Institute for Operations Research
and Computational Finance
University of St.Gallen

Valuation and Bidding of Hydro Power Plants in the EEX

Karl Frauendorfer
(joint work with Gido Haarbrücker, Alvin Schwendener)

ior/cf-HSG

5th IBM Swiss OR Days

Zurich, 27. August 2007

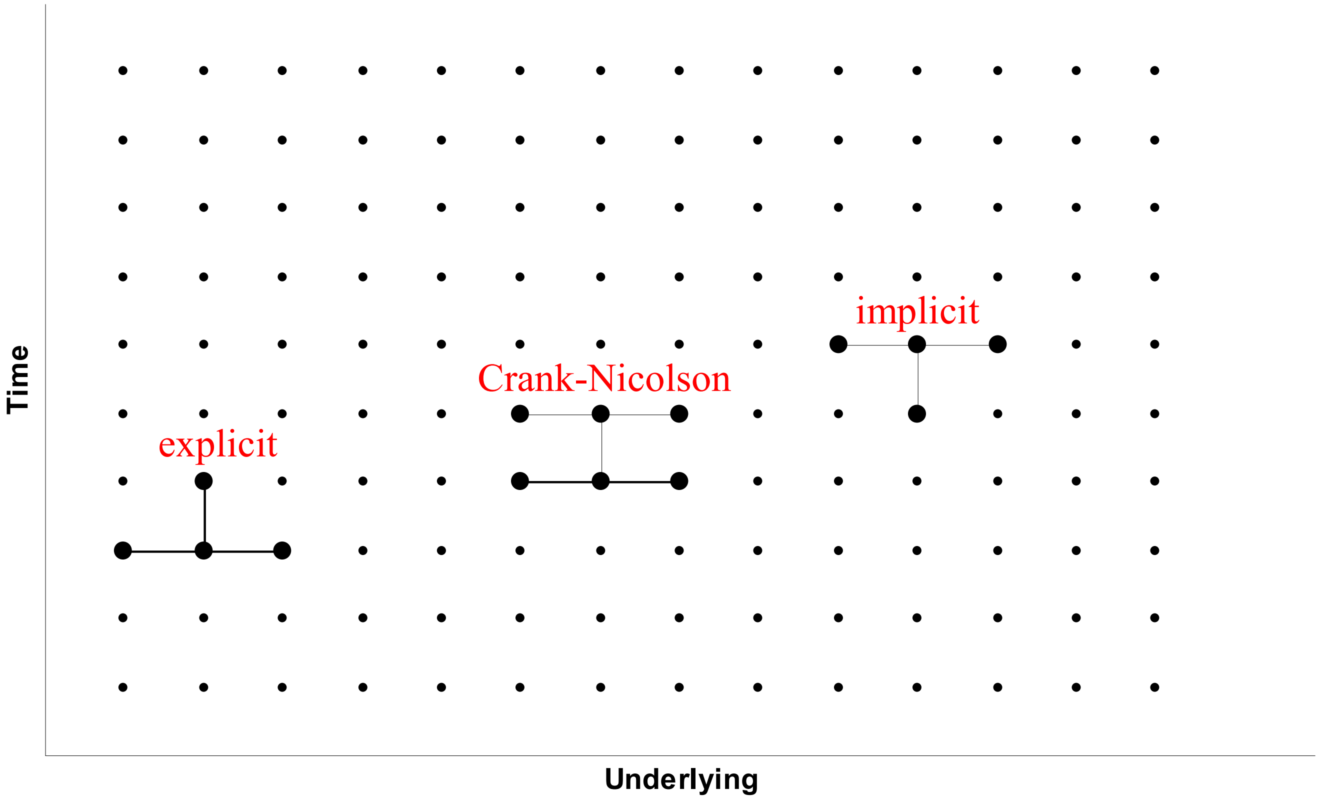
Agenda

1. Motivation
2. Valuation of Swing Options
    1. Structual Properties of the Value Function
    2. Price Dynamics
    3. Influence of Integral and Differential Restrictions
    4. Risk Measurement and P&L Distribution
3. Bidding Strategies of a Storage Power Plant
    1. Model Specification and Input Variables
    2. Outputs and Sensitivity Analysis
    3. Bidding Strategy

© ior/cf - HSG

Finite Difference vs. MSP (I/II)

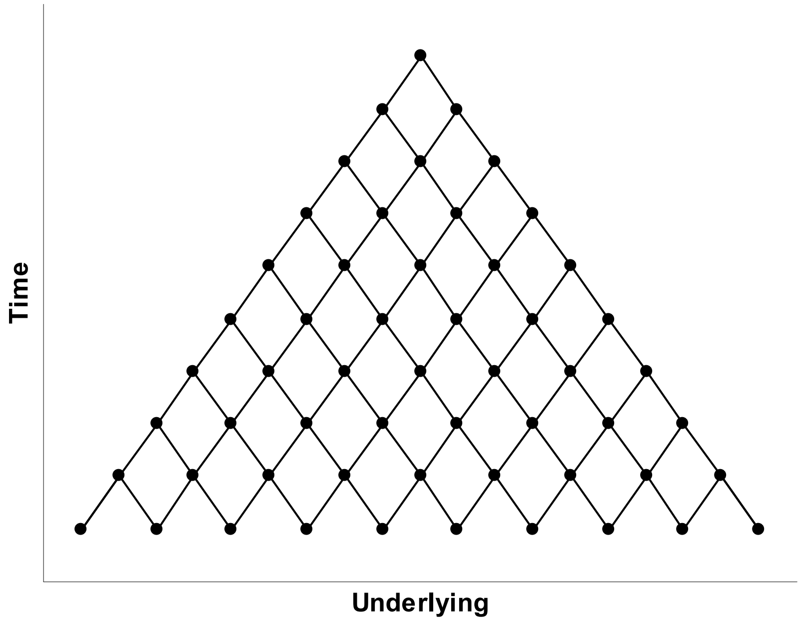
Stability Criterion: $0 < \triangle\tau \leq \frac{\triangle x^2}{2}$



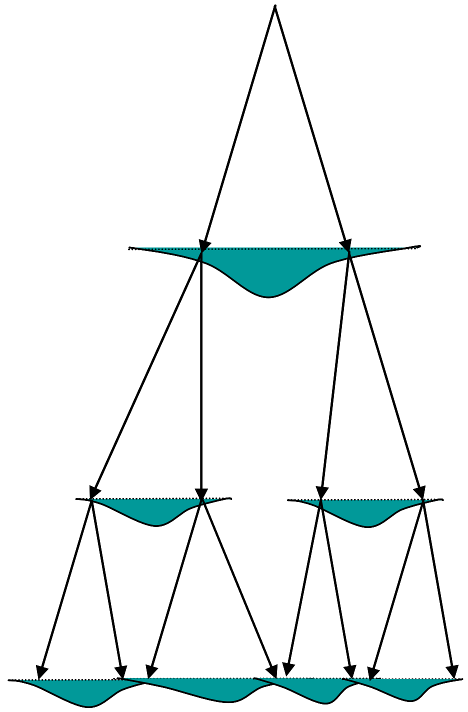
© ior/cf - HSG

Finite Difference vs. MSP (II/II)

Recombining Tree



Non-Recombining Tree



© ior/cf - HSG

Agenda

1. Motivation
2. Valuation of Swing Options
    1. Structual Properties of the Value Function
    2. Price Dynamics
    3. Influence of Integral and Differential Restrictions
    4. Risk Measurement and P&L Distribution
3. Bidding Strategies of a Storage Power Plant
    1. Model Specification and Input Variables
    2. Outputs and Sensitivity Analysis
    3. Bidding Strategy

© ior/cf - HSG

Valuation Problem

$$\max \; E\left(\sum_{t=0 \vee t^-}^{t^+} (S_t - K) \cdot \pi_t \cdot p_t \right)$$

s.t.

$$e^- \le e_{t^+} \le e^+$$ integral restriction

$$e_t - e_{t-1} = \pi_t \cdot p_t$$ energy balance equation

separable restriction $$p^- \le p_t \le p^+$$

differential restriction $$\left| p_t - p_{t-1} \right| \le \rho_t$$

p_t, e_t *nonanticipative*

for all $t = 0 \vee t^-, \ldots, t^+$

Legend:

p_t	power at hour *t* [MW]
e_t	cumulative purchased energy by the end of hour *t* [MWh]
S_t	spot price at hour *t* [€ /MWh]
K	strike price [€ /MWh]
π_t	on-/off-quality at hour *t* [.]

