



Insurance Demand and the Mitigation of Default Risk

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Mossin's Demand Model as Starting Point

Mossin, J. (1968). Aspects of Rational Insurance Purchasing. Journal of Political Economy, 76, 533-568 → *Full coverage is optimal iff premium is actuarially fair*

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Mossin (1968) (default-free setting)

$$W := w - c_I(\alpha_I) + (-1 + \alpha_I)L,$$

$$L = \begin{cases} l, & \text{if loss occurs, prob } p \\ 0, & \text{else, prob } 1 - p \end{cases}$$

Modified Demand Model: The Case of Non-Performance

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↳ **Doherty N., Schlesinger H. (1990).** Rational Insurance Purchasing: Consideration of Contract Nonperformance. Quarterly Journal of Economics, 105, 243-253 → *Given default risk, over- or under-insurance may be optimal even though the premium is actuarially fair*

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Doherty & Schlesinger (1990)

$$W := w - c_I(\alpha_I) + \{-1 + \alpha_I(\mathbf{1} - \chi_D(\mathbf{1} - r))\}L, \quad \chi_D = \begin{cases} 1, & \text{if insurer fails, prob } q \\ 0, & \text{else, prob } 1 - q \end{cases}$$

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↳ **Mahul O., Wright B. D. (2007).** Optimal Coverage for Incompletely Reliable Insurance, Economic Letters 95, 456-461 → *Optimality of under- or over-insurance depends on size of recovery rate r*

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↳ Optimal insurance demand under the risk of contract nonperformance, if default risk can be diversified?
→ *multiple co-insurance as diversification measure*

Our Model Framework

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Our model framework

n co-insurers: each co-insurer holds $\frac{1}{n}$ in premium and losses

$$W_{F,n} := w - c_I(\alpha_I, \mathbf{n}) + \left\{ -1 + \alpha_I \left(1 - \frac{\mathbf{F}}{\mathbf{n}} (1 - r) \right) \right\} L,$$

- $F \in \{0, \dots, n\}$ is the random number of failed insurers
- $F \sim BB \left(q \frac{1-\theta}{\theta}, (1-q) \frac{1-\theta}{\theta} \right)$, θ is the joint default correlation factor
- As $\theta \rightarrow 0$, F approaches a binomial distribution (independent defaults)

α_I is our decision variable:
coverage rate of co-insurance policy

Remarks on The Model: Probability Weights, Utility and Premium Principle

Policyholder's utility (is maximized with respect to α_I)

$$U = (1 - p)u(W_{no\ loss}) + p \sum_{k=0}^n d_{k,n} u(W_{k,n})$$

Probability of k failing co-insurers (given a policy with n co-insurers)

$$d_{k,n} = \mathbb{P}[F = k] = \int_0^1 \binom{n}{k} x^k (1 - x)^{n-k} \psi(x; \alpha, \beta) dx$$

($\psi(x; \alpha, \beta)$ is the density of a Beta distribution with parameters $\alpha = q \frac{1-\theta}{\theta}$ and $\beta = (1 - q) \frac{1-\theta}{\theta}$)

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Assumed premium principle: *Expected Payoff* x *Proportional Cost Loading*

$$\begin{aligned} c_I(\alpha_I, n) &= E[\text{Default Adjusted Indemnification}](1 + \lambda_I) \\ &= \alpha_I L p \{1 - q(1 - r)\} (1 + \lambda_I) = c_I(\alpha_I) \end{aligned}$$

Independent
of n and θ !

