Institute for Operations Research
and Computational Finance

University of St.Gallen

A structural model for electricity forward prices

Florentina Paraschiv, University of St. Gallen, ior/cf

with Fred Espen Benth, University of Oslo

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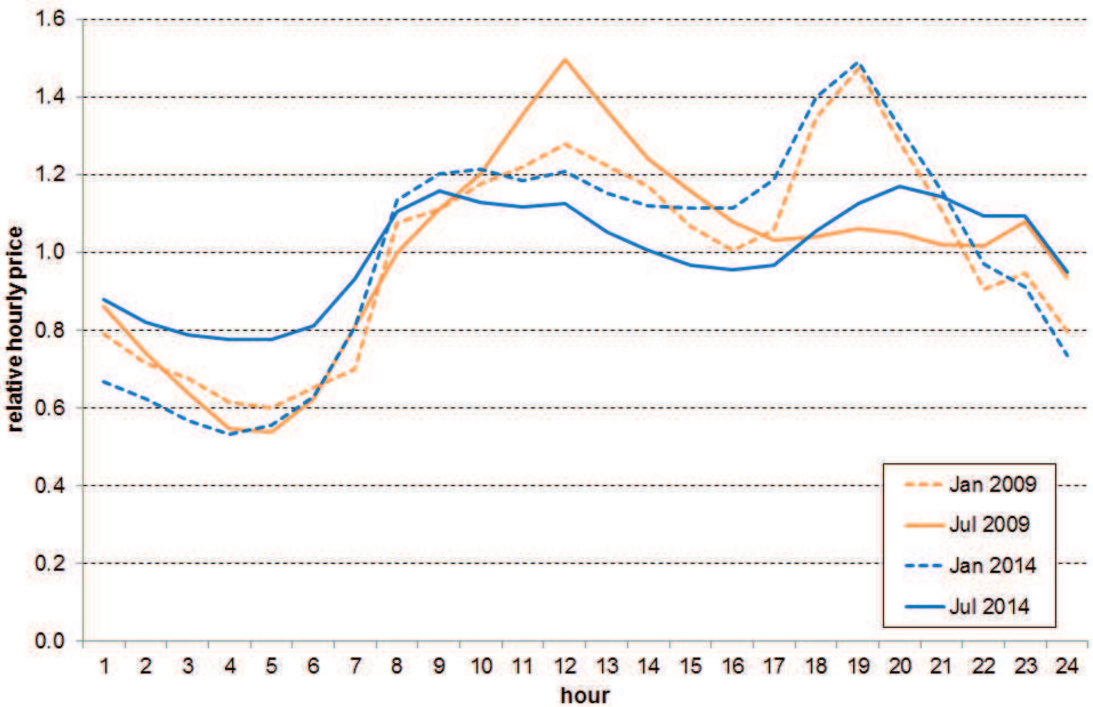
Outlook

- Structural models for forward electricity prices are highly relevant: major structural changes in the market due to the infeed from renewable energy
- Renewable energies impact the market price expectation – **impact on futures (forward) prices?**
- We will refer to a panel of daily price forward curves derived over time: cross-section analysis with respect to the *time dimension* and the *maturity space*
- Examine and model the dynamics of *risk premia*, the *volatility term structure*, *spatial correlations*



Literature review

- Models for forward prices in commodity/energy:
  - Specify one model for the spot price and from this derive for forwards: *Lucia and Schwartz (2002)*; *Cartea and Figueroa (2005)*; *Benth, Kallsen, and Mayer-Brandis (2007)*;
  - Heath-Jarrow-Morton approach – price forward prices directly, by multifactor models: *Roncoroni, Guiotto (2000)*; *Benth and Koekebakker (2008)*; *Kiesel, Schindlmayr, and Boerger (2009)*;
- Critical view of *Koekebakker and Ollmar (2005)*, *Frestad (2008)*
  - Few common factors cannot explain the substantial amount of variation in forward prices
  - Non-Gaussian noise
- Random-field models for forward prices:
  - *Roncoroni, Guiotto (2000)*;
  - *Andresen, Koekebakker, and Westgaard (2010)*;
- Derivation of seasonality shapes and price forward curves for electricity:
  - *Fleten and Lemming (2003)*;
  - *Bloechlinger (2008).*

