Rational Inattention via Ignorance Equivalence

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joint with
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The views expressed here do not reflect those of the Federal Reserve Board.

Main Contribution

- We propose the novel concept of the **ignorance equivalent** as a parsimonious summary of a decision problem with flexible learning.
  - Ability to learn is *'equivalent'* to access to a larger menu
  - Learning analogue of the certainty equivalent for lotteries

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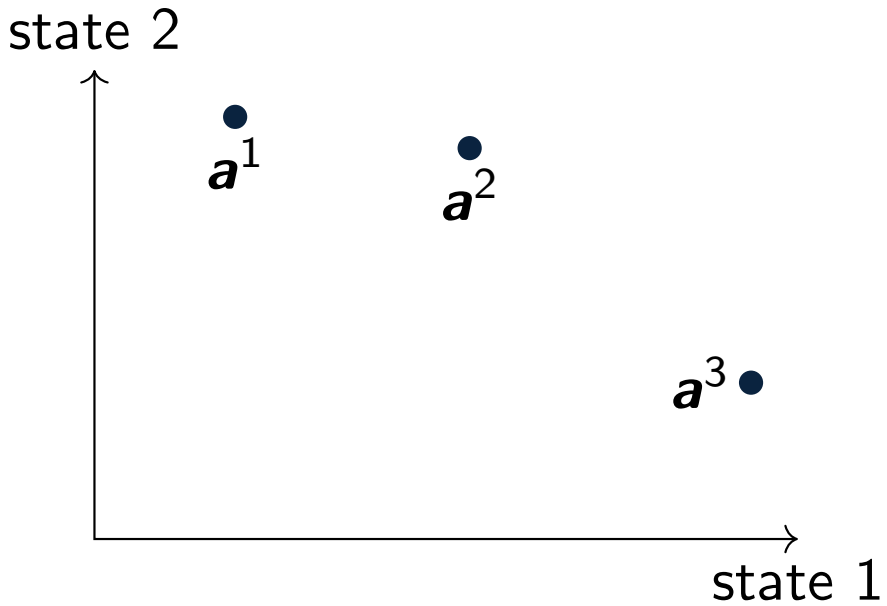
  - Ability to learn is *'equivalent'* to access to a larger menu
  - Learning analogue of the certainty equivalent for lotteries
- The equivalence between learning ability and fictional payoffs simplifies analysis, including in multi-player settings.

Model Setup

- An agent is choosing an action from a finite menu $\mathcal{A}$.
- Action $\boldsymbol{a} \in \mathcal{A} \subseteq \mathbb{R}^I$ yields utility a_i in state $i \in \mathcal{I} = 1, ..., I$.
- The agent has a prior belief $\boldsymbol{\pi} \in \Delta\mathcal{I}$ over states.

- **Payoffs, states and actions**



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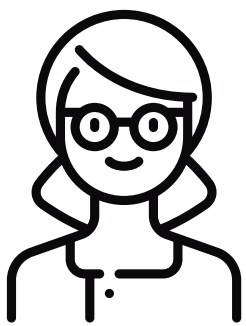
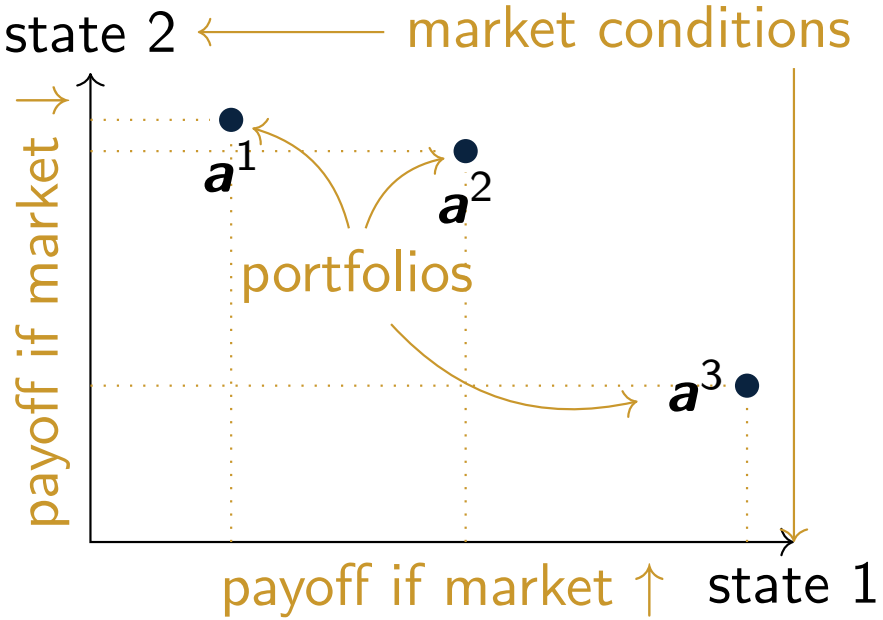


Investor Example



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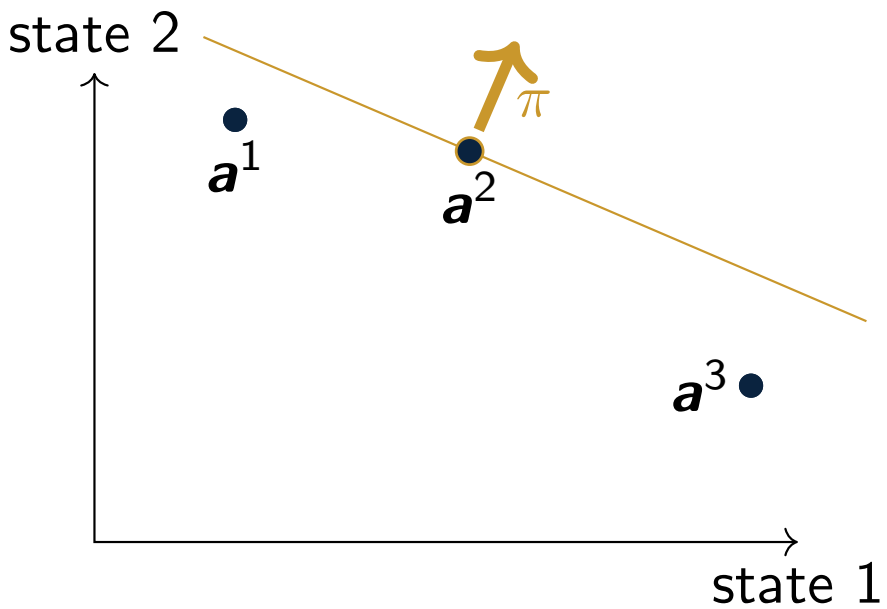
- **No information**
  - Agent maximizes expected utility under prior belief $\boldsymbol{\pi}$.



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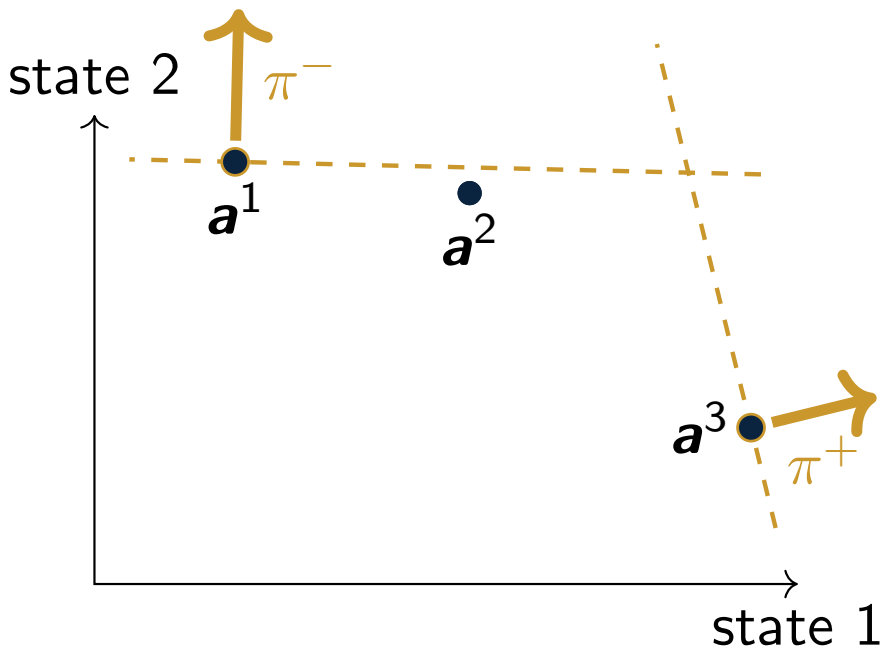
- **Incomplete information**
  - Agent observes signal draw s,
  - updates belief to $\boldsymbol{\pi}^s$,
  - maximizes expected utility under this belief.



Rational Inattention

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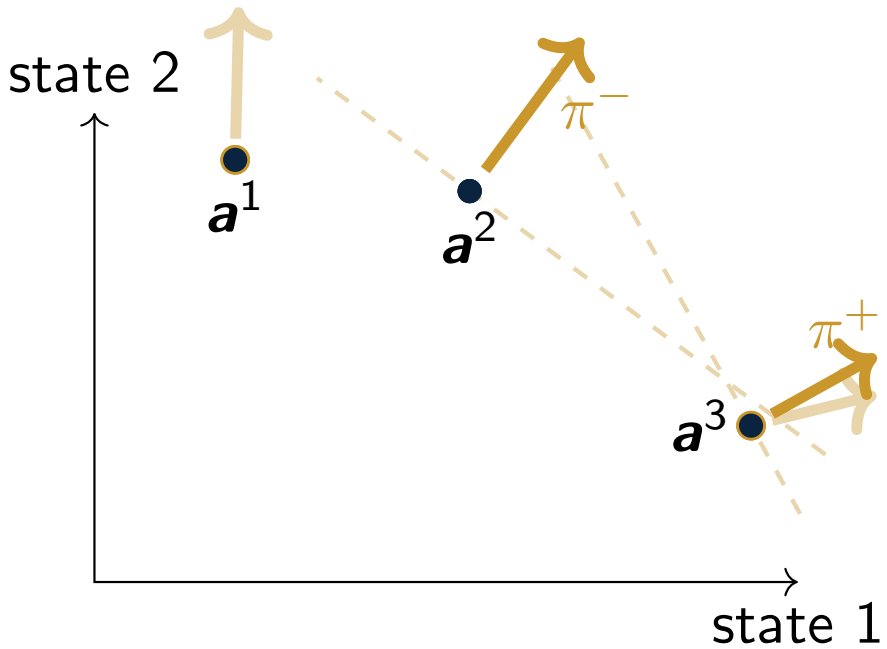
- **Rational Inattention**
  - Agent chooses *which* costly signal to draw,
  - ... and proceeds as before.



Welfare: $W(\mathcal{A}, \boldsymbol{\pi}, c) =$ E[consumption utility] - [cost of signal].

Learning Strategies

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  where $q_i \in \Delta S$ denotes the probability of each realization in state i.
  - Obedience principle: Signal recommends actions, $S = \mathcal{A}$, agent obeys.
    We refer to these as **learning strategies**.

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- Admissible costs include all differentiable & prior-concave UPS costs.
  Prominent examples:
  - Mutual Information *(Sims JME'03)*,
  - some Tsallis costs *(Caplin-Dean-Leahy '19)*
  - Total Information *(Bloedel-Zhong '20)*, which subsumes
    - Wald costs *(Morris-Strack '19)*, Fisher Information *(Hébert-Woodford '20)*.