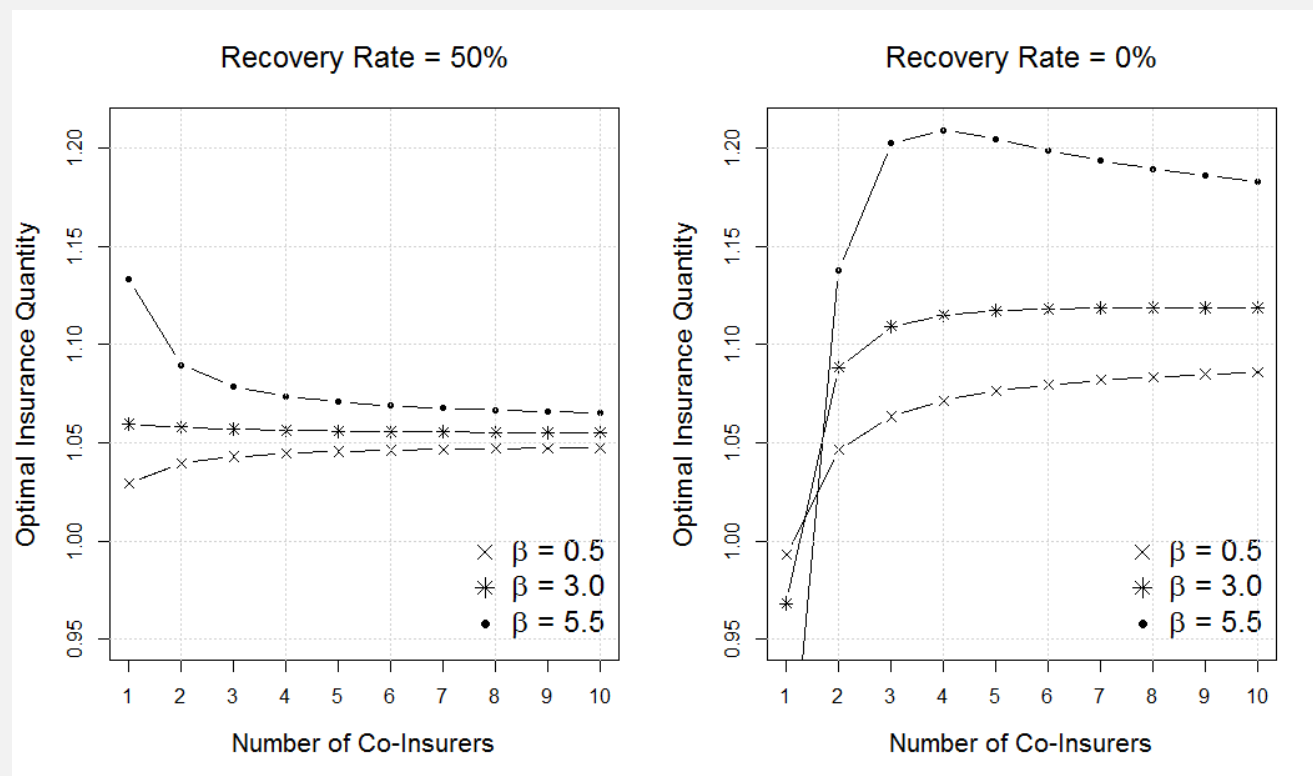
Each insurer receives $\frac{1}{n} c_I(\alpha_I) \rightarrow 0$, as n becomes large: fixed running costs?

Effect of Diversification on Optimal Demand

- Assumed the insurer can increase the number of co-insurers from n to $n+1 \rightarrow$
Natural question: Is it optimal to **increase** or to **decrease** insurance **coverage**?
- First intuition: Given two policies, it seems to be nearby that it is optimal to take up more of the policy that provides higher utility.

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Numeric Example:

$$u(x) = -\exp(-\beta x)$$

Initial wealth w	1.5
Loss prob. p	5.0 %
Loss size l	1.0
Default prob. q	1.0 %
Correlation θ	15 %
Cost loading λ_I	0.0

Monotonicity Criterion

Let $\alpha_{I,n}^*$ be the optimal insurance demand for n co-insurers and set

$$w_n^*(x) = w - l - c(\alpha_{I,n}^*) + \alpha_{I,n}^* l - \alpha_{I,n}^* (1 - r) l x.$$

Then, $\alpha_{I,n+1}^* \geq (\leq) \alpha_{I,n}^*$ holds true, if

$$\left(1 - \frac{w-l}{w_n^*(x)}\right) \eta(w_n^*(x)) \leq (\geq) 2, \text{ for all } x \in [0, 1],$$

where $\eta(x) := -xu'''(x)/u''(x)$ is the policyholder's measure of relative prudence.