Problem statement

- Previous models model forward prices evolving over time (time-series) along the **time at maturity T**: *Andresen, Koekebakker, and Westgaard (2010)*
- Let $F_t(T)$ denote the forward price at time $t \geq 0$ for delivery of a commodity at time $T \geq t$
- Random field in t:

$$t \mapsto F_t(T), \qquad t \geq 0 \tag{1}$$

- Random field in both t and T:

$$(t, T) \mapsto F_t(T), \qquad t \geq 0, \qquad t \leq T \tag{2}$$

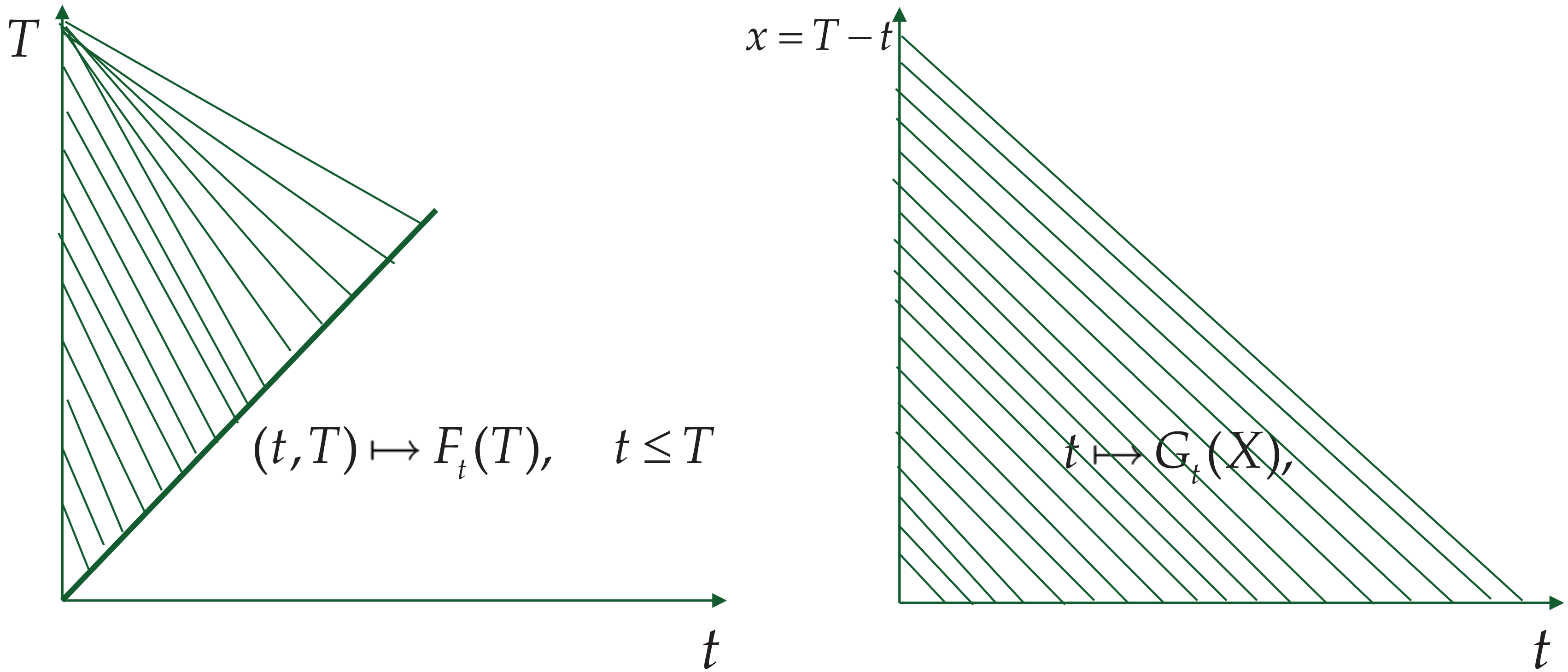
- Get rid of the second condition: **Musiela parametrization** $x = T - t$, $x \geq 0$.