UPS : uniformly posterior separable

Ignorance Equivalent

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- the agent is willing to commit to *never* implement α,

$$W(\mathcal{A}, \pi, c) \geq W(\mathcal{A} \cup \{\alpha\}, \pi, c).$$

Agent is willing to forgo learning opportunities that arise when α is added to $\mathcal{A}$.

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Since larger menus are always better,

$$\{\alpha\} \sim \mathcal{A} \sim \mathcal{A} \cup \{\alpha\}$$

in terms of the agent's preference over menus given cost c and prior π.

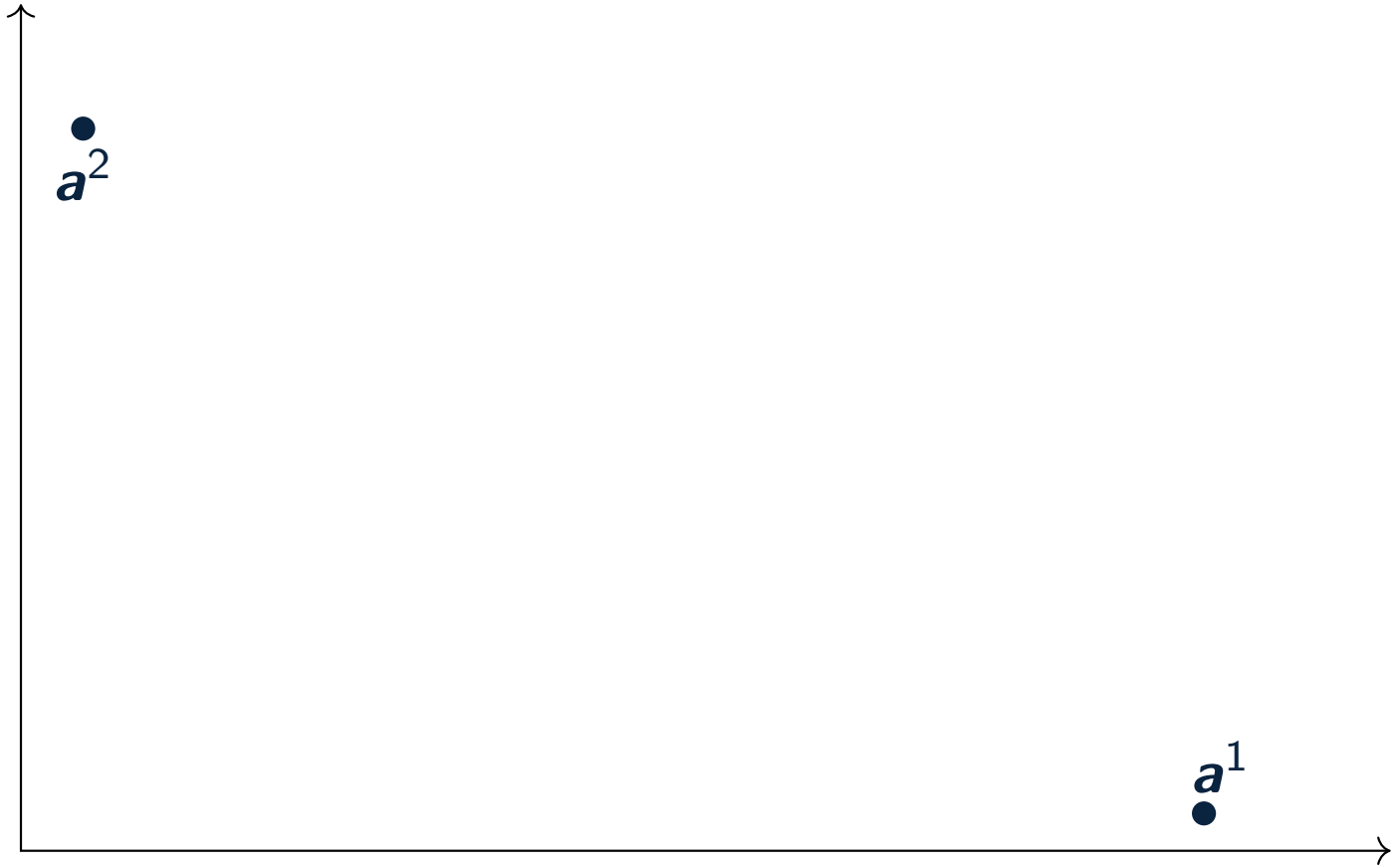
Existence and Uniqueness

Theorem 1

Each RI problem $(\mathcal{A}, \boldsymbol{\pi}, c)$ admits a unique ignorance equivalent $\boldsymbol{\alpha}$.
The ignorance equivalent can be constructed from any optimal signal $\mathcal{S}$.

Ignorance equivalence: $W(\{\boldsymbol{\alpha}\}, \boldsymbol{\pi}, c) = W(\mathcal{A}, \boldsymbol{\pi}, c) = W(\mathcal{A} \cup \{\boldsymbol{\alpha}\}, \boldsymbol{\pi}, c)$.

Locating the Ignorance Equivalent



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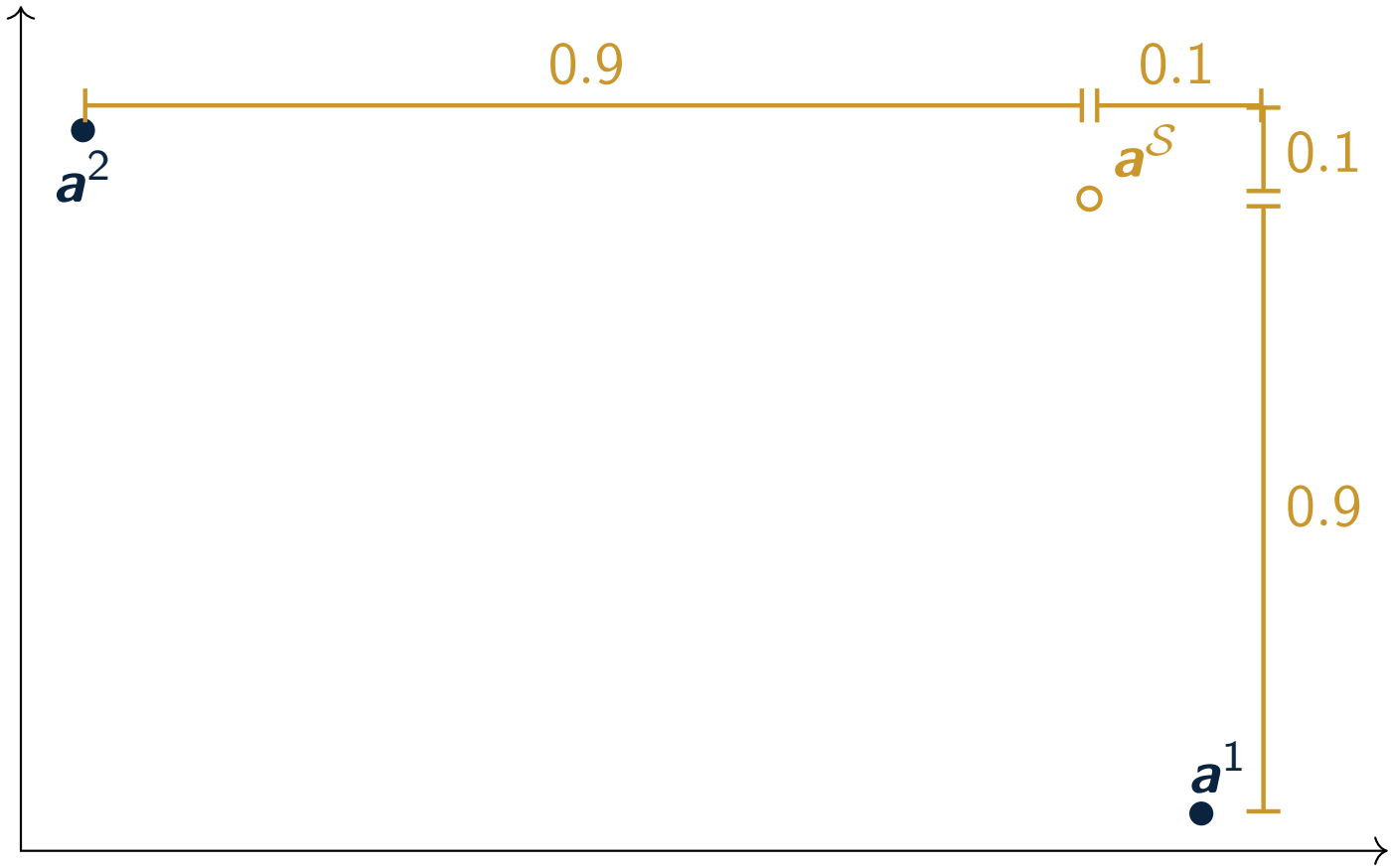

- Strategy $\mathcal{S}$ always implements the high-payoff action with $q_i(\boldsymbol{a}^i) = 0.9$.
- In each state i, the expected consumption payoff is $a_i^{\mathcal{S}} = \sum_{\boldsymbol{a} \in \mathcal{A}} q_i(\boldsymbol{a}) a_i$.

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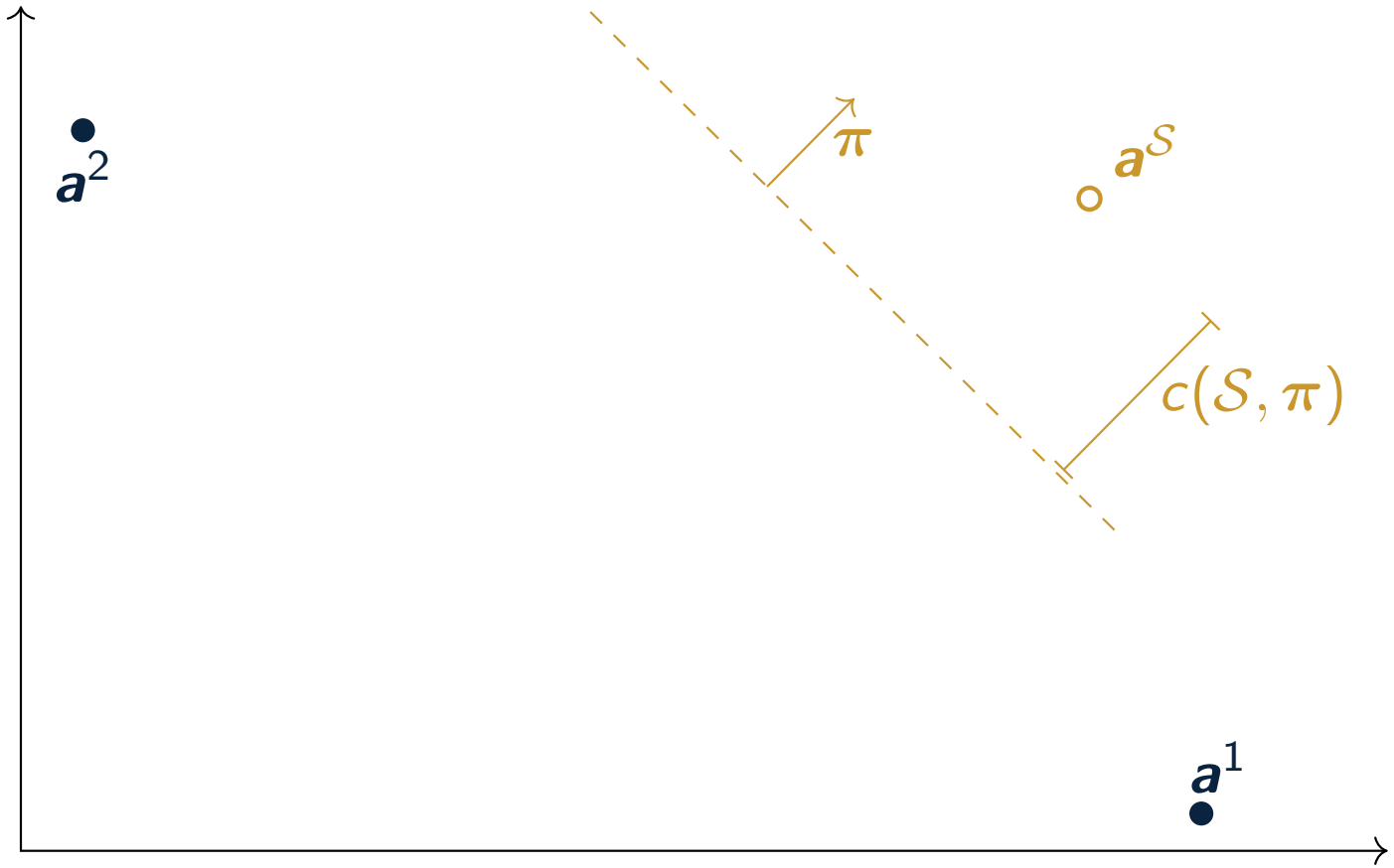

- Following $\mathcal{S}$ yields net utility $\pi \cdot a^{\mathcal{S}} - c(\mathcal{S}, \pi)$ under prior π.