© ior/cf - HSG

Dynamic representation

- Dynamic representation in terms of value functions for $t = 1, ..., T$ (with $\omega_t := (\eta_t , \xi_t)$, $\omega^t := (\omega_1 , ..., \omega_t)$, $\Omega_t := \Theta_t \times \Xi_t$, $\phi_{T+1}(\cdot) := 0$):

$$\begin{aligned}\phi_t(x^{t-1},\omega^t) &:= \max \rho_t(x^{t-1},x_t,\eta^t) + \int_{\Omega_{t+1}} \phi_{t+1}(x^{t-1},x_t,\omega^t,\omega_{t+1})dP_{t+1}(\omega_{t+1} \mid \omega^t) \\ \text{s.t.} \quad & f_t(x^{t-1},x_t) \le h_t(\xi^{t-1},\xi_t) \\ & x_t \ge 0\end{aligned}$$

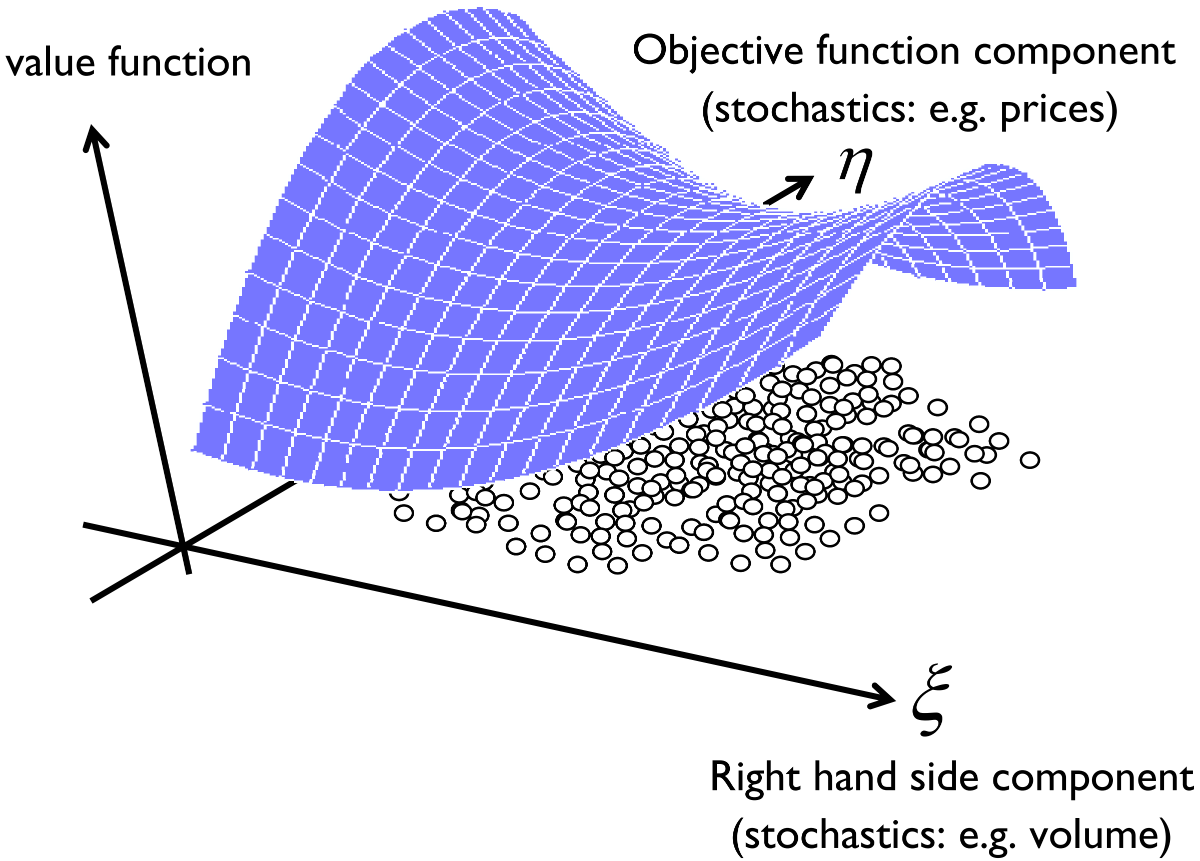
- The problem at $t = 0$ depends on the observations $\omega_0 := (\eta_0 , \xi_0)$ which are usually assumed deterministic:

$$\begin{aligned}&\max \rho_0(x_0,\eta_0) + \int_{\Omega_1} \phi_1(x_0,\omega_0)dP_1(\omega_1 \mid \omega_0) \\ &\text{s.t.} \quad f_0(x_0) \le h_0(\xi_0) \\ &\qquad x_0 \ge 0\end{aligned}$$

© ior/cf - HSG

Saddle Property

- Under reasonable assumptions we get a value function with saddle structure:



© ior/cf - HSG

2-Factor-Pilipovic Model (generalized)

The two risk-factors of the generalized Pilipovic model:

$$\begin{aligned} dX_1(t) &= \overbrace{\kappa(X_2(t) - X_1(t))dt}^{\text{Mean Reverting}} + \sigma_1 dW_1(t), \\ dX_2(t) &= \sigma_2 dW_2(t). \end{aligned}$$

with:

X_1 : Risk-factor 1 (describes short term dynamics)

X_2 : Risk-factor 2 (describes long term dynamics)

κ : Mean Reverting Parameter

σ_1 : Volatility-term of risk-factor X_1

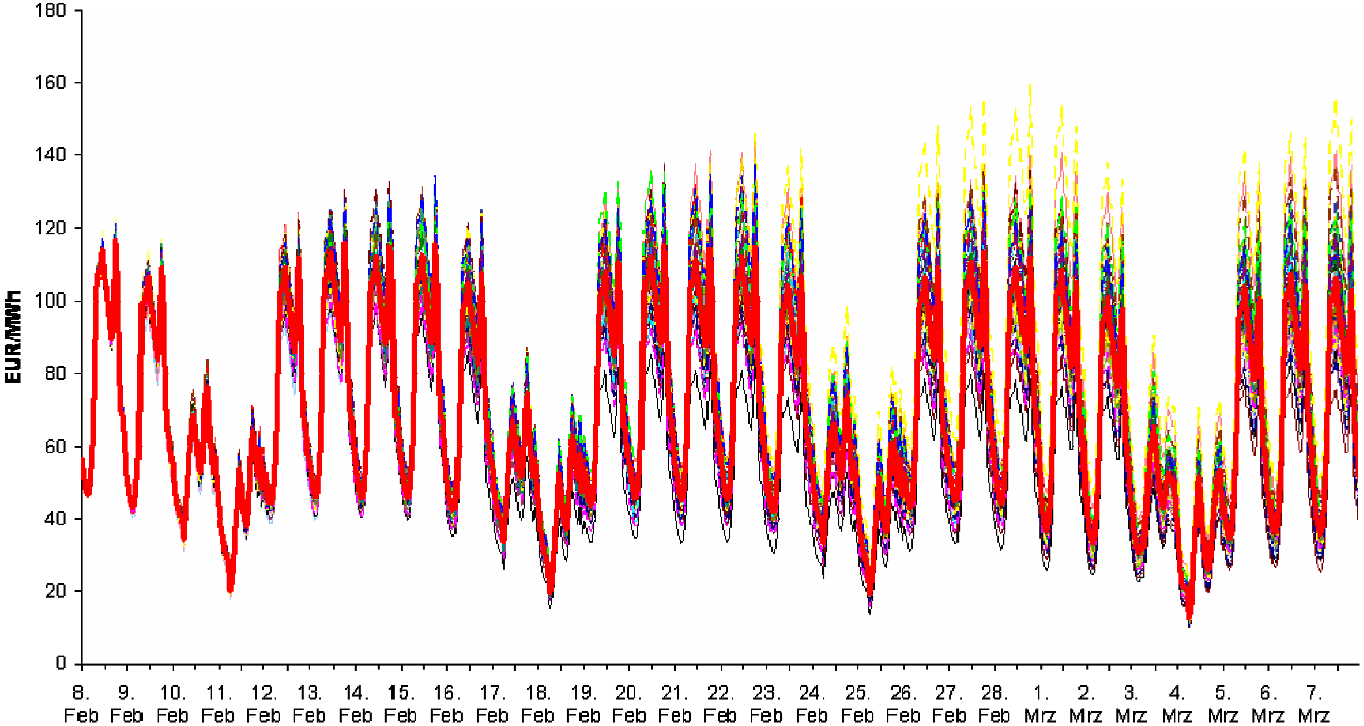
σ_2 : Volatility-term of risk-factor X_2