$$F_t(t + x) = F_t(T), \qquad t \geq 0 \tag{3}$$

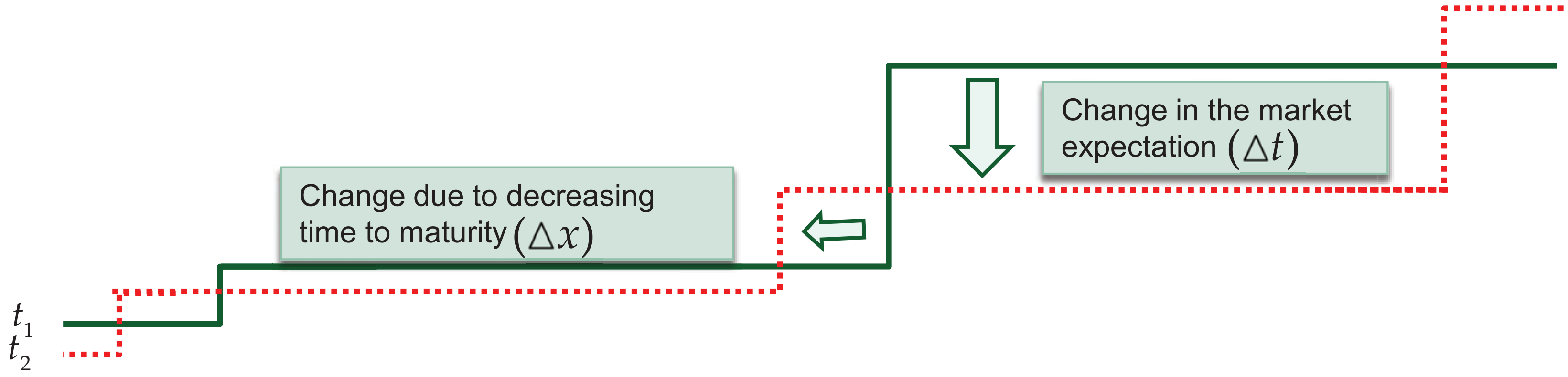
- Let $G_t(x)$ be the forward price for a contract with time to maturity $x \geq 0$. Note that:

$$G_t(x) = F_t(t + x) \tag{4}$$

Graphical interpretation



Influence of the “time to maturity”



Model formulation: Heath-Jarrow-Morton (HJM)

- The stochastic process $t \mapsto G_t(x), \quad t \geq 0$ is the solution to:

$$dG_t(x) = (\partial_x G_t(x) + \beta(t,x))\ dt + dW_t(x) \tag{5}$$

  - Space of curves are endowed with a Hilbert space structure $\mathcal{H}$
  - ∂_x differential operator with respect to time to maturity
  - β spatio-temporal random field describing the market price of risk
  - W Spatio-temporal random field describing the randomly evolving residuals

- Discrete structure:

$$G_t(x) = f_t(x) + s_t(x)\,, \tag{6}$$

  - $s_t(x)$ deterministic seasonality function $\mathbb{R}^2_+ \ni (t,x) \mapsto s_t(x) \in \mathbb{R}$

Model formulation (cont)

We furthermore assume that the deasonalized forward price curve, denoted by $f_t(x)$, has the dynamics:

$$df_t(x) = (\partial_x f_t(x) + \theta(x) f_t(x))\, dt + dW_t(x)\,, \tag{7}$$

with $\theta \in \mathbb{R}$ being a constant. With this definition, we note that

$$\begin{aligned} dF_t(x) &= df_t(x) + ds_t(x) \\ &= (\partial_x f_t(x) + \theta(x) f_t(x))\, dt + \partial_t s_t(x)\, dt + dW_t(x) \\ &= (\partial_x F_t(x) + (\partial_t s_t(x) - \partial_x s_t(x)) + \theta(x)(F_t(x) - s_t(x)))\, dt + dW_t(x)\,. \end{aligned}$$

In the natural case, $\partial_t s_t(x) = \partial_x s_t(x)$, and therefore we see that $F_t(x)$ satisfy (5) with $\beta(t,x) := \theta(x) f_t(x)$.

The market price of risk is proportional to the deseasonalized forward prices.