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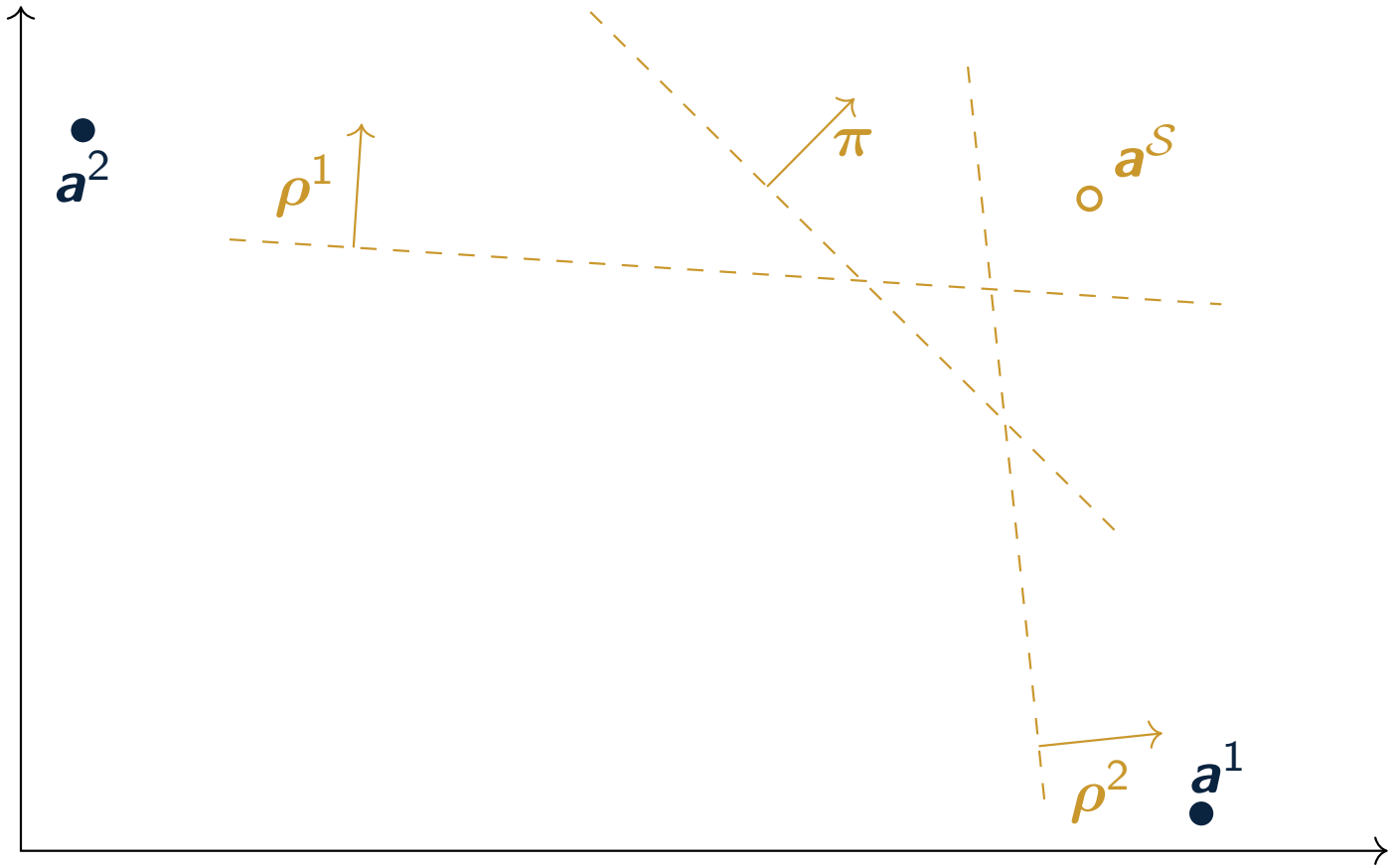

- The same construction shows net utility from $\mathcal{S}$ under any belief ρ.

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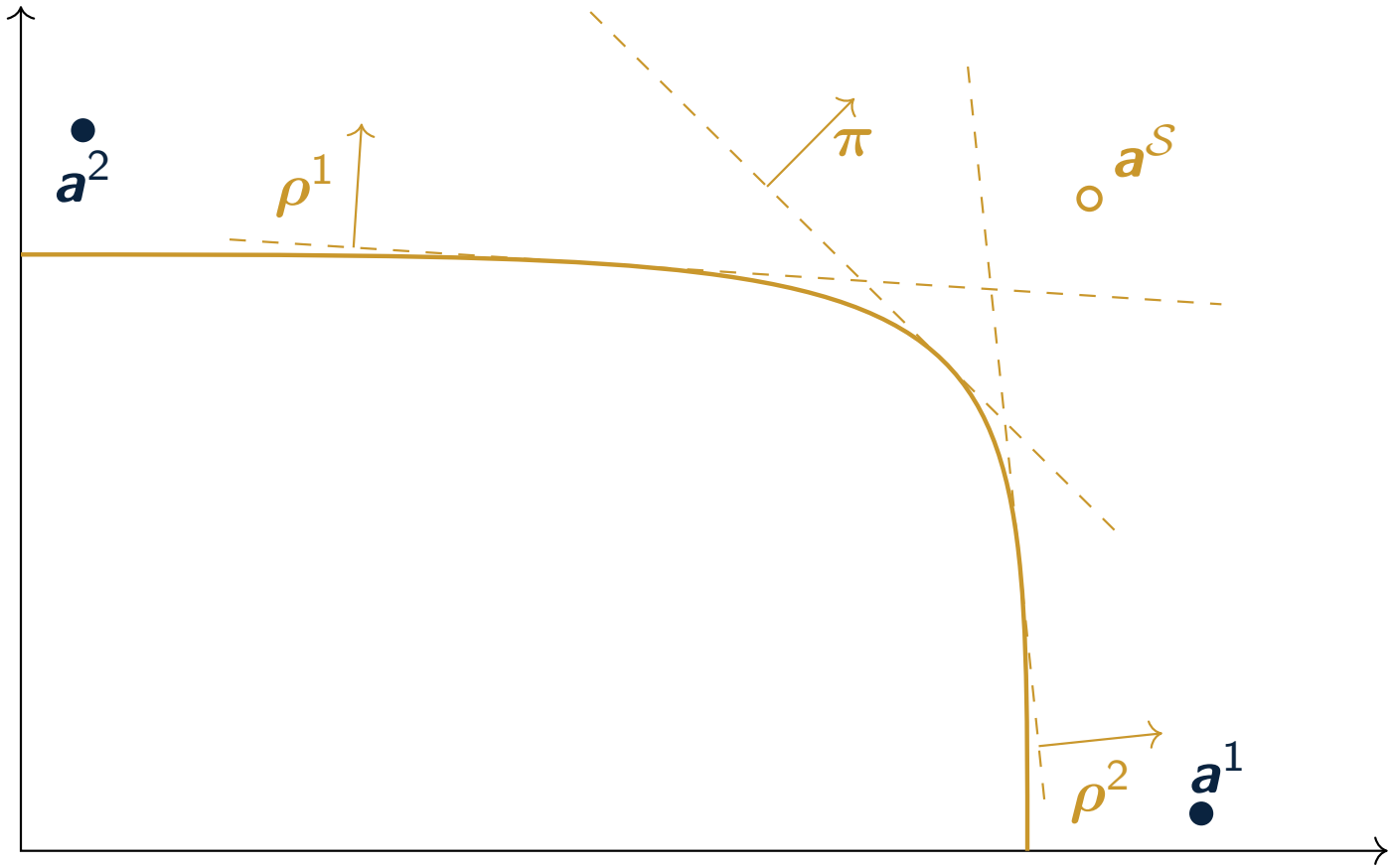

- Dominance: $\boldsymbol{x} \precsim \mathcal{S} \iff \boldsymbol{\rho} \cdot \boldsymbol{x} \leq \boldsymbol{\rho} \cdot \boldsymbol{a}^{\mathcal{S}} - c(\mathcal{S}, \boldsymbol{\rho}) \ \forall \rho.$