$W_1(t),\ W_2(t)$: Brownian motion

© ior/cf - HSG

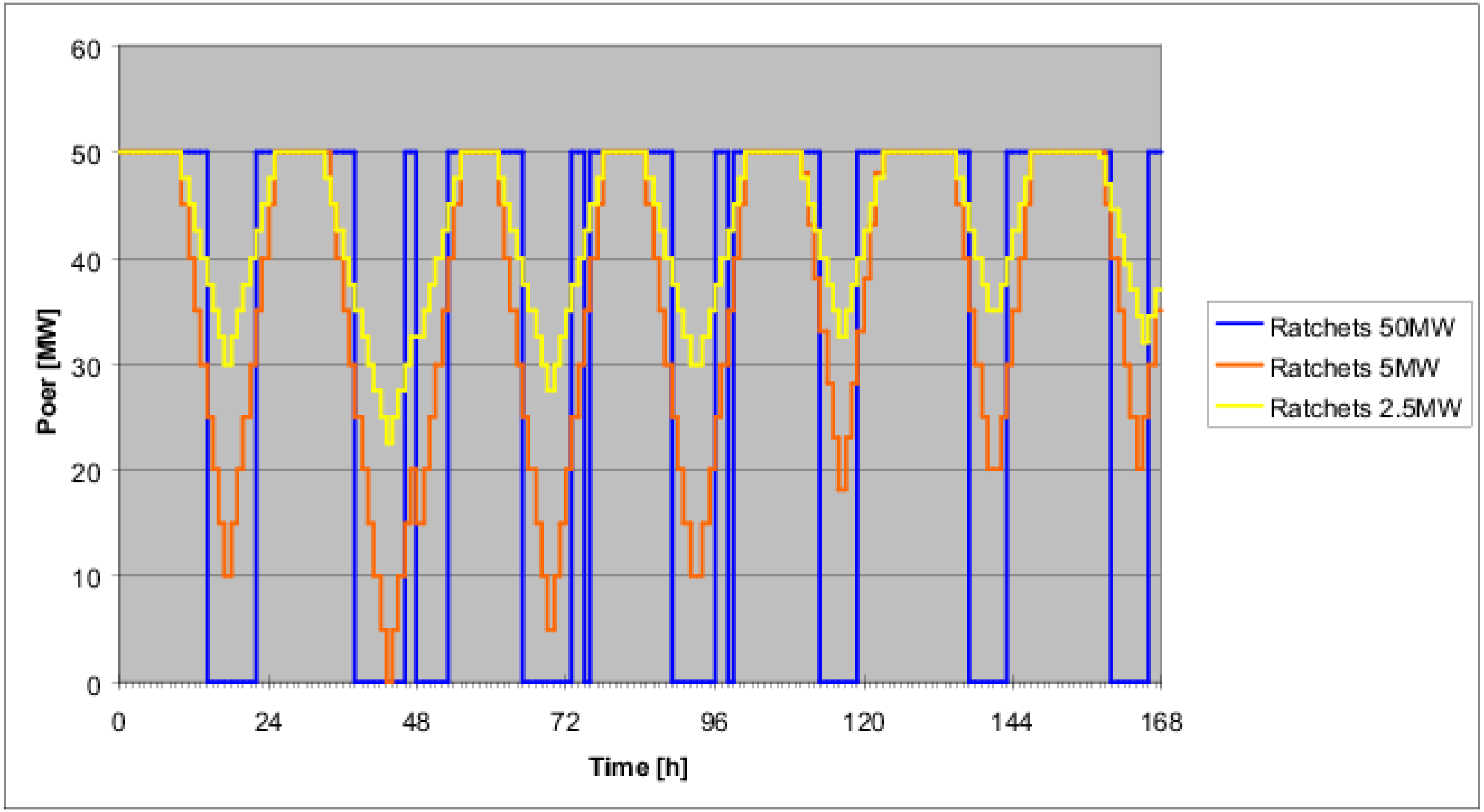
Input: HPFC & Price Dynamics

- **HPFC:** arbitragefree, hourly price forward curve
  - seasonal, weekly and daily pattern
- **Price dynamics:** 2-factor-Pilipovic model ($\kappa_1, \sigma_1, \sigma_2$)



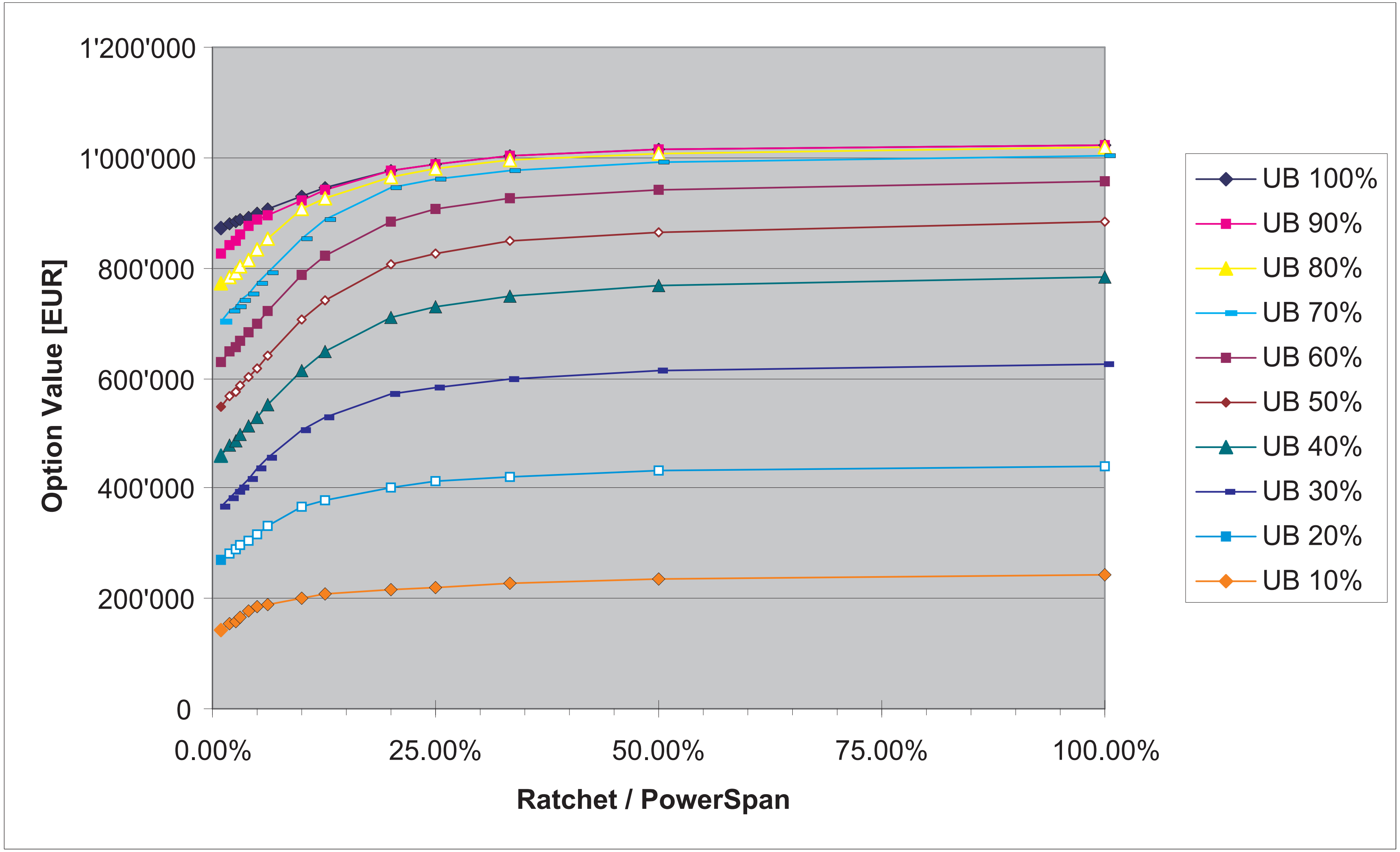
© ior/cf - HSG

Influence of Ratchets: On Deterministic Exercise Strategy



© ior/cf - HSG

Influence of Upper Energy Limit and Ratchets



© ior/cf - HSG

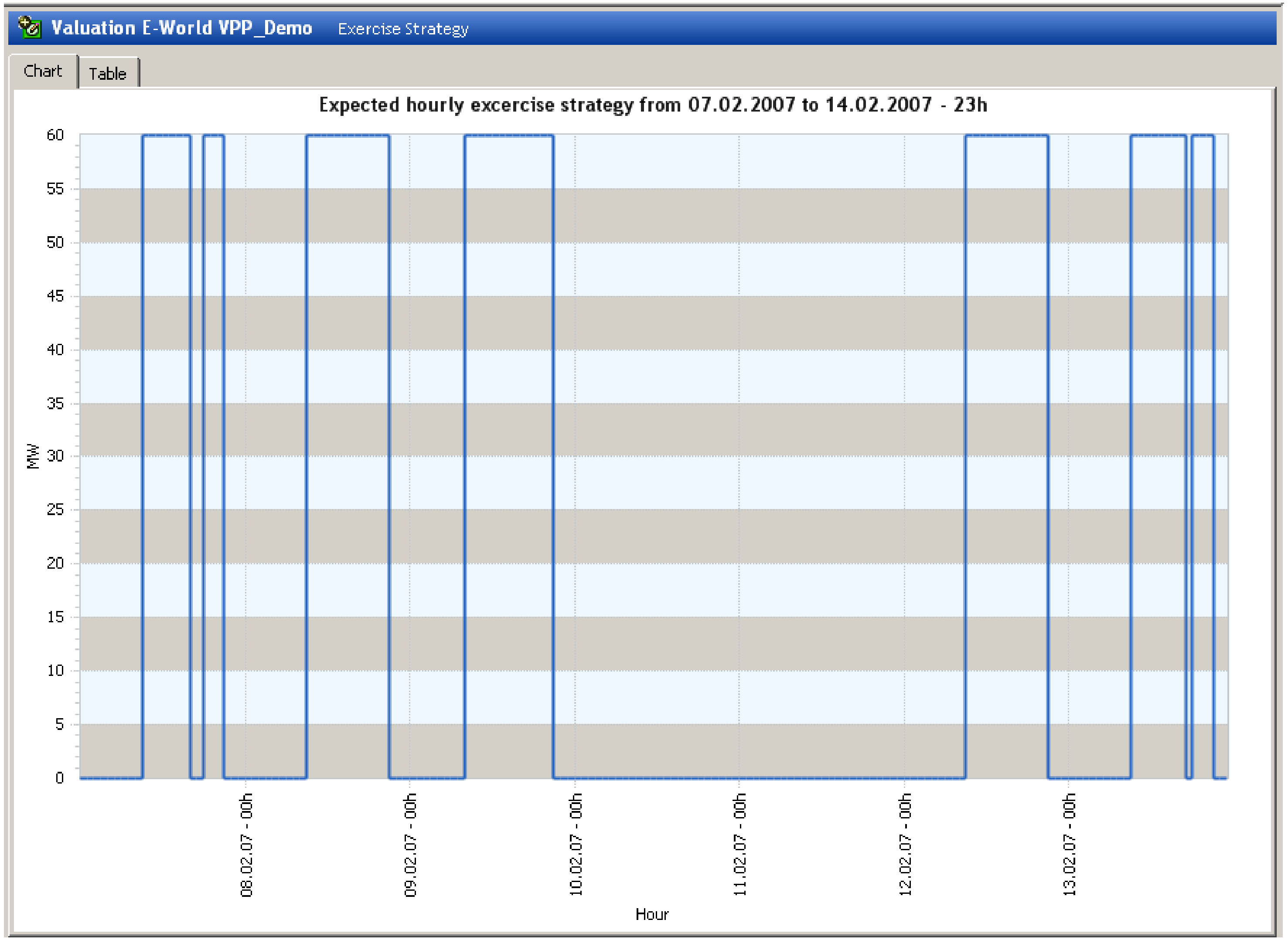
Output (BIT@EPI.VPP) – Contract Specifications

