} \frac{\partial \xi}{\partial \pi} = \frac{a\omega \mathcal{P}^* L^*(aF + \mathcal{P} - \tau^2 \mathcal{P})}{((1-\pi)(aF + \mathcal{P}) + \tau \mathcal{P}^*)^2} > 0 \\ \frac{\partial N_N}{\partial \pi} &= -\frac{a\omega \mathcal{P} L^*}{aF + \mathcal{P}} \frac{\partial \xi}{\partial \pi} = -\frac{a\omega \mathcal{P}^* L^*}{aF + \mathcal{P}} \frac{(aF + \mathcal{P} + \tau \mathcal{P}^*)(aF + \mathcal{P} - \tau^2 \mathcal{P})}{((1-\pi)(aF + \mathcal{P}) + \tau \mathcal{P}^*)^2} < 0 \\ \frac{\partial N^*}{\partial \pi} &= 0 \end{aligned}$$

C Proof of Proposition 4

Part (a): The two corner solutions are given by $\frac{\partial p^*}{\partial \tau} \Big|_{\pi=0} = \frac{1}{a} \left[\frac{\mathcal{P}^{*2} - \mathcal{P}^2 - aF\mathcal{P}}{(\mathcal{P}^* + \tau \mathcal{P})^2} \right] \geq 0$ and $\frac{\partial p^*}{\partial \tau} \Big|_{\pi=1} = \frac{1}{a} > 0$. For the case of $\pi = 0$, the derivative is negative as long as the relative population sizes are not too different. The cross-derivative is

$$\frac{\partial^2 p^*}{\partial \tau \partial \pi} = \frac{1}{a} \left[\frac{2\mathcal{P} \mathcal{P}^* ((1-\pi)(aF + \mathcal{P}) + \tau \mathcal{P}^*)}{(\mathcal{P}^* + (1-\pi)\tau \mathcal{P})^3} \right] > 0$$

Part (b): The two corner solutions are $\left. \frac{\partial \xi}{\partial \tau} \right|_{\pi=0} = \frac{(aF+\mathcal{P})(aF+\mathcal{P}+2\tau\mathcal{P}^*)+\tau^2\mathcal{P}^{*2}}{(aF+\mathcal{P}+\tau\mathcal{P}^*)^2} > 0$ and $\left. \frac{\partial \xi}{\partial \tau} \right|_{\pi=1} = -\frac{(aF+\mathcal{P})}{\tau^2\mathcal{P}^*} < 0$. The cross-derivative is given by:

$$\frac{\partial^2 \xi}{\partial \tau \partial \pi} = \frac{\mathcal{P}^{*2}((aF+\mathcal{P}-\tau^2\mathcal{P})((1-\pi)(aF+\mathcal{P})+\tau\mathcal{P}^*)-2(aF+\mathcal{P})(aF+\mathcal{P}+\tau\mathcal{P}^*)(1+\tau(1-\pi)))}{P((1-\pi)(aF+\mathcal{P})+\tau\mathcal{P}^*)^3} < 0$$

Part (c): The case of $\pi = 0$ was already proven in proposition 3 of FHZ. For $\pi = 1$, we can show that:

$$\left. \frac{\partial \phi}{\partial \tau} \right|_{\pi=1} = -\frac{2\mathcal{P}^2 L^*(aF+\mathcal{P})}{(L+L^*)(aF+(1+\tau)\mathcal{P})^2} < 0$$

D Proof of Proposition 5

The welfare effects of trade costs τ in the case of no parallel trade restrictions are given by:

$$\begin{aligned} \left. \frac{\partial U(\pi, \tau)}{\partial \tau} \right|_{\pi=0} &= -\frac{a\mathcal{P}^*L^*}{aF+\mathcal{P}} \left[\frac{\mathcal{P}(\tau^2\mathcal{P}+2a\tau F+2\tau\mathcal{P}^*+aF+\mathcal{P})}{(aF+\mathcal{P}^*+\tau\mathcal{P})^2} \right] < 0 \\ \left. \frac{\partial U^*(\pi, \tau)}{\partial \tau} \right|_{\pi=0} &= \frac{a^2 F \mathcal{P} L^*}{(aF+\mathcal{P}^*+\tau\mathcal{P})^2} > 0 \end{aligned}$$

In the case of forbidden parallel trade, the welfare effects of τ are given by:

$$\left. \frac{\partial U^*(\pi, \tau)}{\partial \tau} \right|_{\pi=1} = -\frac{aL^*(\tau^2\mathcal{P}\mathcal{P}^*+2a\tau F\mathcal{P}+2\tau\mathcal{P}^2+a^2F^2+aF\mathcal{P}^*+aF\mathcal{P}+\mathcal{P}\mathcal{P}^*)}{\tau^2(aF+\mathcal{P}^*+\tau\mathcal{P})^2} < 0$$