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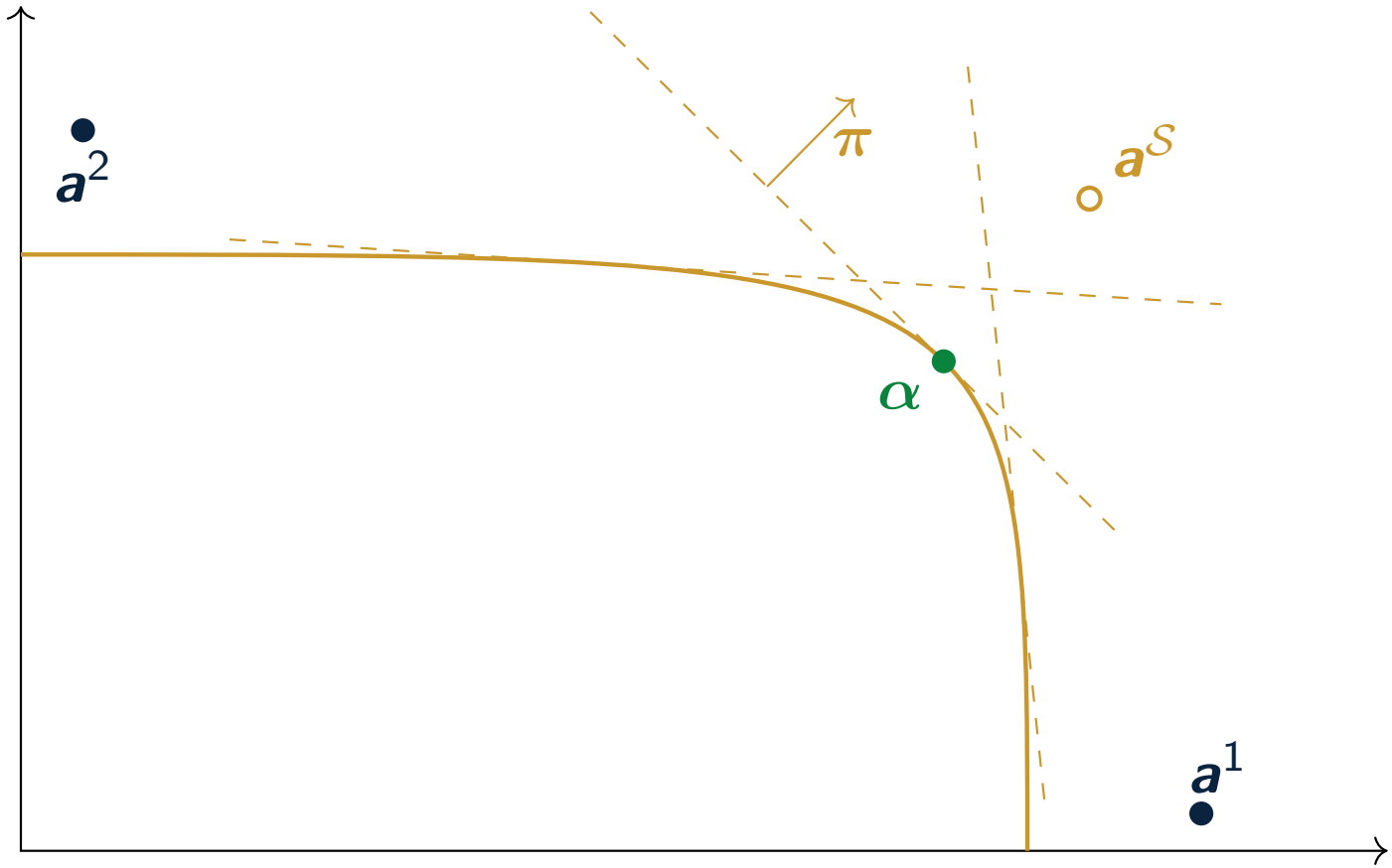
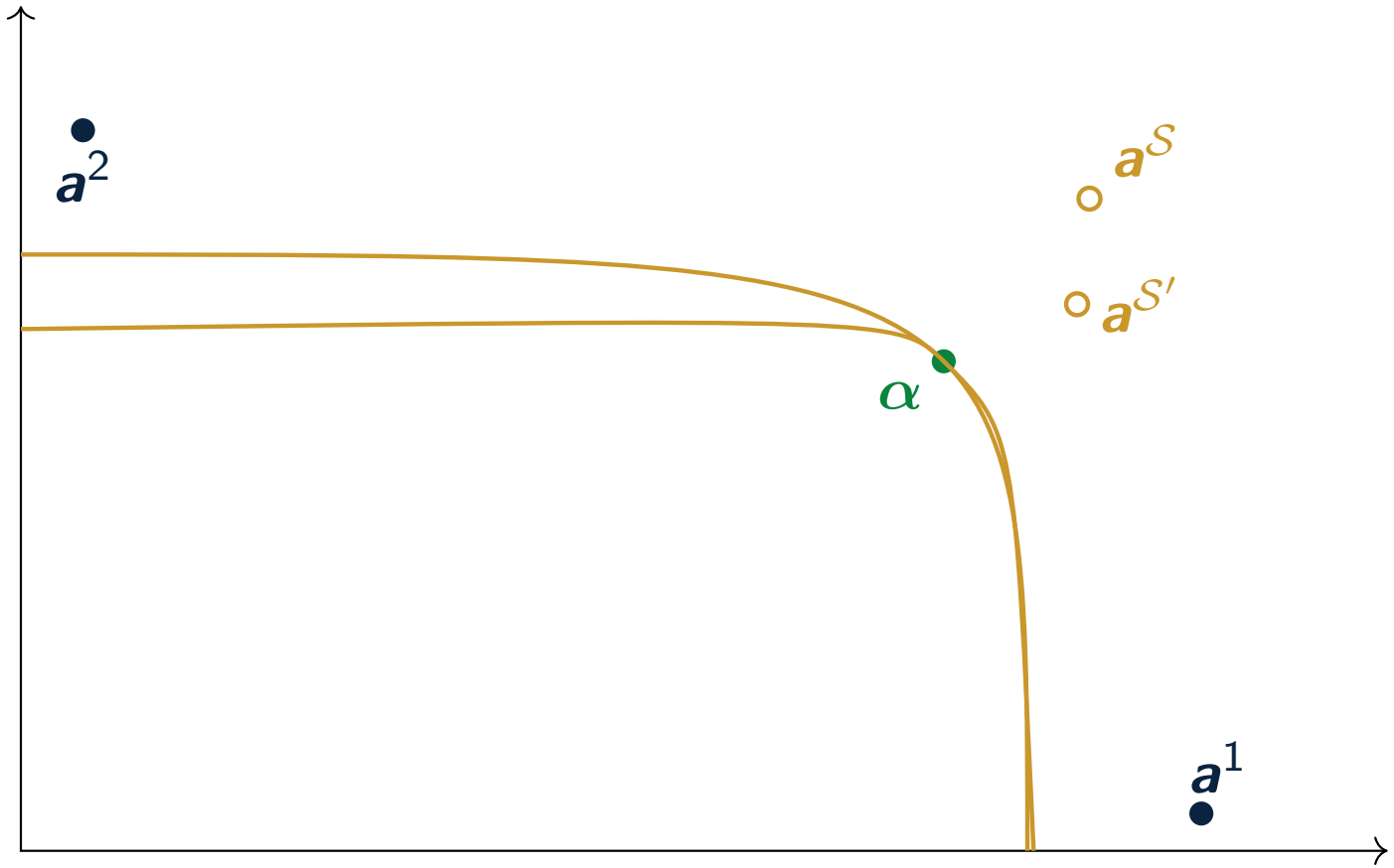


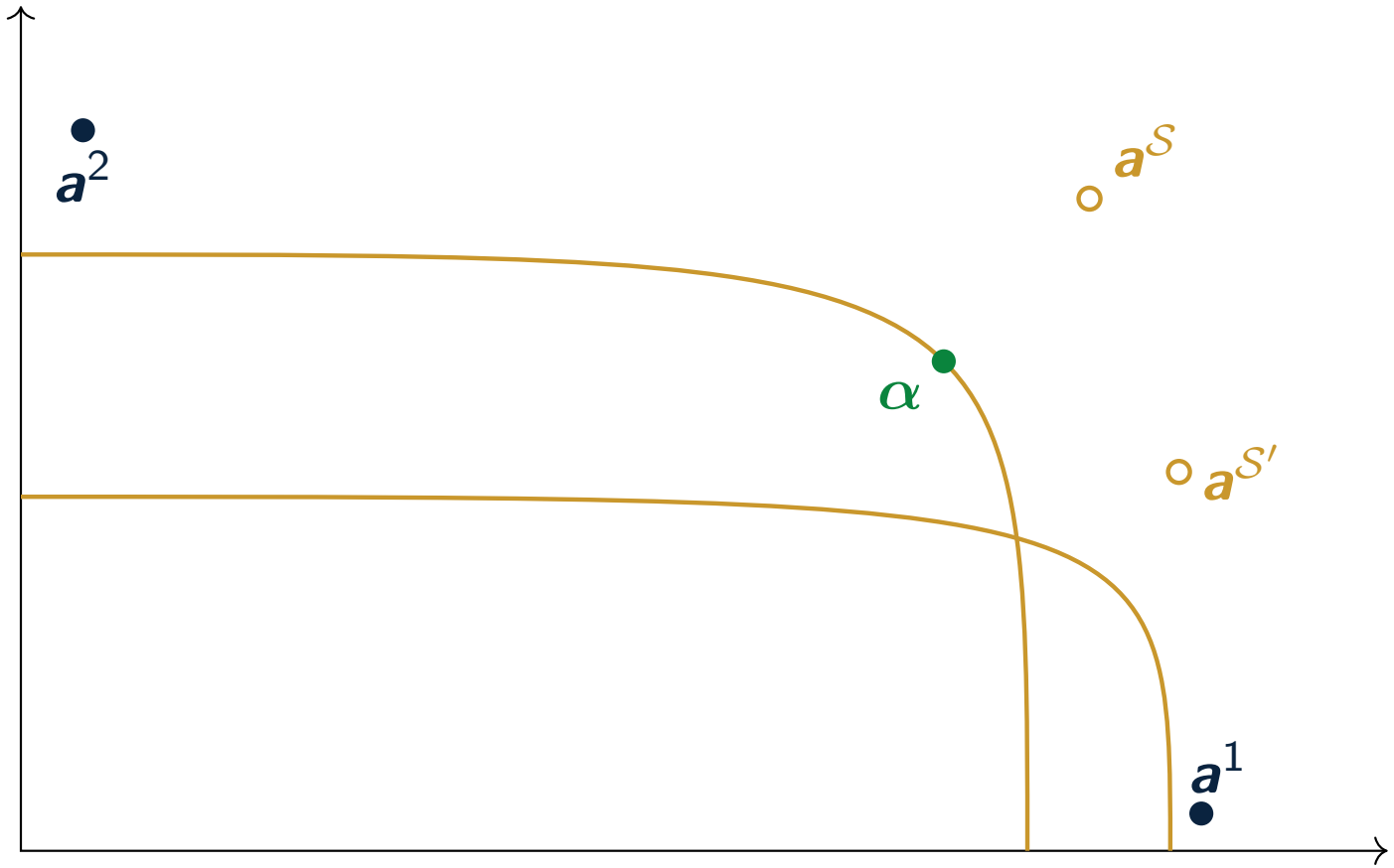
- Unconditional implementation of the ignorance equivalent α
  - is just as good as the optimal $\mathcal{S}$ under the prior.
  - is no better than $\mathcal{S}$ under *any* belief.

Identifying optimal signals



- Optimal signals are exactly those that beat α under any belief.

A suboptimal signal



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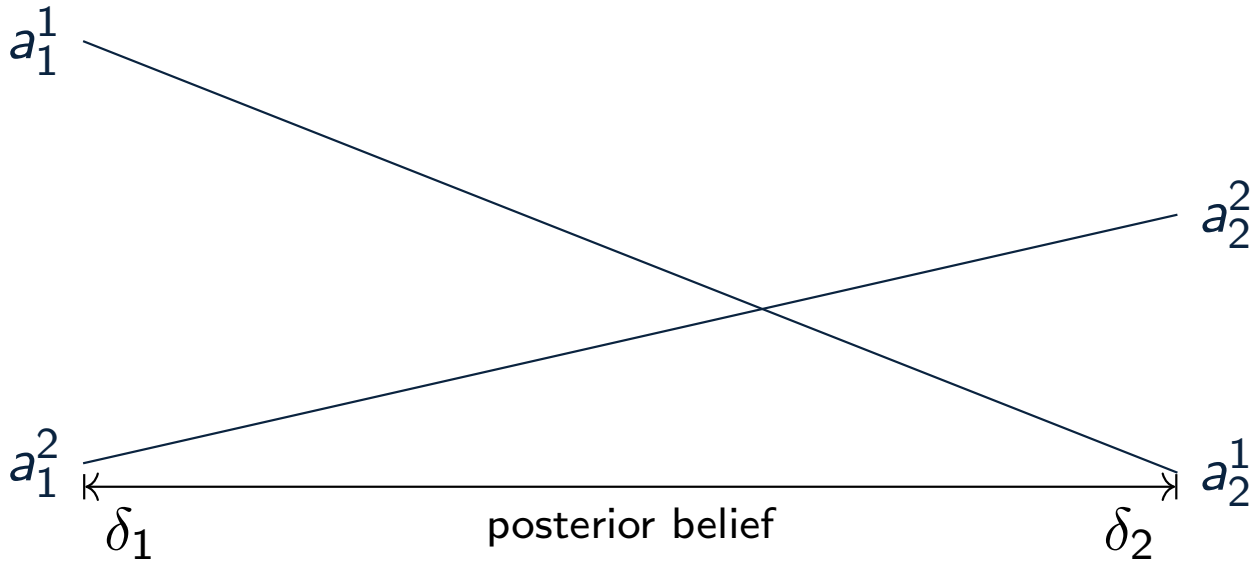
Posterior-based construction (Kamenica&Gentzkow'11, Caplin&Dean'13)



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- For each action, draw a line to represent expected utility.