Call Option on VPP

• Delivery Period:	01. February 2007 - 07. February 2008
• Evaluation Date:	07. February 2007
• Strike Price:	40.00 Euro
• Maximum Power:	60 MW
• Energy Exercised:	2000 MWh
• Exercise Quality:	Peak: Mon – Fri, 8h - 20h

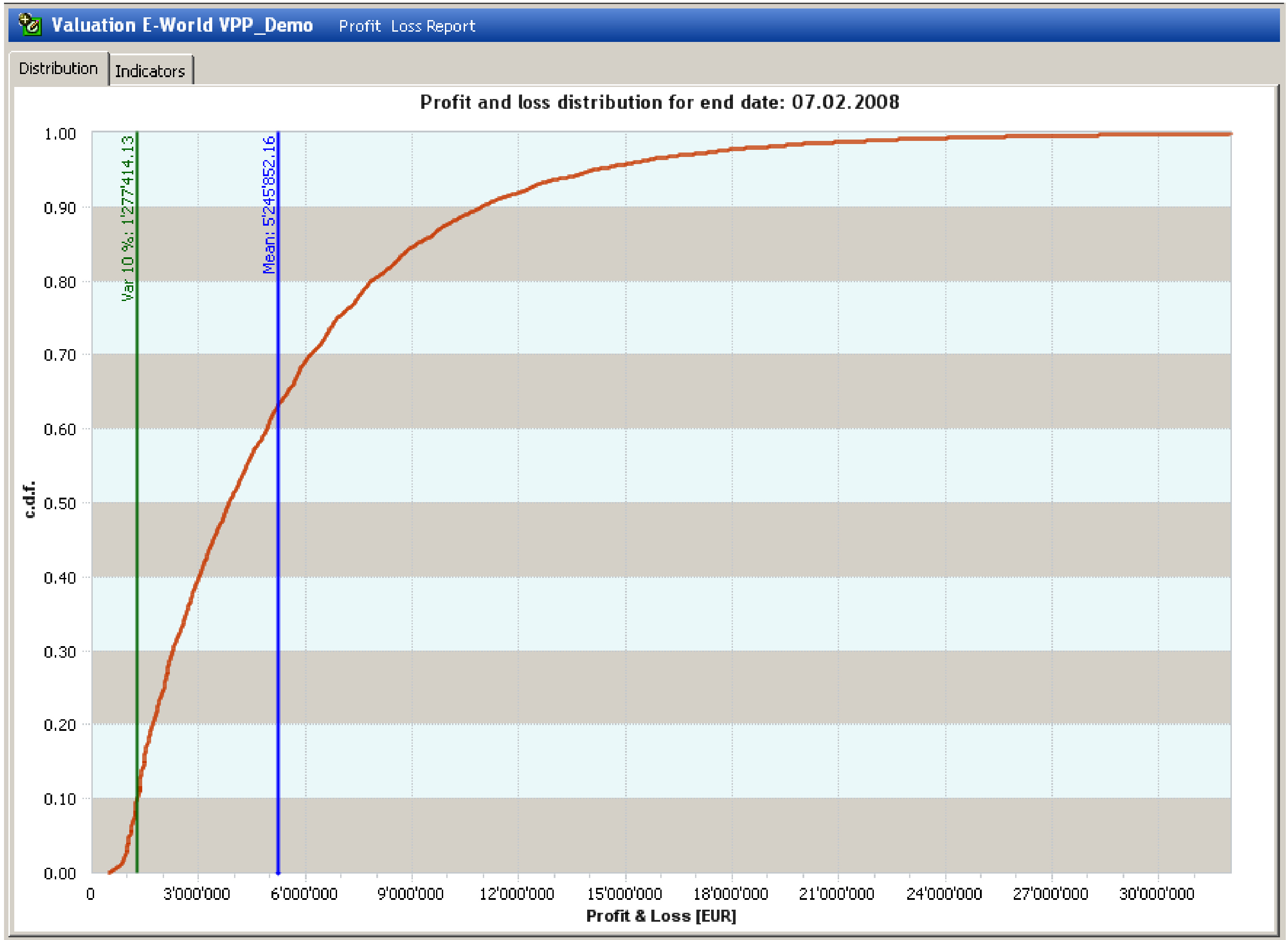
© ior/cf - HSG

Output (BIT@EPI.VPP) – Exercise Strategy



© ior/cf - HSG

Output (BIT@EPI.VPP) – P&L-Distribution



© ior/cf - HSG

Output (BIT@EPI.VPP) – P&L-Indicators

Valuation E-World VPP_Demo Profit Loss Report

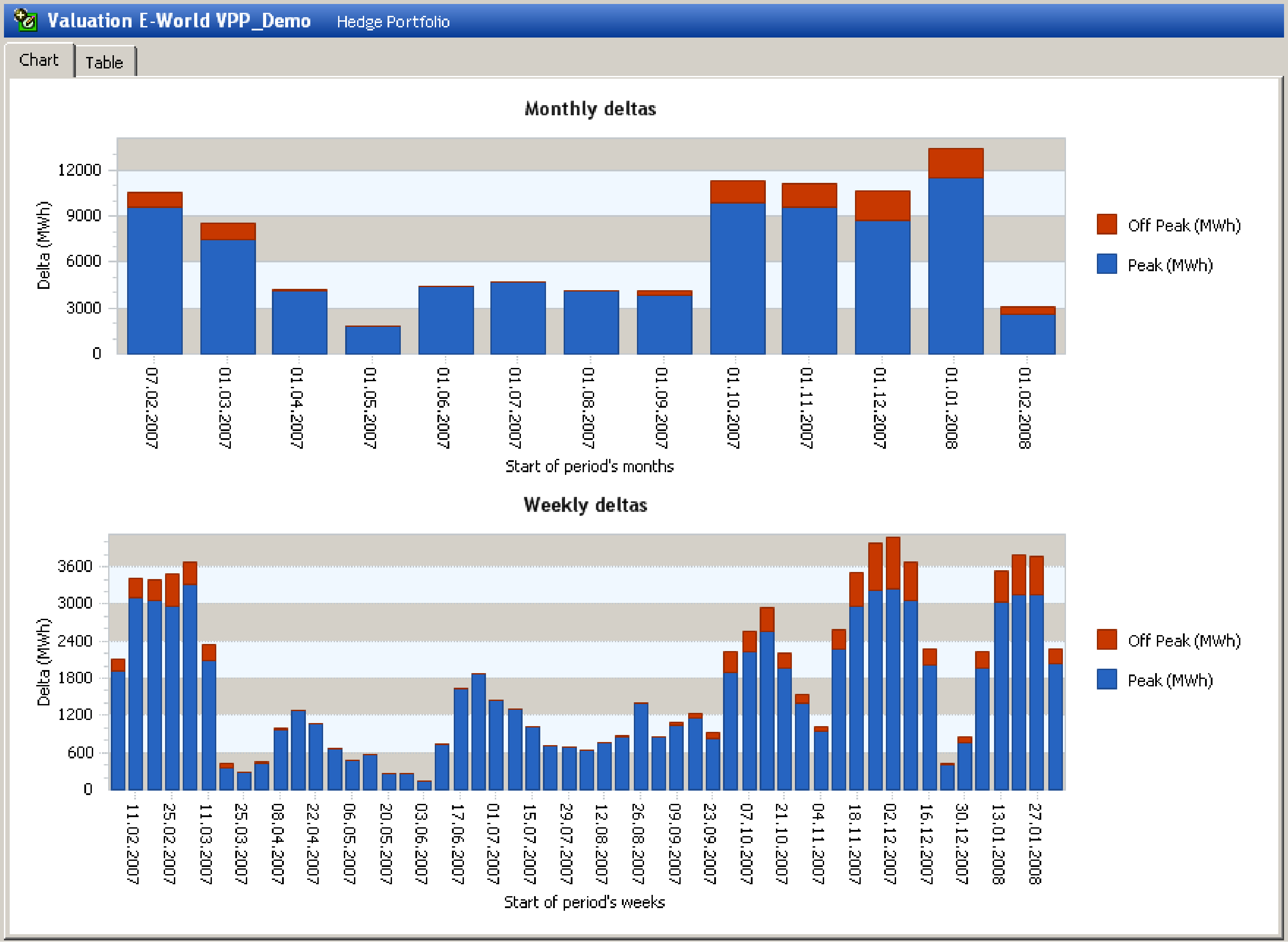
Distribution | Indicators

Level [%]	VaR [EUR]	TailVaR [EUR]
0.05	1'067'380.36	895'965.70
0.10	1'277'414.13	1'036'959.81
0.15	1'464'483.38	1'147'496.07
0.20	1'704'944.89	1'253'066.45
0.25	2'039'724.42	1'375'041.77
0.30	2'281'235.33	1'502'291.50
0.35	2'647'990.27	1'639'851.70
0.40	3'007'008.75	1'787'711.58
0.45	3'404'861.33	1'946'108.29
0.50	3'849'479.46	2'115'584.55
0.55	4'352'634.49	2'297'286.48
0.60	4'926'782.29	2'491'297.19
0.65	5'515'386.73	2'698'070.68
0.70	6'141'677.96	2'920'832.42
0.75	6'907'856.82	3'163'280.11
0.80	7'856'795.18	3'429'734.85
0.85	9'121'601.74	3'728'840.18
0.90	10'951'754.18	4'077'314.96
0.95	14'087'168.38	4'513'557.47
1.00	116'305'237.97	5'245'852.16