In the general case, the effect of trade costs on welfare is:

$$\begin{aligned} \frac{\partial U}{\partial \tau} &= \frac{\partial N^*}{\partial \tau} \left(1 + \xi \frac{\mathcal{P}}{\mathcal{P}^*} \right) + \frac{\partial \xi}{\partial \tau} \left(\frac{\partial N_N}{\partial \xi} + N^* \frac{\mathcal{P}}{\mathcal{P}^*} \right) + \frac{\partial N_N}{\partial \omega} \frac{\partial \omega}{\partial \tau} \leq 0 \\ \frac{\partial U^*}{\partial \tau} &= \frac{\partial N^*}{\partial \tau} \left(1 + \xi \frac{\mathcal{P}}{\mathcal{P}^*} \right) + \frac{\partial \xi}{\partial \tau} N^* \frac{\mathcal{P}}{\mathcal{P}^*} \geq 0 \end{aligned}$$

where the inequalities follow from appendix C. Hence, what remains to be shown is $\partial^2 U / \partial \tau \partial \pi > 0$ and $\partial^2 U^* / \partial \tau \partial \pi < 0$. From (7), (10) and (11), we get:

$$\begin{aligned} \frac{\partial^2 N_T}{\partial \tau \partial \pi} &= \frac{a\mathcal{P}L^*}{(aF+\mathcal{P}^*+\tau\mathcal{P})^2} \left((aF+\mathcal{P}^*+\tau\mathcal{P})\xi_{\tau,\pi} - \mathcal{P}\xi_\pi \right) < 0 \\ \frac{\partial^2 N_N}{\partial \tau \partial \pi} &= -\frac{a\mathcal{P}L^*}{aF+\mathcal{P}} \left(\omega_\tau \xi_\pi + \omega \xi_{\tau,\pi} \right) > 0 \end{aligned}$$

where $\xi_\pi = \frac{\partial \xi}{\partial \pi} > 0$ and $\xi_{\tau,\pi} = \frac{\partial^2 \xi}{\partial \tau \partial \pi} < 0$. From this, it can be verified that $\frac{\partial^2 (N_N+N_T)}{\partial \tau \partial \pi} > 0$, which concludes the proof.

E Optimal Trade Policy

Given that $\frac{\partial \xi}{\partial \pi} > 0$ by appendix A, all that remains to be shown is that

$$\begin{aligned}
\mathcal{P} \frac{\partial N_N}{\partial \xi} + \frac{\mathcal{P}(\mathcal{P} + \mathcal{P}^*)}{\mathcal{P}^*} N^* &= -\frac{a\mathcal{P}L^*}{aF + \mathcal{P}} \frac{aF + \mathcal{P} + \tau\mathcal{P}^*}{aF + \mathcal{P}^* + \tau\mathcal{P}} + \frac{\mathcal{P} + \mathcal{P}^*}{\mathcal{P}^*} \frac{a\mathcal{P}^*L^*}{aF + \mathcal{P}^* + \tau\mathcal{P}} \\
&= \frac{aL^*}{(aF + \mathcal{P}^* + \tau\mathcal{P})(aF + \mathcal{P})} \left[-\mathcal{P}(aF + \mathcal{P} + \tau\mathcal{P}^*) + (\mathcal{P} + \mathcal{P}^*)(aF + \mathcal{P}) \right] \\
&= \frac{a\mathcal{P}^*L^*}{(aF + \mathcal{P}^* + \tau\mathcal{P})(aF + \mathcal{P})} \left[aF + \mathcal{P} - \tau\mathcal{P} \right] > 0
\end{aligned}$$

where the inequality follows from the trade condition (12).

F Proof Proposition 6

Part (a):

$$\frac{\partial \xi}{\partial \mathcal{P}^*/\mathcal{P}} = \frac{(1 - \pi) \left(\frac{aF + \mathcal{P}}{\mathcal{P}} \right) \left(\frac{aF + \mathcal{P}}{\mathcal{P}} - \tau^2 \right)}{\left((1 - \pi) \left(\frac{aF + \mathcal{P}}{\mathcal{P}} \right) + \tau \frac{\mathcal{P}^*}{\mathcal{P}} \right)^2} \geq 0$$

Part (b): It can be shown that $\frac{\partial N^*}{\partial L^*} = \frac{a\mathcal{P}^*}{aF + \mathcal{P}^* + \tau\mathcal{P}} > 0$ and $\frac{\partial N_T}{\partial L^*} = \frac{a\mathcal{P}\xi}{aF + \mathcal{P}^* + \tau\mathcal{P}} > 0$, which ensures that $\partial U^*/\partial L^* > 0$. For the North, we additionally have $\frac{\partial N_N}{\partial L^*} = -\left(\frac{a\mathcal{P}}{aF + \mathcal{P}} \right) \left(\frac{aF + \mathcal{P} + \tau\mathcal{P}^*}{aF + \mathcal{P}^* + \tau\mathcal{P}} \right) \xi < 0$. However, in sum:

$$\frac{\partial U}{\partial L^*} = a\mathcal{P}^* \left[\frac{((1 - \pi)(aF + \mathcal{P}) + \tau\mathcal{P}^*)(aF + \mathcal{P}) - \tau\mathcal{P} \left((aF + \mathcal{P}) \left(\pi \frac{\mathcal{P}^*}{\mathcal{P}} + (1 - \pi)\tau \right) + (1 - \pi)\mathcal{P}^*\tau^2 \right)}{(1 - \pi)(aF + \mathcal{P}) + \tau\mathcal{P}^*} \right] \geq 0$$