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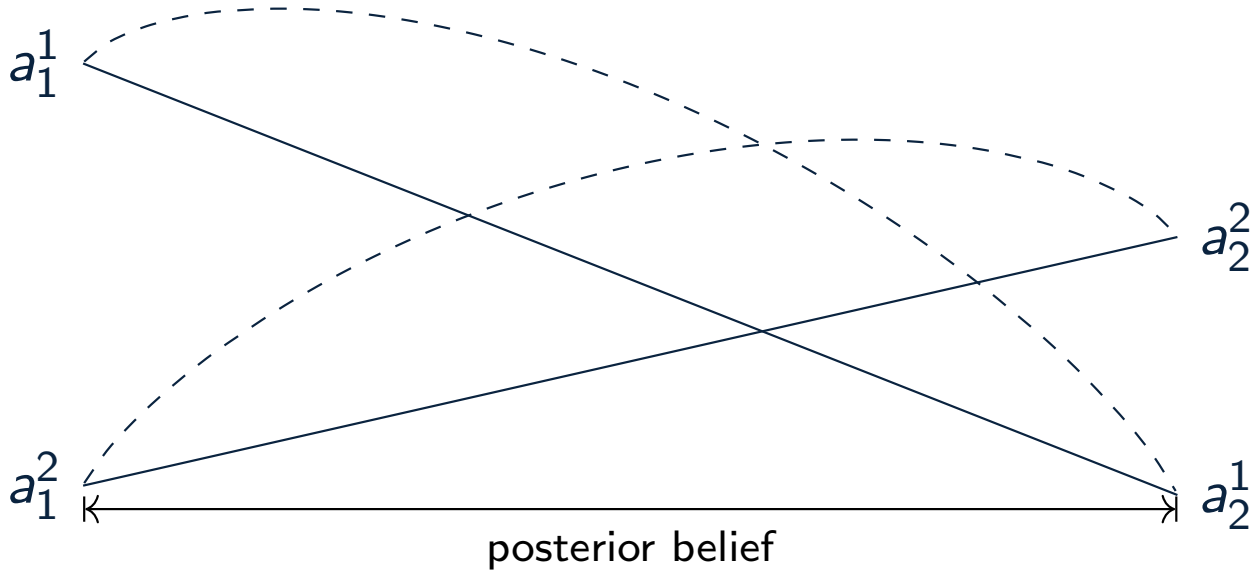

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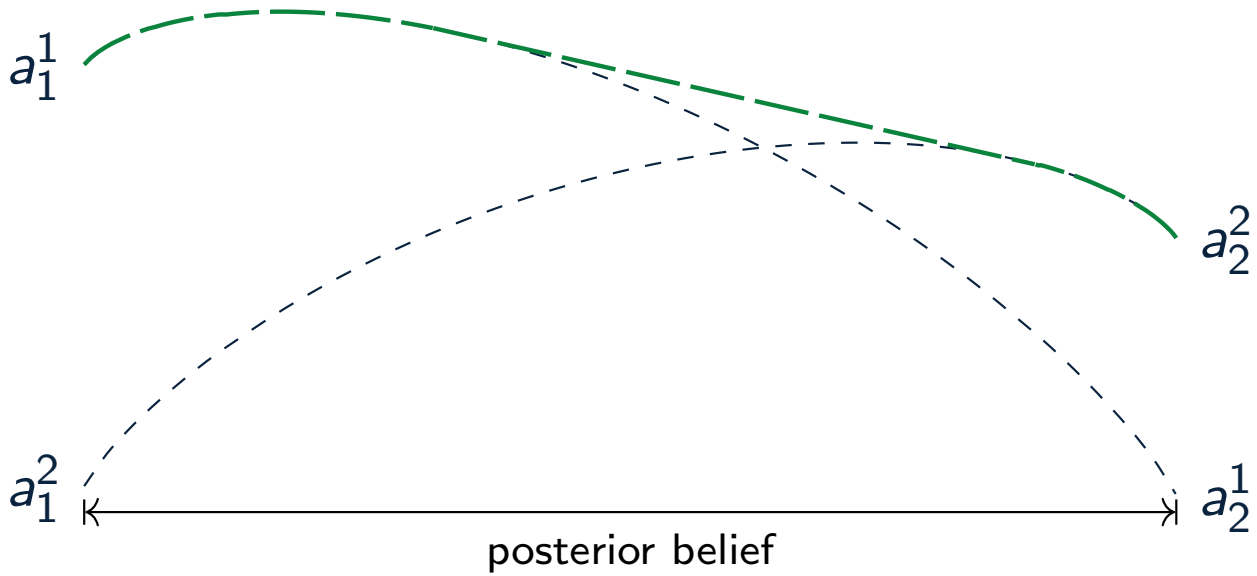

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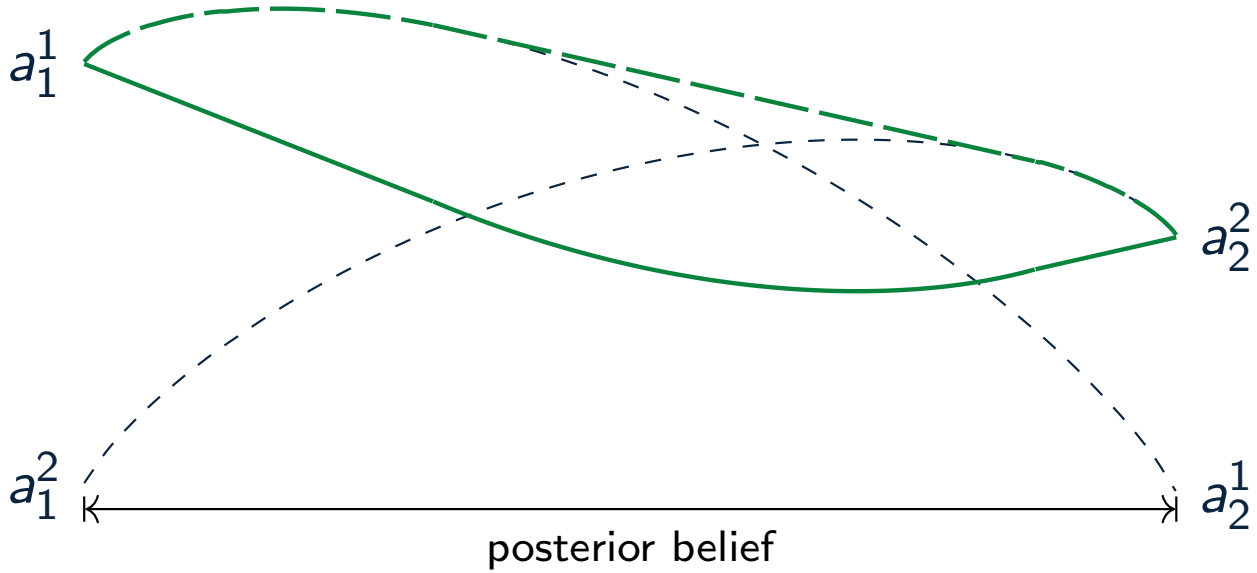

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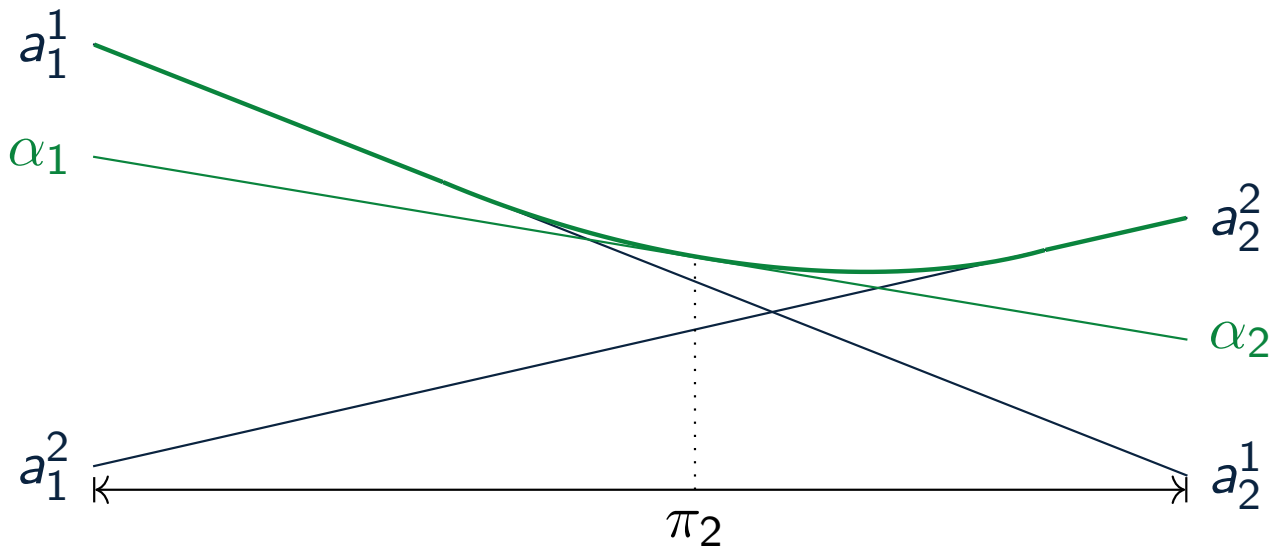

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- Supporting hyperplane at prior $\boldsymbol{\pi} \Rightarrow$ ignorance equivalent $\boldsymbol{\alpha}$.