Model formulation (cont)

We discretize the dynamics in Eq. (7) by an Euler discretization

$$df_t(x) = (\partial_x f_t(x) + \theta(x) f_t(x))\; dt + dW_t(x)$$

$$\partial_x f_t(x) \approx \frac{f_t(x+\Delta x) - f_t(x)}{\Delta x}$$

$$f_{t+\Delta t}(x) = (f_t(x) + \frac{\Delta t}{\Delta x}(f_t(x+\Delta x) - f_t(x)) + \theta(x) f_t(x)\Delta t + \epsilon_t(x) \tag{8}$$

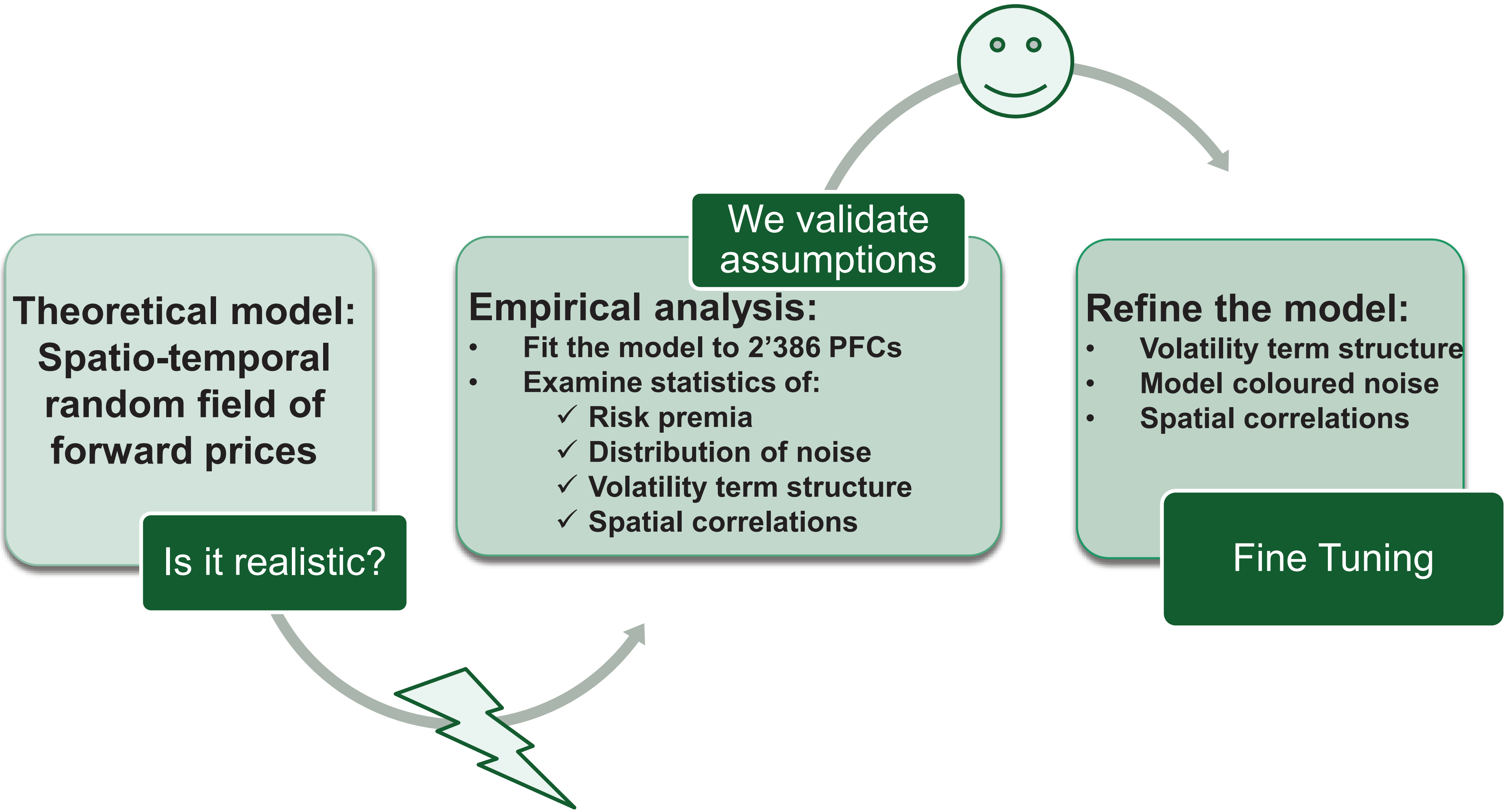
with $x \in \{x_1, \ldots, x_N\}$ and $t = \Delta t, \ldots, (M-1)\Delta t$, where $\epsilon_t(x) := W_{t+\Delta t}(x) - W_t(x)$.