Fair Price	**5'245'852.16**	**EUR**
VaR 3%	986'218.88	EUR
Tail VaR 3%	818'237.73	EUR
VaR 10%	1'277'414.13	EUR
Tail VaR 10%	1'036'959.81	EUR
Median	3'849'479.46	EUR
Tail VaR 50%	2'115'584.55	EUR
ADev	3'317'110.83	EUR
SDev	4'588'229.02	EUR
Skewness	2.27	
Kurtosis	8.82	

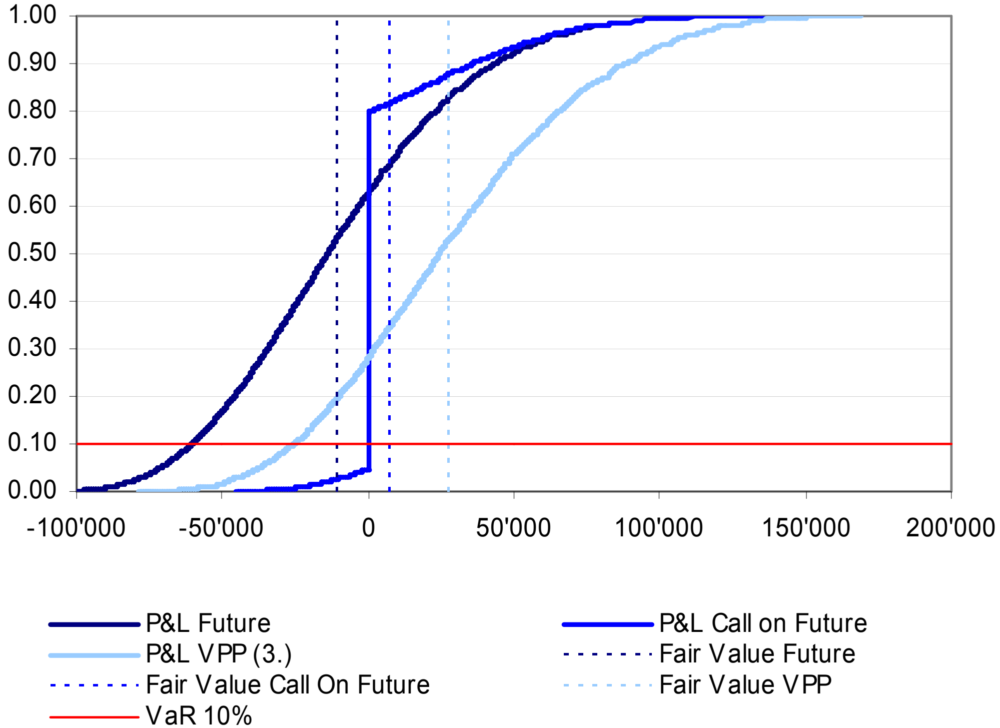
© ior/cf - HSG

Output (BIT@EPI.VPP) – Hedge Portfolio



© ior/cf - HSG

Comparison of the P&L distribution



Contract	1. Future	2. Call on Future	3. VPP	4. VPP
Energy constraints	3'132 MWh	3'132 MWh	3'132 MWh	3'000-3264 MWh
Price	-10'693.61	7'501.30	26'229.17	28'274.06
VaR (10%)	-60'383.11	0.00	-28'313.58	-25'489.52

© ior/cf - HSG

Analytically Tractable Benchmark Model (I/II)

Market Specification

- Forward curve: flat 55.00 EUR/MWh
- Price process: GBM with volatility of 20%
- Zero interest rate

Contract Specification Swing Option

- Start/End Date: 01/01/07 - 31/01/07
- Strike: 55.00 EUR/MWh
- # of exercise rights: n = 1, ..., 30 call rights (day-ahead)
- Valuation Date 01/01/07

© ior/cf - HSG

Analytically Tractable Benchmark Model (II/II)

- Under these model assumptions the optimal exercise strategy for a swing option is to hold on to the exercise rights as long as possible, i.e. exercise the last right on the maturity date (if in the money), the last but one right on the day before the maturity date, . . . (see Carmona/Touzi).

- The price of the swing option with n exercise rights $C^n(T,K)$ is given by

$$C^n(T,K) = \sum_{k=0}^{n-1} C(T-k,K)$$

- where C(T,K) is the standard Black-Scholes price for the call option with strike K and maturity T = 30 days.

© ior/cf - HSG

BIT@EPI.VPP vs. Black Scholes

# rights	analytic	BIT@EPI.VPP	rel. error
1	30.190	30.090	0.333%
2	59.874	60.170	-0.495%
3	89.040	88.809	0.259%
4	117.684	117.452	0.197%
5	145.790	145.450	0.232%
6	173.352	173.442	-0.052%
7	200.354	199.878	0.236%
8	226.792	226.320	0.206%
9	252.639	252.063	0.231%
10	277.900	277.790	0.041%
11	302.555	301.829	0.239%
12	326.580	325.872	0.218%
13	349.973	349.115	0.242%
14	372.694	372.386	0.085%
15	394.740	393.735	0.255%
16	416.096	415.104	0.237%
17	436.713	435.591	0.259%
18	456.588	456.066	0.117%
19	475.684	474.373	0.276%
20	493.980	492.700	0.260%
21	511.413	509.943	0.287%
22	527.934	527.186	0.144%
23	543.536	541.834	0.312%
24	558.120	556.488	0.292%
25	571.625	569.725	0.333%
26	583.960	582.946	0.173%
27	594.972	592.704	0.381%
28	604.520	602.476	0.338%
29	612.335	609.377	0.479%
30	617.820	616.290	0.248%
31	617.830	616.311	0.248%