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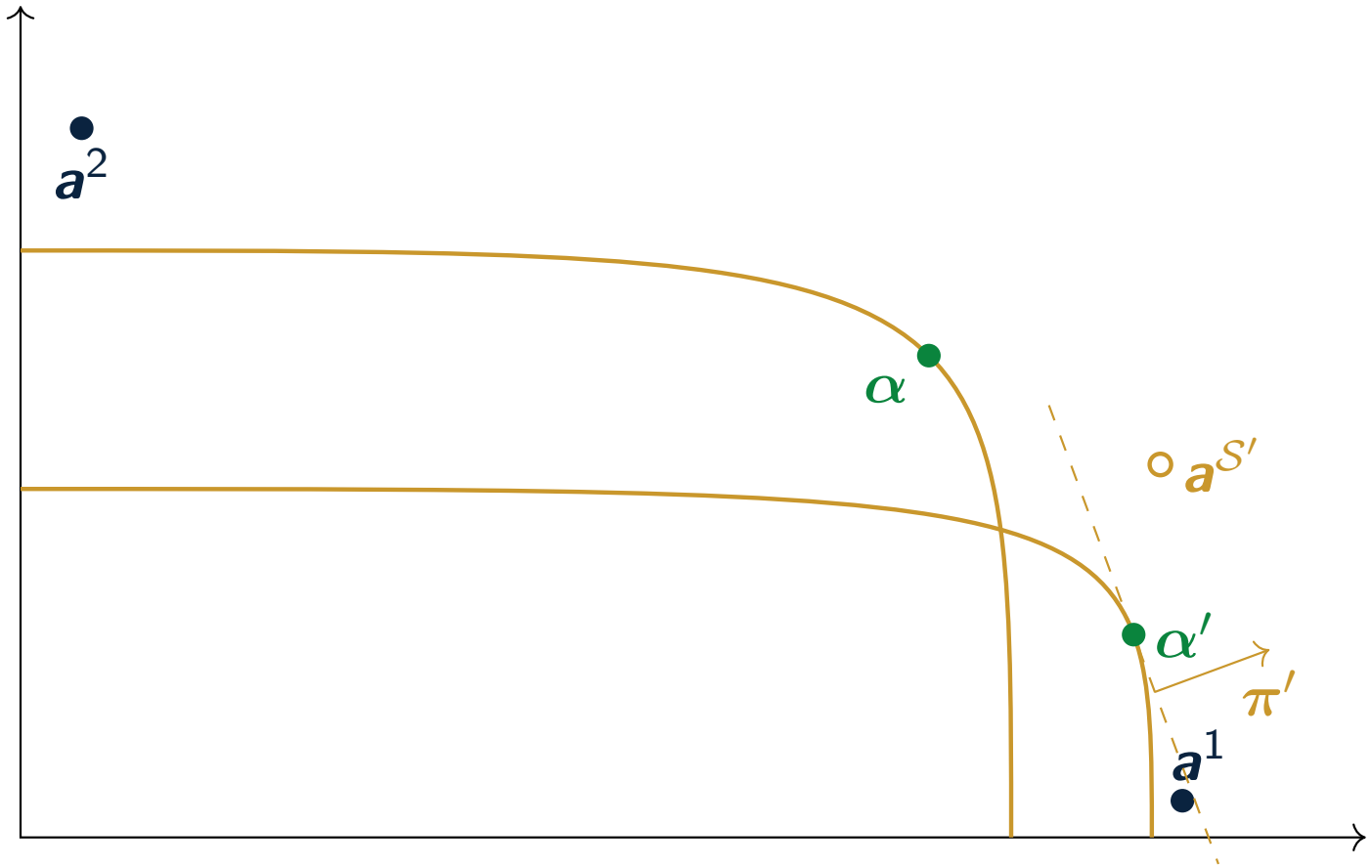
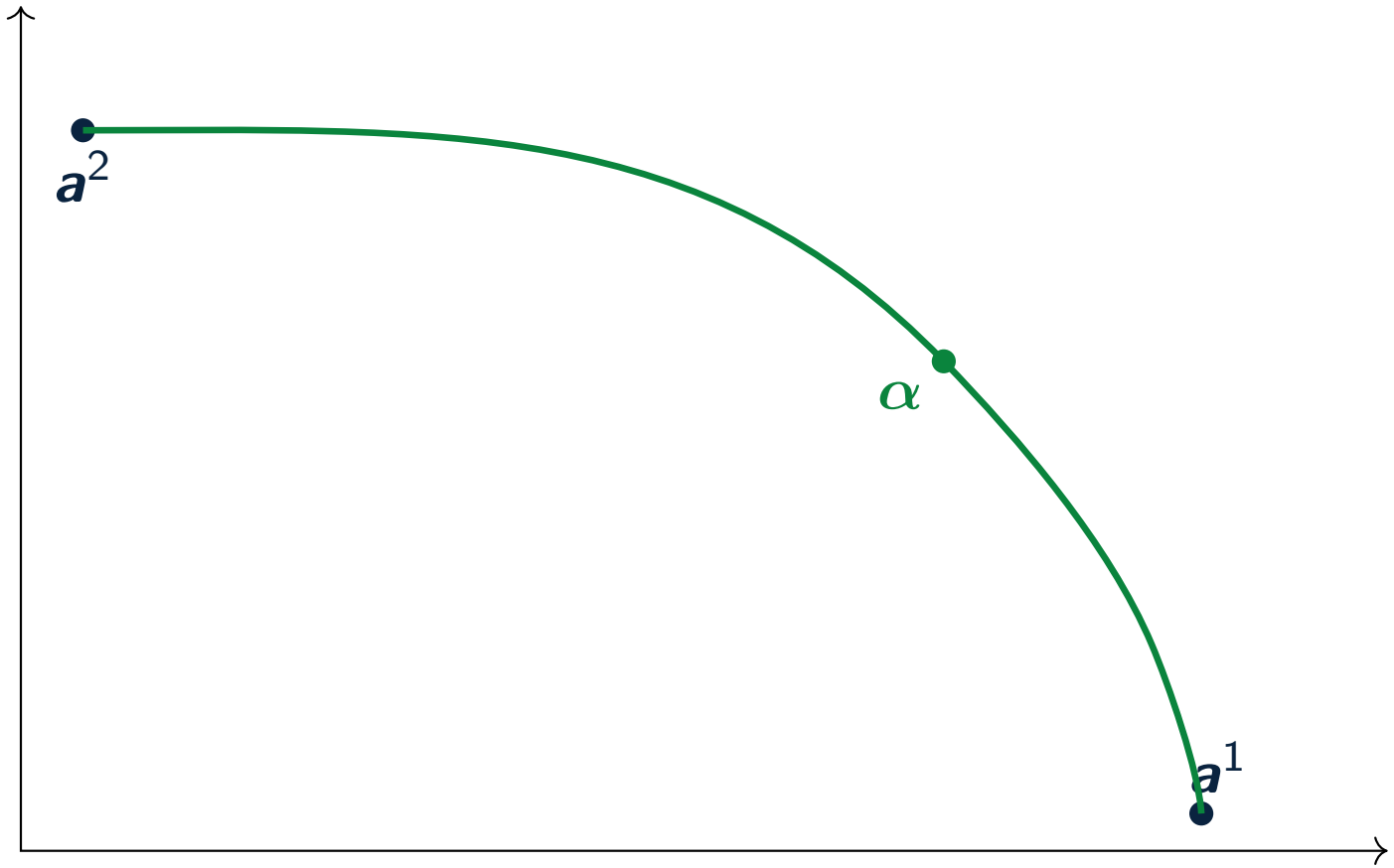
Self-selection Property (Corollary 3)



- Stopping at α' is no better than continuing with $\mathcal{S}'$.

Learning-Proof Menu



- Together, ignorance equivalents form the **Learning-Proof menu** $\bar{\mathcal{A}}$.

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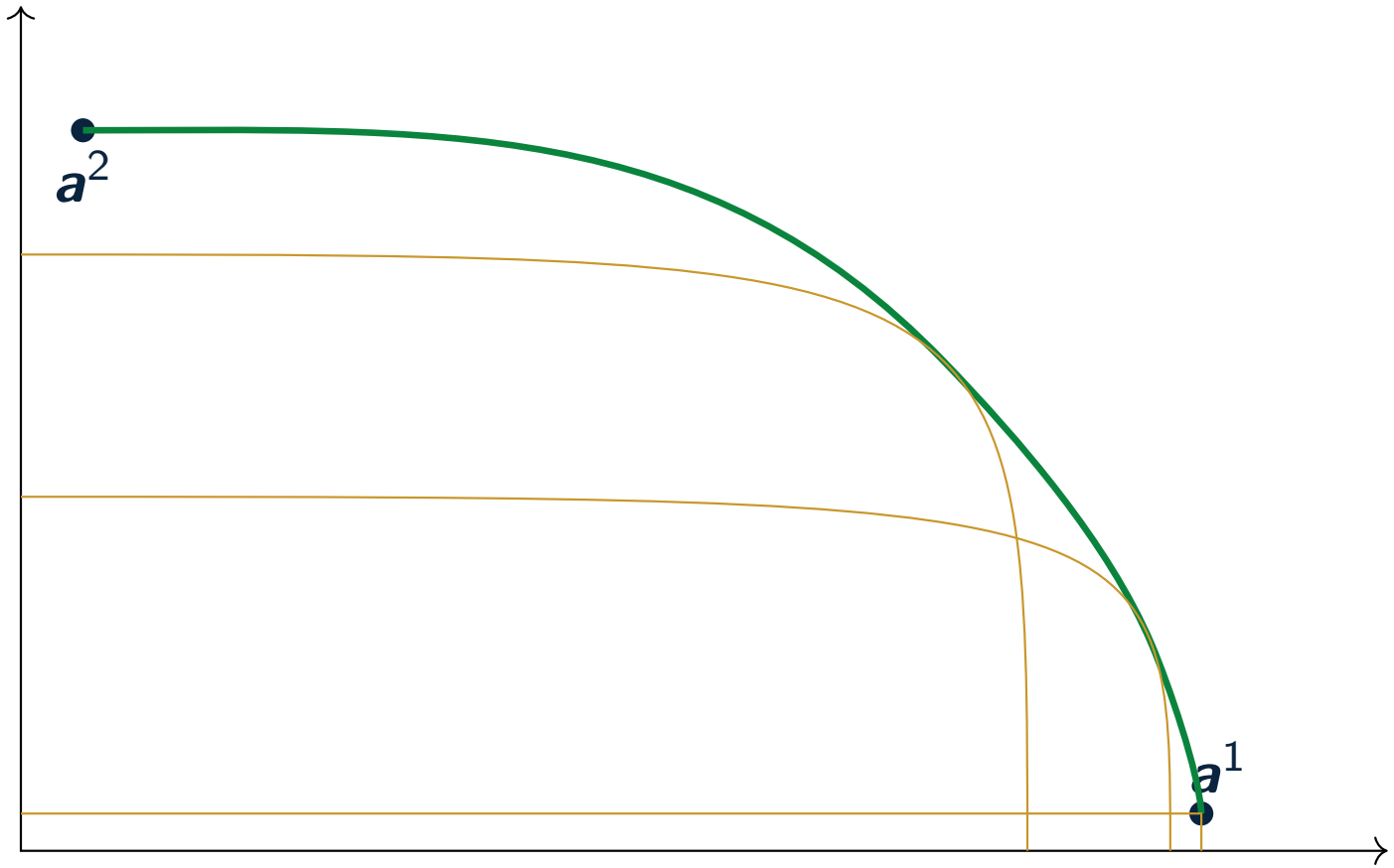
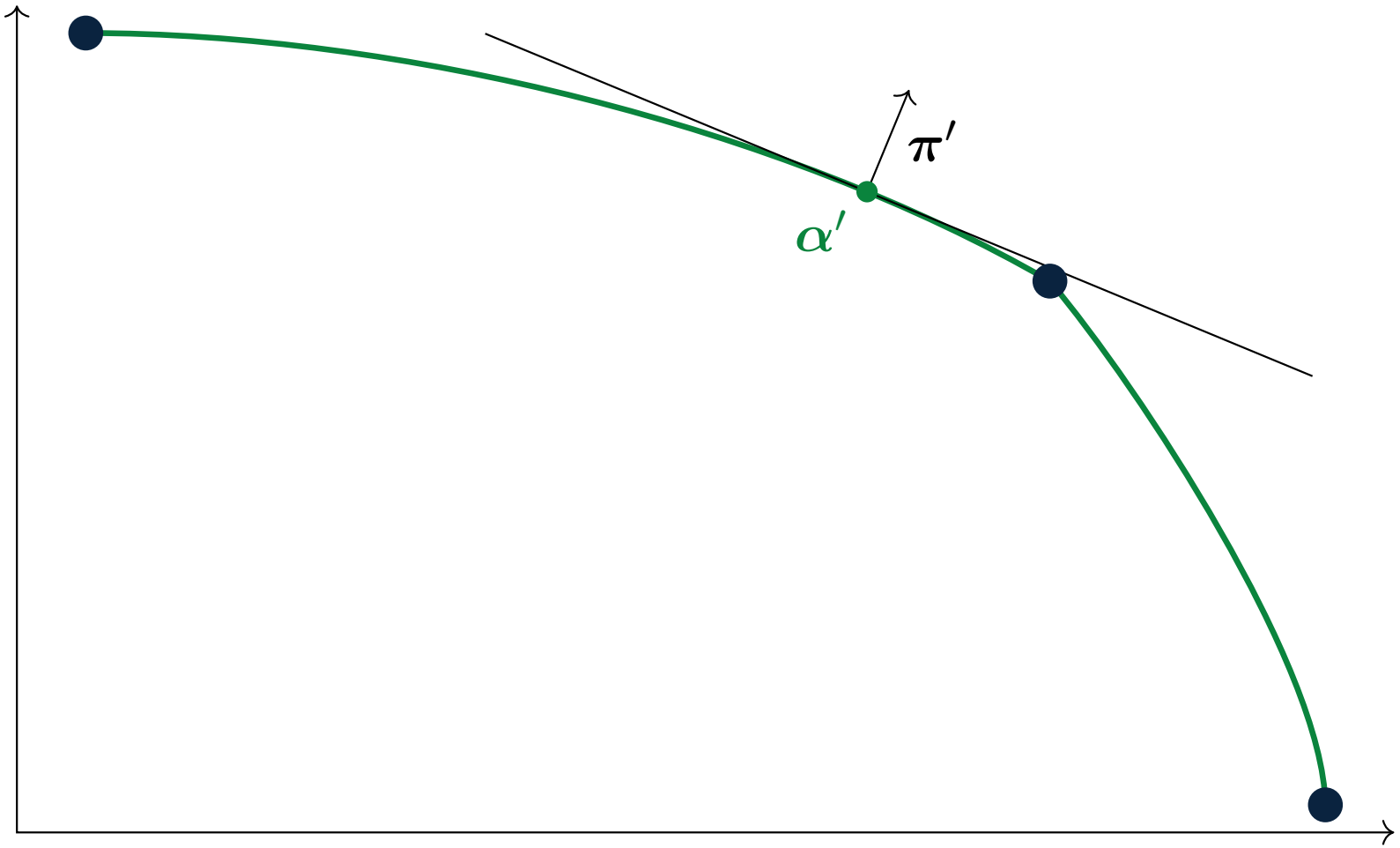