Possible cases (heuristically)

(1) Low degree of prudence $\rightarrow$ " $\leq$ " is fulfilled $\rightarrow$ optimal coverage **non-decreasing**

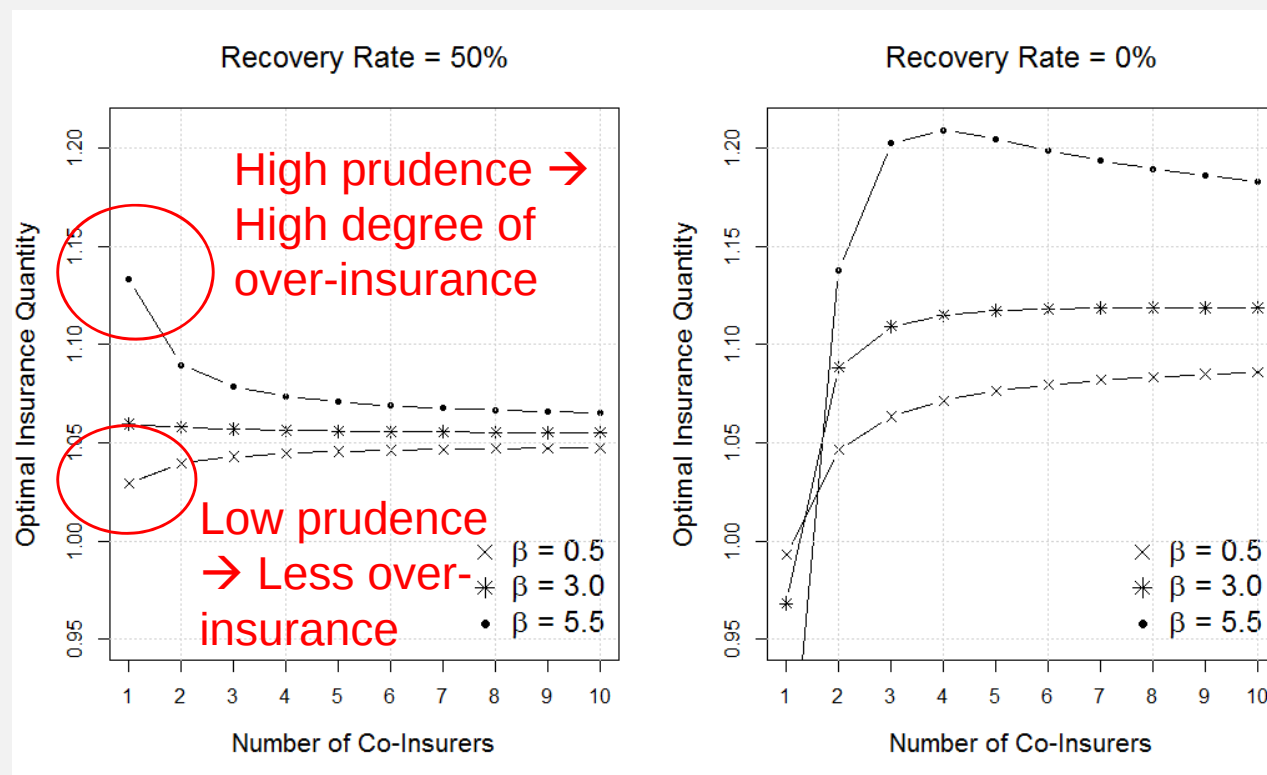
(2) High degree of prudence

& high recovery rate $\rightarrow$ " $\geq$ " is fulfilled $\rightarrow$ optimal coverage **non-increasing**

& low recovery rate $\rightarrow$ neither " $\geq$ " nor " $\leq$ " is fulfilled $\rightarrow$ ambiguous monoton.

Monotonicity Criterion: Influence of Prudence

- Assumed the insurer can increase the number of co-insurers from n to $n+1 \rightarrow$ Natural question: Is it optimal to **increase** or to **decrease** insurance **coverage**?
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(1) Analogous result for default correlation: Let $\theta_1 \leq \theta_2$

$$\left(1 - \frac{w-l}{w_n^*(x)}\right) \eta(w_n^*(x)) \leq (\geq) 2, \text{ for all } x \in [0, 1], \Rightarrow \alpha_{I,n}^*(\theta_1) \geq (\leq) \alpha_{I,n}^*(\theta_2)$$

A rising default correlation thus (rather) results in a

- decreased coverage, when policyholder is low-prudent
 - increased coverage, when policyholder is high-prudent and recovery rate is high
-

(2) Implication for the single-insurer policy

$$\left(1 - \frac{w-l}{w_1^*(x)}\right) \eta(w_1^*(x)) \leq (\geq) 2, \text{ for all } x \in [0, 1], \Rightarrow \alpha_{I,1}^* \leq (\geq) \frac{\text{Optimal demand in a default-free setting}}{1-q(1-r)}$$

- Low prudence $\rightarrow$ (Rather) less coverage than “benchmark”
- High prudence and high recovery rate $\rightarrow$ (Rather) more coverage than “benchmark”

Thank You

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