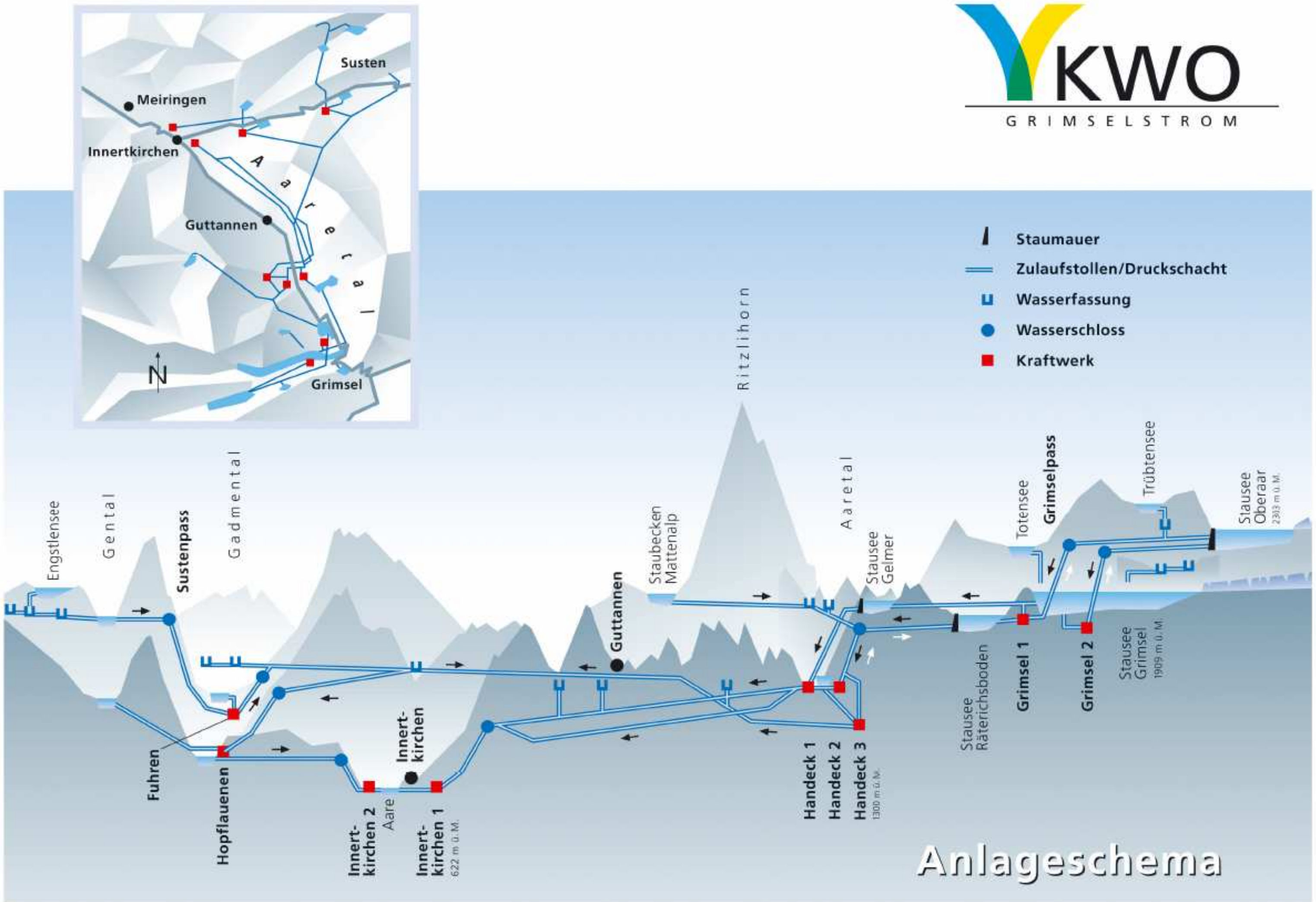
© ior/cf - HSG

Agenda

1. Motivation
2. Valuation of Swing Options
    1. Structual Properties of the Value Function
    2. Price Dynamics
    3. Influence of Integral and Differential Restrictions
    4. Risk Measurement and P&L Distribution
3. Bidding Strategies of a Storage Power Plant
    1. Model Specification and Input Variables
    2. Outputs and Sensitivity Analysis
    3. Bidding Strategy

© ior/cf - HSG

Hydro Power System – KWO Switzerland



© ior/cf - HSG

Motivation (I/II)

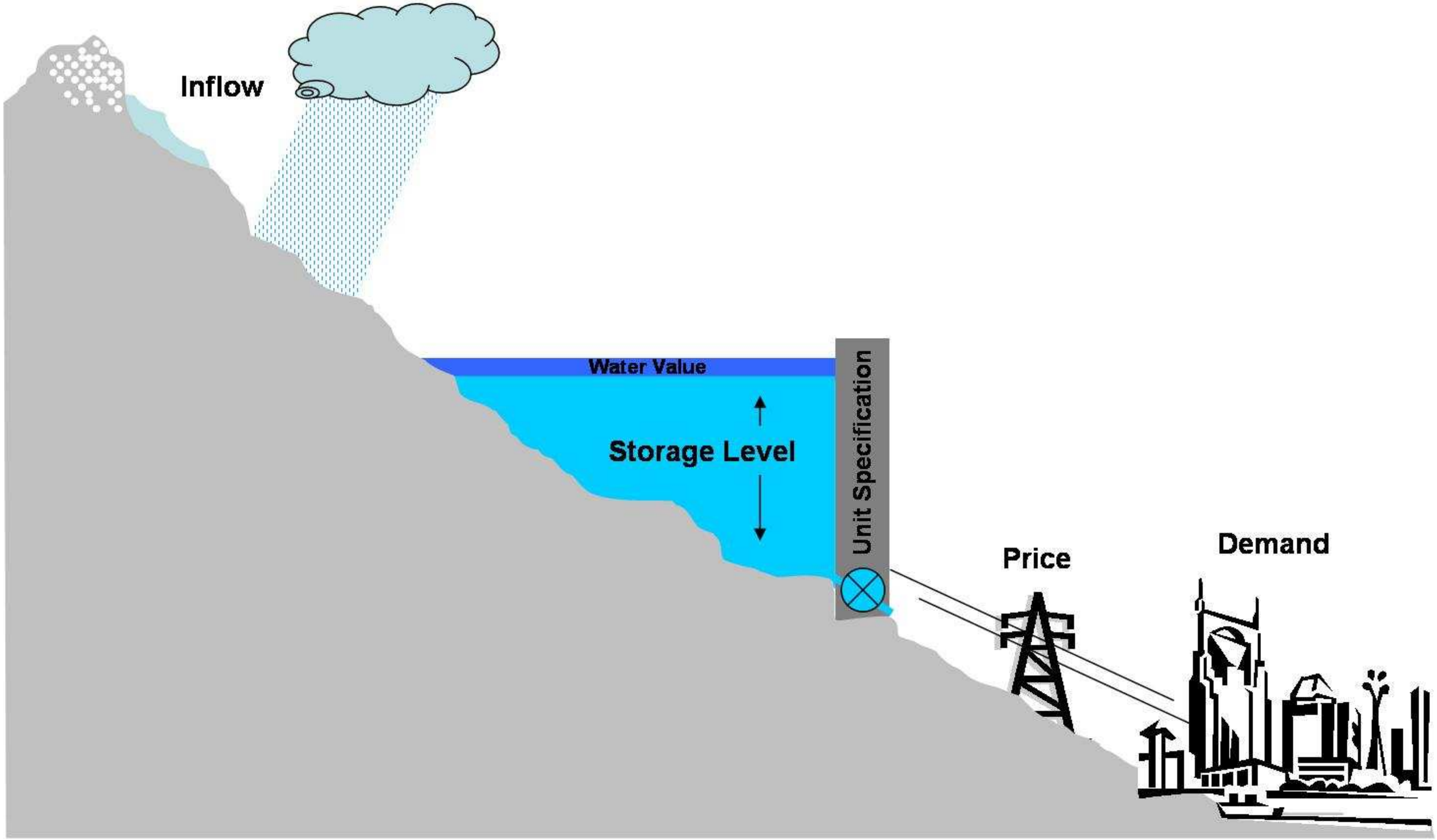
- Power Producers have to take decisions in the presence of multidimensional uncertain influences.

- Already the optimal regulation of a single hydro storage without any cascade and pump ends up in a complex problem.

- Such problems require sophisticated valuation tools:
  - for American type options.
  - considering dynamics and correlations of different risk factors.

© ior/cf - HSG

Motivation (II/II)



© ior/cf - HSG

Model Design

- Model is based on multistage stochastic optimization.

- Model specification:
  - time period: years 2004 to 2006
  - storage limit: 175‘000 MWh
  - planning horizon: 1 year (52 weeks)

- Three cases:

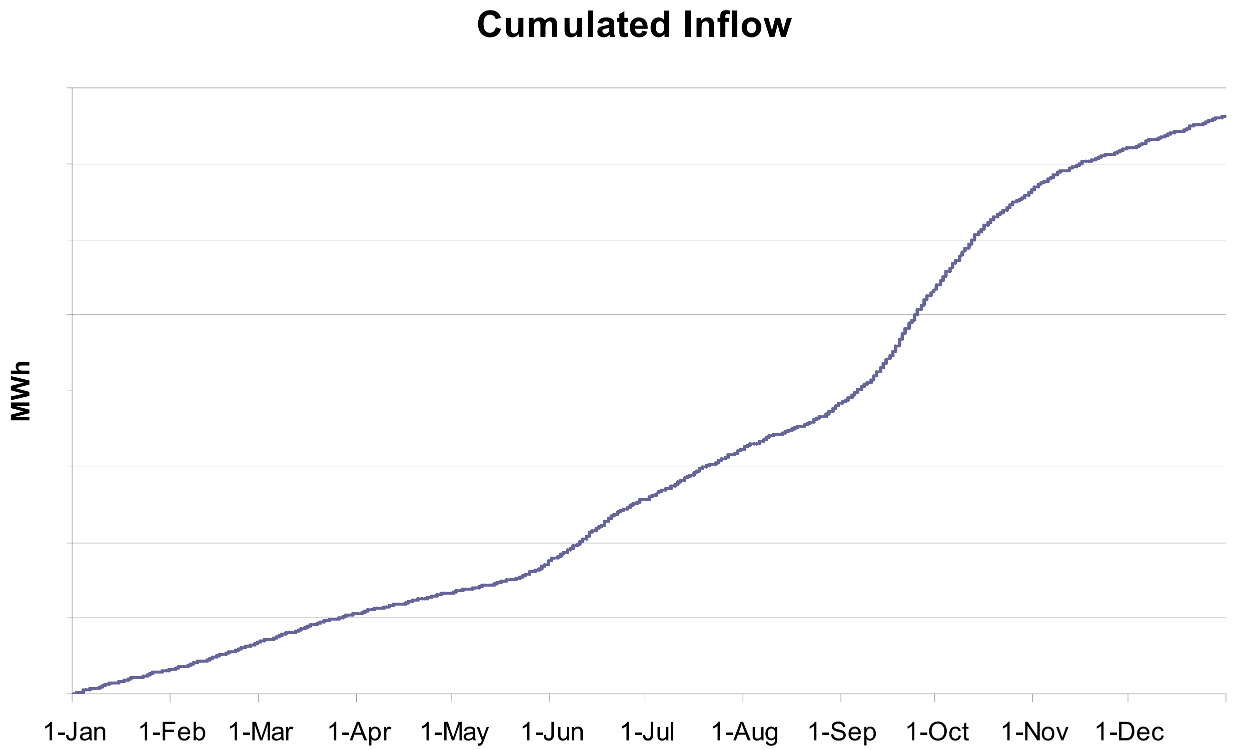
  Case 1:
  - initial case

  Case 2:
  - reduced price volatility (σ_1): σ_1(Case 2) = ⅔ * σ_1(Case 1)