- The learning-proof menu can also be constructed from the signals.

Learning-Proof Menu



The Learning-Proof Menu is a **EU representation** of the RI problem: Ability to learn is *as if* agent has access to $\bar{\mathcal{A}}$ rather than $\mathcal{A}$.

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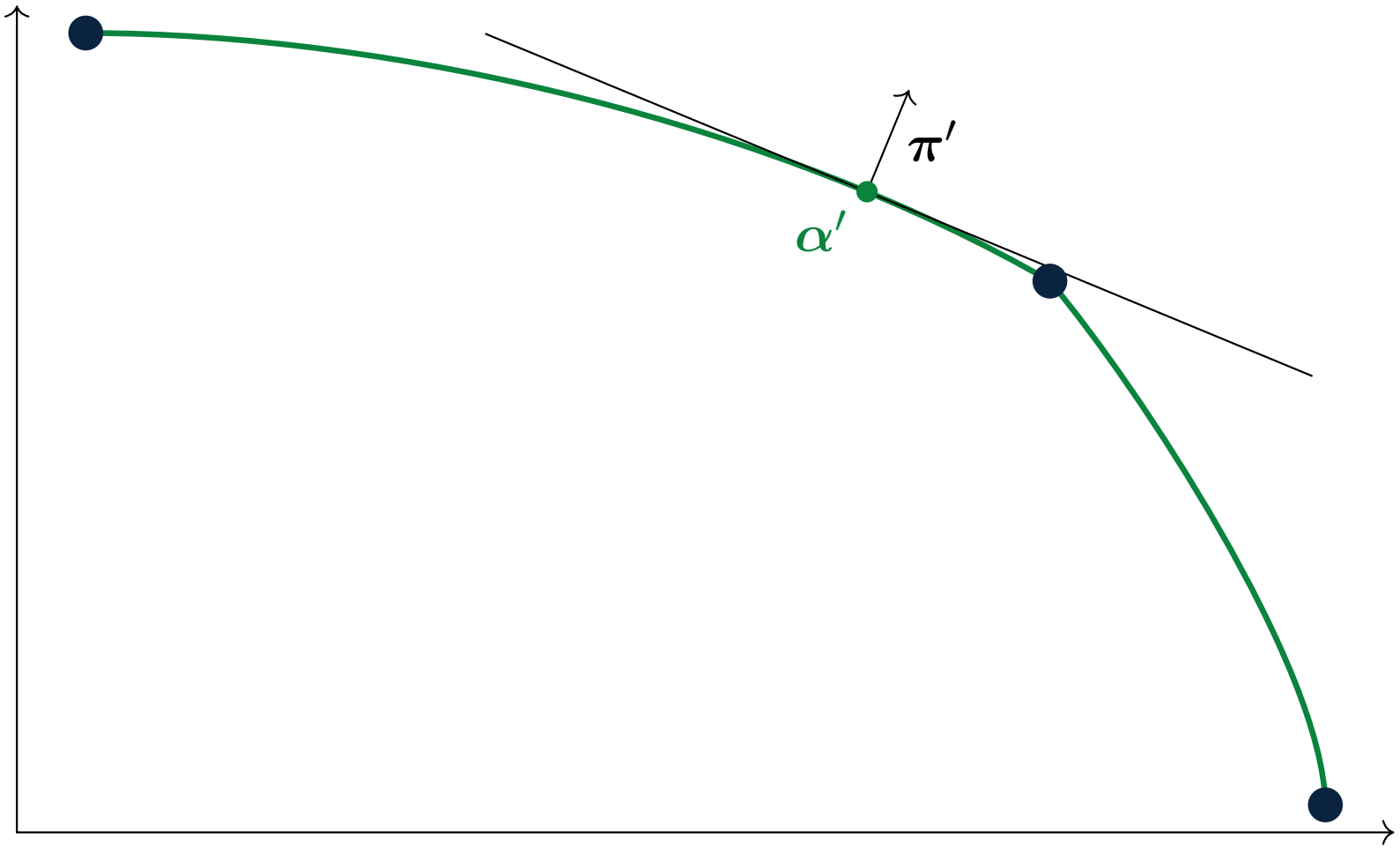
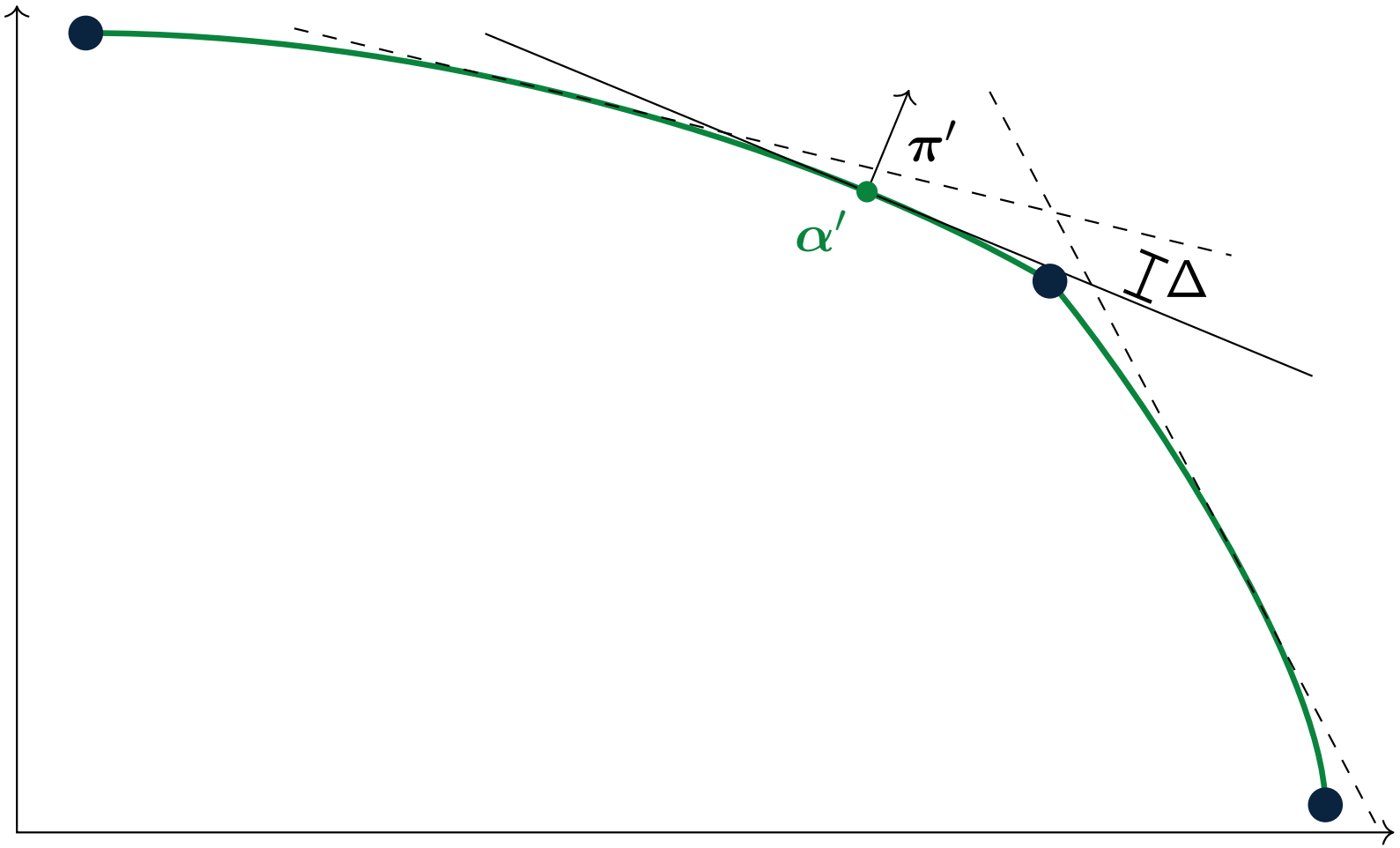


Offering the learning-proof menu $\bar{\mathcal{A}}$ is helpful for belief elicitation.

Learning-Proof Menu



Welfare impact Δ of exogenous information is intuitive in $\bar{\mathcal{A}}$.

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Ignorance equivalent:

- Condenses the entire menu $\mathcal{A}$ into one payoff vector $\boldsymbol{\alpha}$.
- Retains enough detail to capture learning opportunities.
- Useful for comparisons across signals, menus, and beliefs.