© ior/cf - HSG

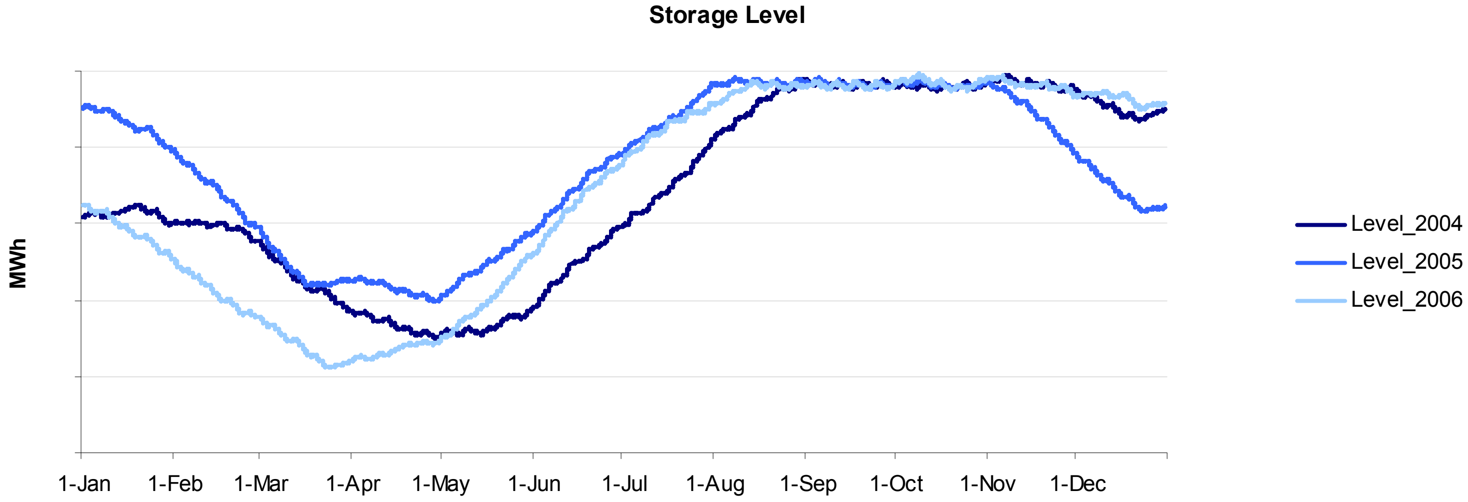
Input: Uncertain Inflow (Central America)

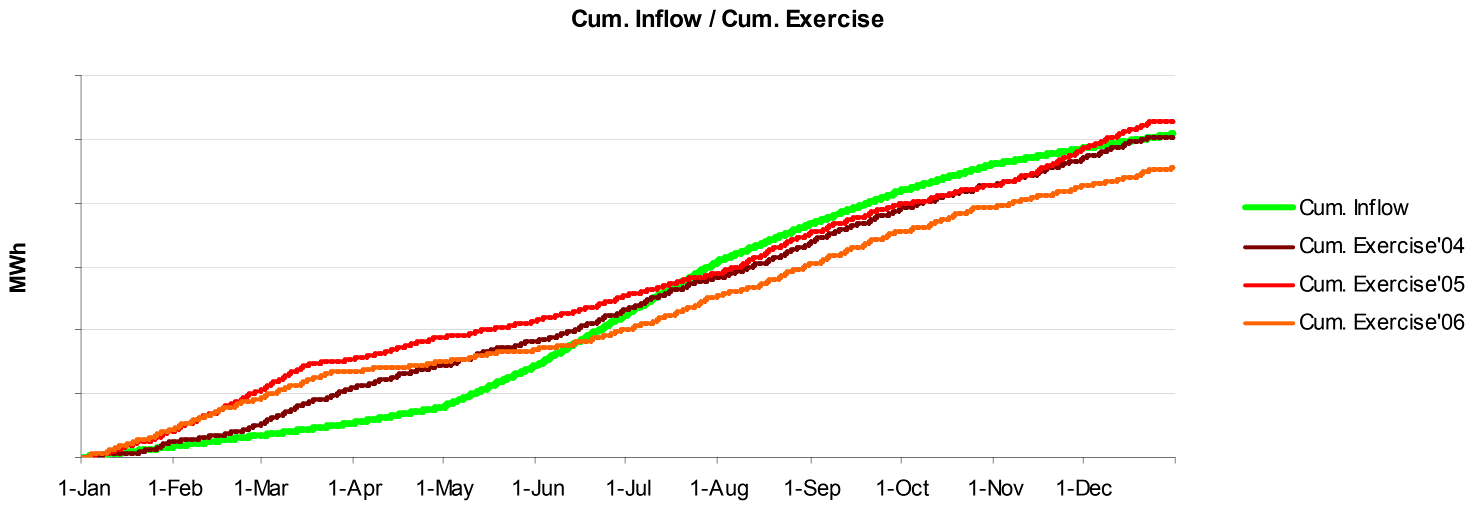
- changing over time
- seasonal pattern



© ior/cf - HSG

Storage Level / Cum. Inflow / Cum. Exercise

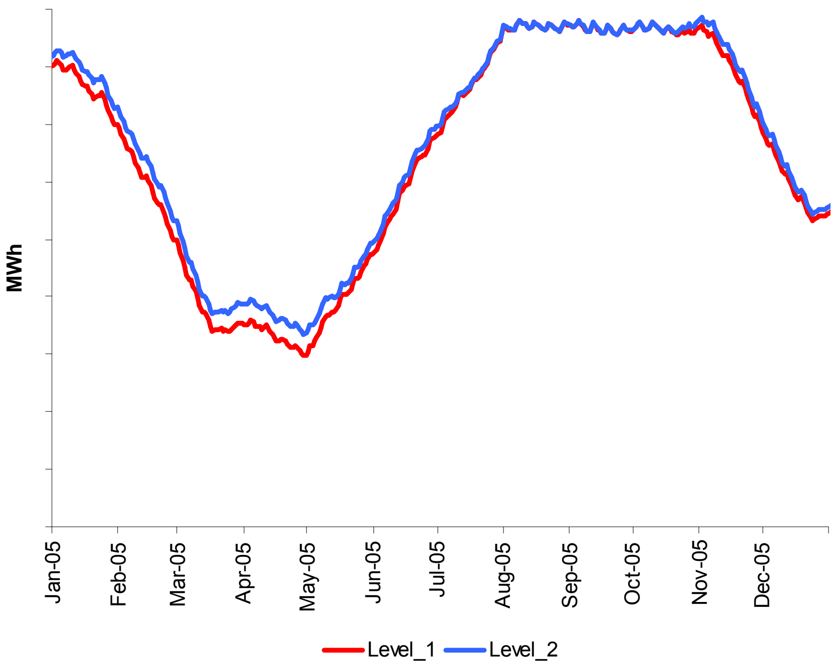


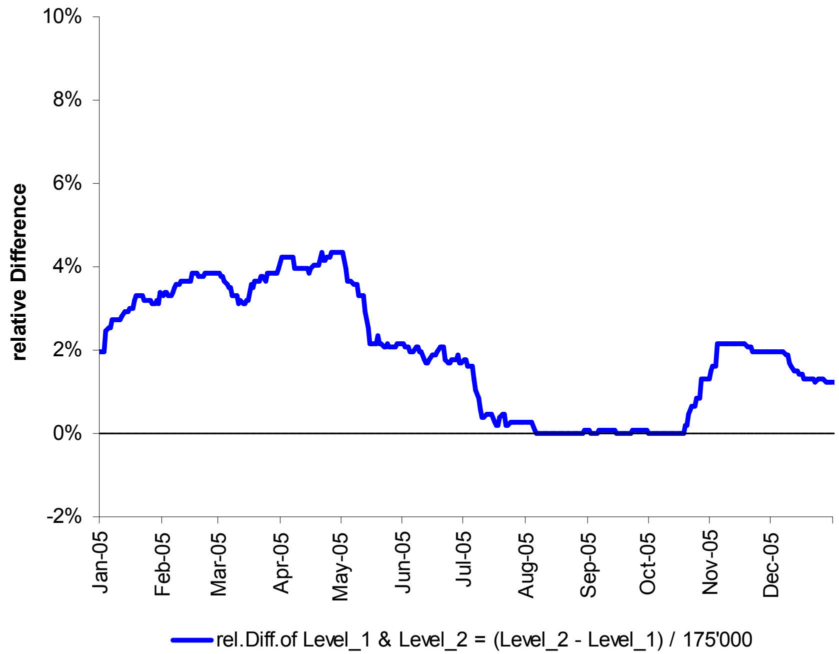


© ior/cf - HSG

Storage Levels

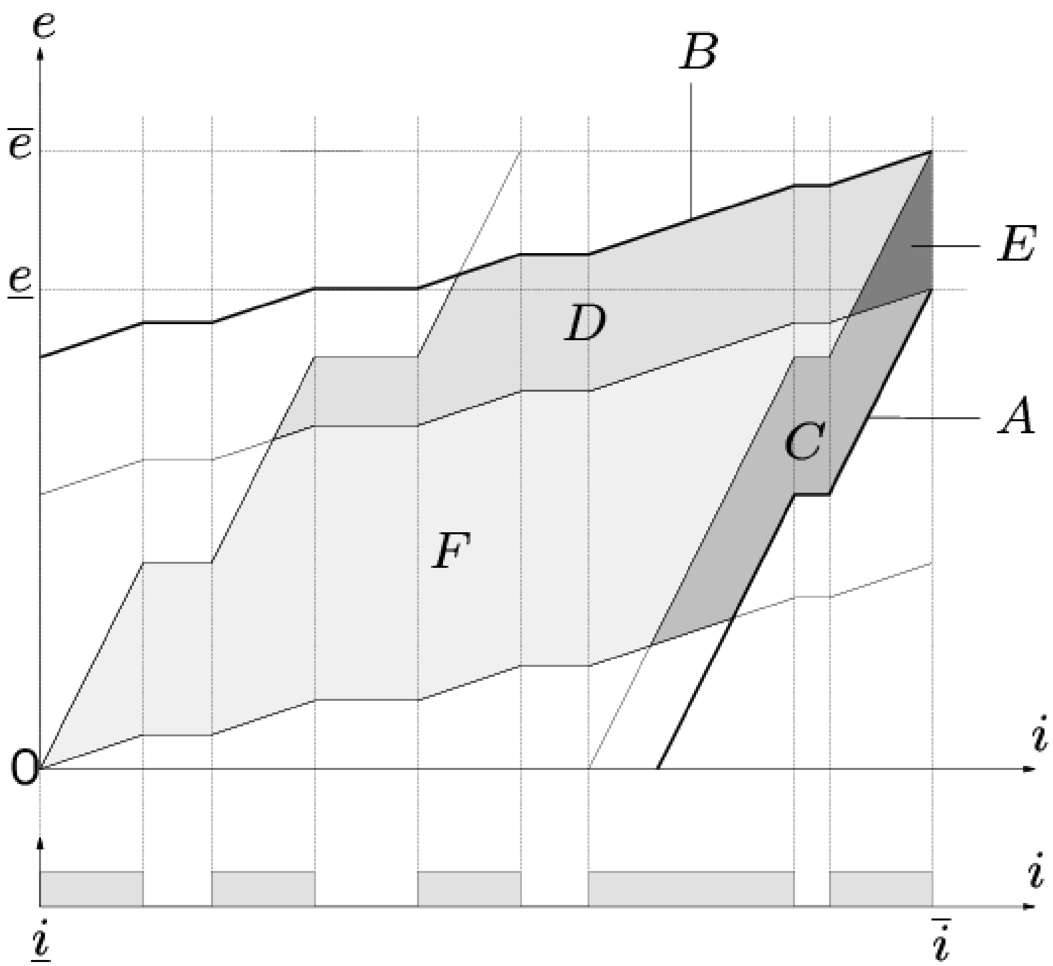
- The reservoir level of case 2 evolves mostly above the level of case 1.
- Subject to the maximum level of 175‘000 MWh, the relative differences move between 0% and 4%.
- The higher volatility in case 1 leads to higher watervalues. Case 1 takes advantage of these higher values.





© ior/cf - HSG

A Time/Energy Analysis for VPPs



© ior/cf - HSG

A Priori Analysis – Regions and Decision Strategies

Easy cases: optimal exercise strategy fully pre-determined

- ***A*** : always p**+**
- ***B*** : always p**-**
- ***E*** : p**-** if Spot < Strike; p**+** if Spot > Strike

Semi-easy cases: optimal exercise strategy partially pre-determined

- ***C*** : p**+** if Spot ≥ Strike; no a-priori statement for Spot < Strike
- ***D*** : p**-** if Spot ≤ Strike; no a-priori statement for Spot > Strike

The difficult case: no pre-determination at all

- ***F*** : no a-priori statement available

© ior/cf - HSG

Bidding Strategy – Example Cerron Grande
(single power plant with installed power 175 MW)

Hour \ Price	0.0	91.7	91.9	93.1	93.8	94.1	94.4	95.4	95.5	96.1	96.5	98.0	98.1	98.4	99.0	100.3	101.2	101.7	102.2	105.4	106.7
1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	175.0	175.0	175.0	175.0	175.0	175.0
2	0.0	0.0	0.0	0.0	0.0	175.0	175.0	175.0	175.0	175.0	175.0	175.0	175.0	175.0	175.0	175.0	175.0	175.0	175.0	175.0	175.0
3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	175.0	175.0	175.0	175.0	175.0	175.0	175.0	175.0	175.0	175.0
4	0.0	0.0	175.0	175.0	175.0	175.0	175.0	175.0	175.0	175.0	175.0	175.0	175.0	175.0	175.0	175.0	175.0	175.0	175.0	175.0	175.0
5	0.0	0.0	0.0	0.0	175.0	175.0	175.0	175.0	175.0	175.0	175.0	175.0	175.0	175.0	175.0	175.0	175.0	175.0	175.0	175.0	175.0
6	0.0	0.0	0.0	0.0	0.0	175.0	175.0	175.0	175.0	175.0	175.0	175.0	175.0	175.0	175.0	175.0	175.0	175.0	175.0	175.0	175.0
7	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	175.0	175.0	175.0	175.0	175.0	175.0	175.0	175.0	175.0
8	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	175.0	175.0	175.0	175.0	175.0
9	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	175.0	175.0	175.0	175.0	175.0	175.0	175.0	175.0
10	0.0	0.0	0.0	0.0	0.0	0.0	0.0	175.0	175.0	175.0	175.0	175.0	175.0	175.0	175.0	175.0	175.0	175.0	175.0	175.0	175.0
11	0.0	0.0	175.0	175.0	175.0	175.0	175.0	175.0	175.0	175.0	175.0	175.0	175.0	175.0	175.0	175.0	175.0	175.0	175.0	175.0	175.0
12	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	175.0	175.0	175.0	175.0	175.0	175.0	175.0	175.0	175.0	175.0	175.0
13	0.0	0.0	0.0	0.0	175.0	175.0	175.0	175.0	175.0	175.0	175.0	175.0	175.0	175.0	175.0	175.0	175.0	175.0	175.0	175.0	175.0
14	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	175.0	175.0	175.0	175.0	175.0	175.0	175.0	175.0	175.0	175.0	175.0	175.0	175.0
15	0.0	0.0	0.0	0.0	0.0	175.0	175.0	175.0	175.0	175.0	175.0	175.0	175.0	175.0	175.0	175.0	175.0	175.0	175.0	175.0	175.0
16	0.0	175.0	175.0	175.0	175.0	175.0	175.0	175.0	175.0	175.0	175.0	175.0	175.0	175.0	175.0	175.0	175.0	175.0	175.0	175.0	175.0
17	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	175.0
18	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	175.0	175.0	175.0	175.0
19	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	175.0	175.0	175.0
20	0.0	0.0	0.0	0.0	0.0	0.0	175.0	175.0	175.0	175.0	175.0	175.0	175.0	175.0	175.0	175.0	175.0	175.0	175.0	175.0	175.0
21	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	175.0	175.0	175.0	175.0	175.0	175.0	175.0	175.0	175.0	175.0	175.0	175.0
22	0.0	0.0	0.0	175.0	175.0	175.0	175.0	175.0	175.0	175.0	175.0	175.0	175.0	175.0	175.0	175.0	175.0	175.0	175.0	175.0	175.0
23	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	175.0	175.0	175.0	175.0	175.0	175.0	175.0
24	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	175.0	175.0
total	**0.0**	**175.0**	**525.0**	**700.0**	**1'050.0**	**1'575.0**	**1'750.0**	**1'925.0**	**2'100.0**	**2'275.0**	**2'450.0**	**2'625.0**	**2'800.0**	**2'975.0**	**3'150.0**	**3'325.0**	**3'500.0**	**3'675.0**	**3'850.0**	**4'025.0**	**4'200.0**

© ior/cf - HSG

Summary

Swing Options

- Swing options and their complexity may be viewed in between the classical plain vanilla options of Finance and a park of hydro power plants of the Power Industry.

- The complexity of a Swing Option is best illustrated by the Time-Energy Diagram.

Storage Power Plant

- The contract values as well as the water values are well explained by standard month and quarter futures traded on EEX. Hence, power producers are able to hedge their risk based on a hedging portfolio consisting of standard future products (monthly rebalancing proposed).

- Particularly for hydro power system, considering the uncertainty is of high relevance, since the power units are flexible enough to respond to changes in price and inflow.

© ior/cf - HSG

BIT@EPI.VPP

Implementation is based on:

Gido Haarbrücker, Daniel Kuhn: *Valuation of Electricity Swing Options by Multistage Stochastic Programming.* Working Paper, Institute of Operations Research and Computational Finance, University of St.Gallen, 2006 (submitted).

Daniel Kuhn: *Convergent Bounds for Stochastic Programs with Expected Value Constraints*. The Stochastic Programming E-Print Series (SPEPS), 2006.

Karl Frauendorfer, Daniel Kuhn, Michael Schürle: *Bounding Methods in Stochastic Optimization – Theory and Application*. Working Paper, Institute for Operations Research and Computational Finance, University of St.Gallen, 2005 (submitted).

Daniel Kuhn: *Aggregation and discretization in multistage stochastic programming.* The Stochastic Programming E-Print Series (SPEPS), 2005.

Daniel Kuhn: *Generalized Bounds for Convex Multistage Stochastic Programs.* Vol. 548 of Lecture Notes in Economics and Mathematical Systems. Springer-Verlag, Berlin, 2004.

Haarbrücker, Gido: *Sequentielle Optimierung verfeinerter Approximationen in der mehrstufigen stochastischen Programmierung.* Doctoral Thesis, University of St.Gallen, 2000.

© ior/cf - HSG