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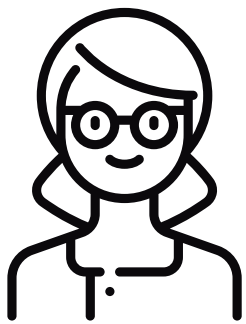
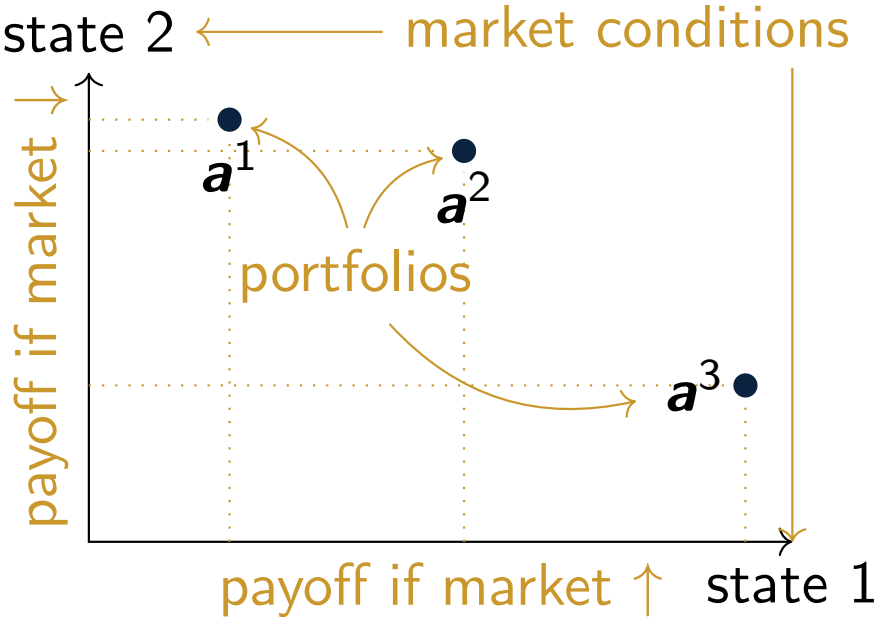
Toy example. A risk-neutral RI investor (she) wants to purchase one of the portfolios in $\mathcal{A}$.





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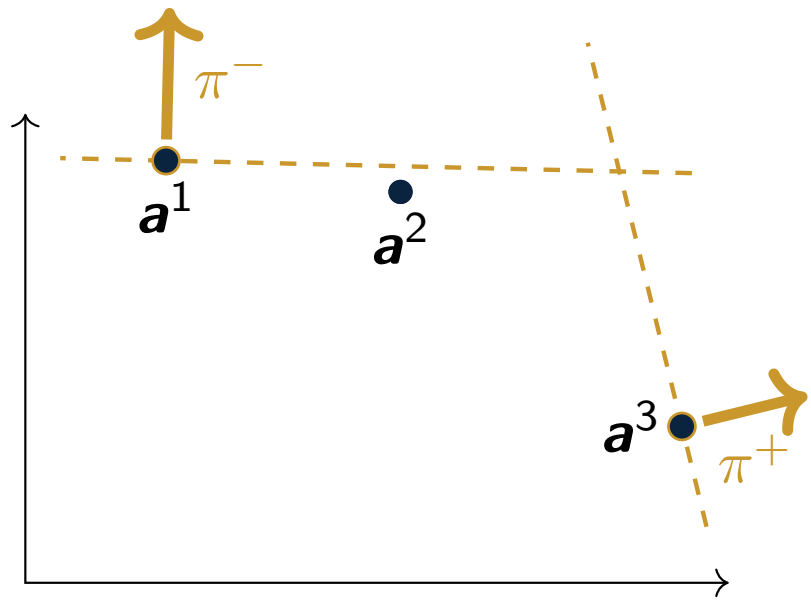

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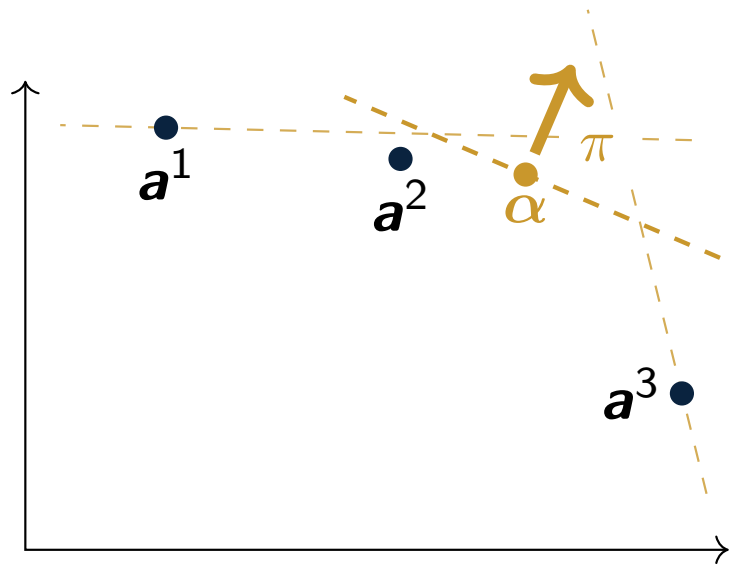
- α describes a portfolio that
  - is purchased unconditionally
  - makes her no better off.

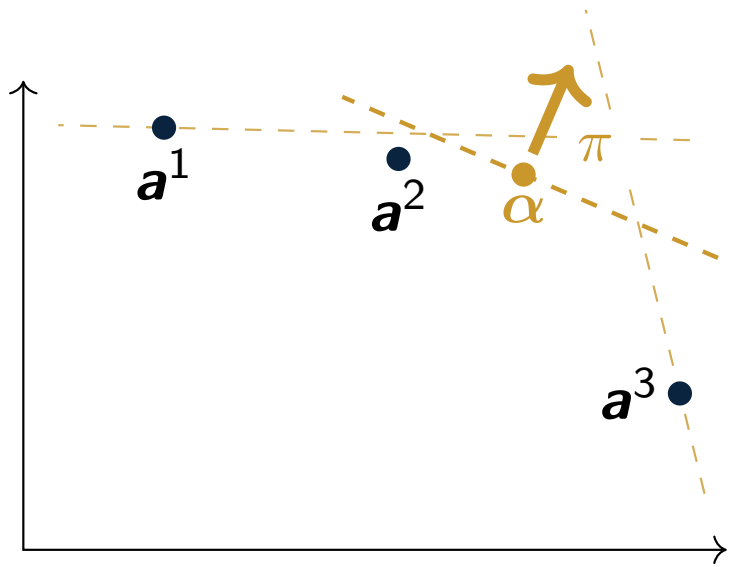


Economic Relevance

The ignorance equivalent and the learning proof menu are not just 'mental shortcuts' for tractability. They arise naturally in economic settings where the menu is designed strategically.

Toy example. A risk-neutral RI investor (she) wants to purchase one of the portfolios in $\mathcal{A}$.

An informed fund manager (he) can offer any payouts $\boldsymbol{a} \in \mathbb{R}^I$.



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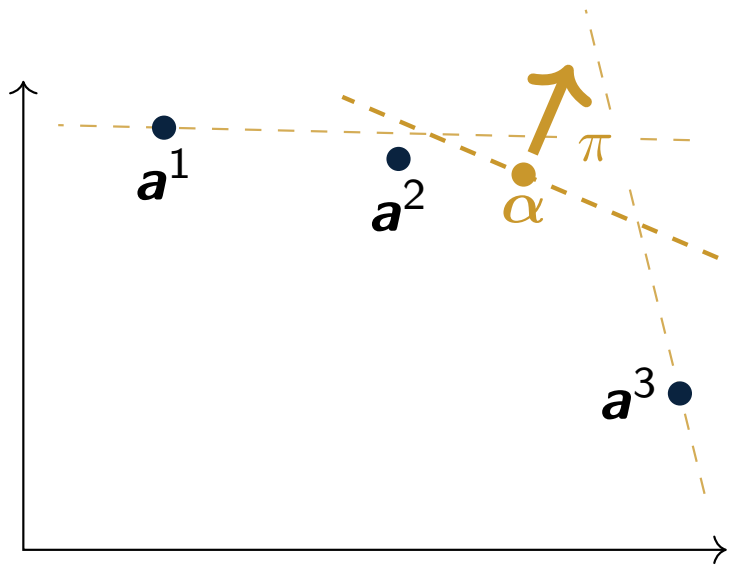

- α is what he wants to offer!
  - Unconditional acceptance.
  - Avoids adverse selection.
  - Reveals no free information.
  - Socially optimal.

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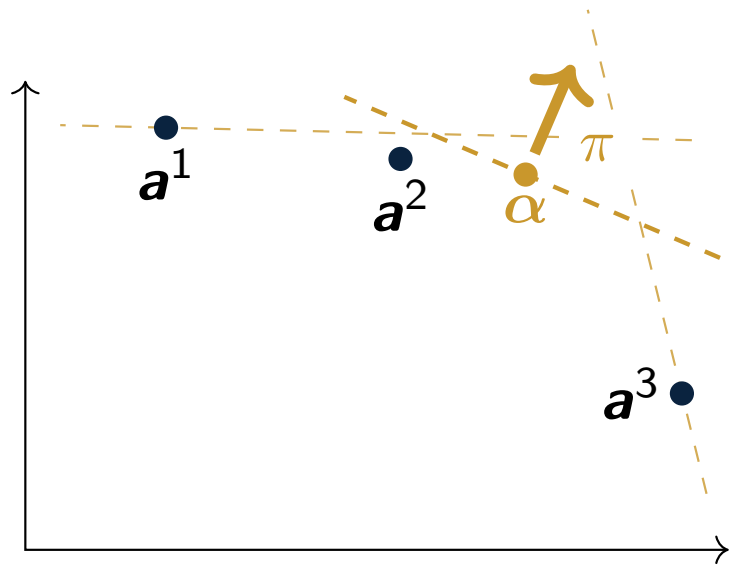

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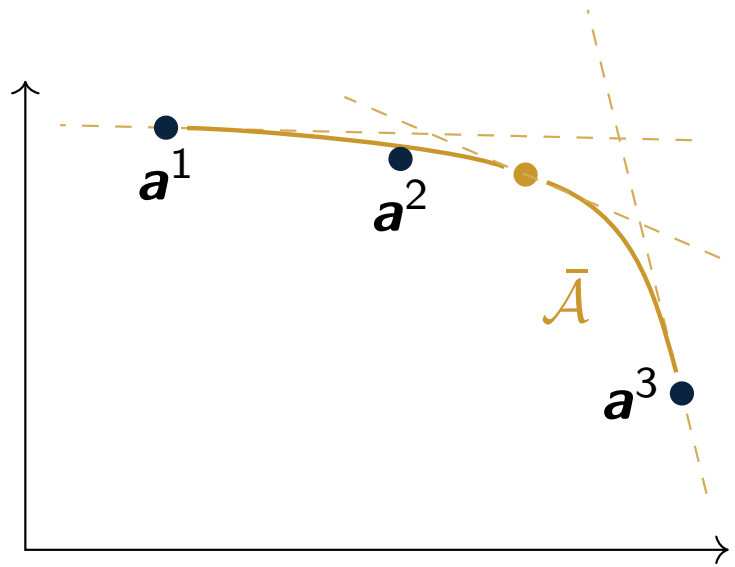
  - The manager can offer $\bar{\mathcal{A}}$.



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- What if agents can access different menus and have comparative advantages in learning?
  – For example, teams problem in technology acquisition.

short version | long version

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