$$Z_t(x) := f_{t+\Delta t}(x) - f_t(x) - \frac{\Delta t}{\Delta x}(f_t(x+\Delta x) - f_t(x)) \tag{9}$$

which implies

$$Z_t(x) = \theta(x) f_t(x)\Delta t + \epsilon_t(x)\,, \tag{10}$$

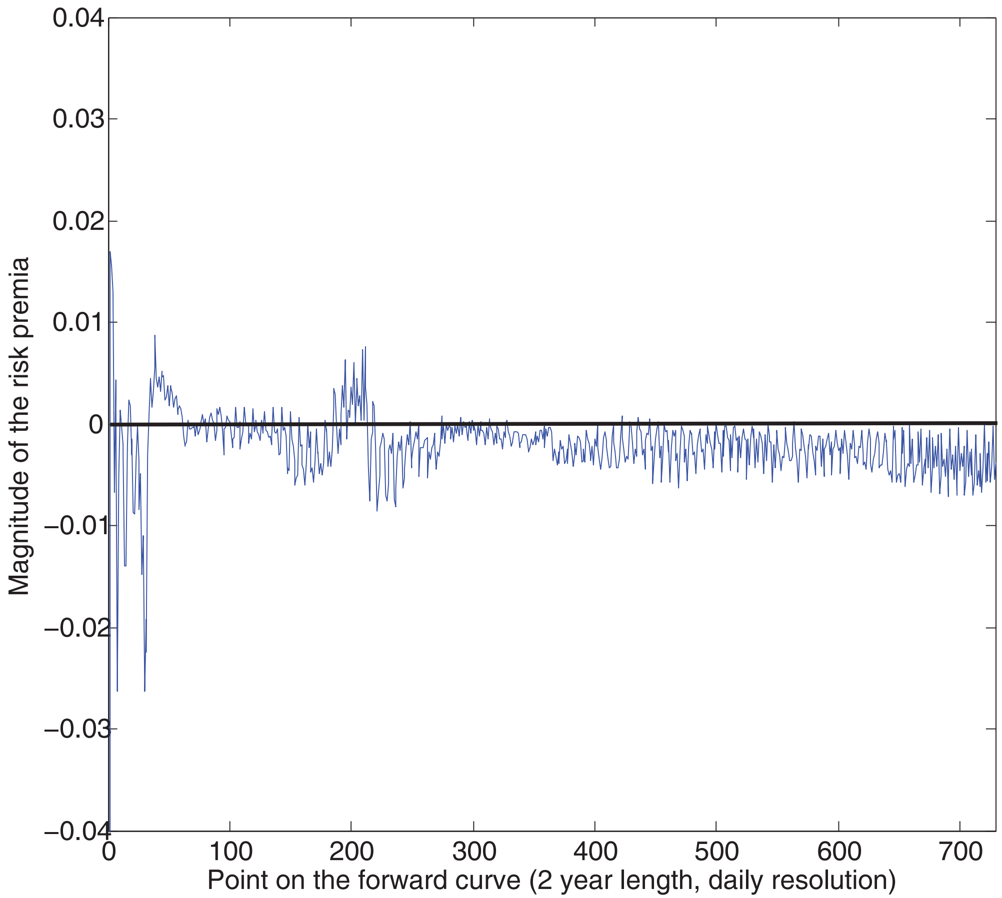
$$\epsilon_t(x) = \sigma(x)\tilde{\epsilon}_t(x) \tag{11}$$



Theoretical model: Spatio-temporal random field of forward prices
Is it realistic?
We validate assumptions
Empirical analysis:
• Fit the model to 2’386 PFCs
• Examine statistics of:
✓ Risk premia
✓ Distribution of noise
✓ Volatility term structure
✓ Spatial correlations
Refine the model:
• Volatility term structure
• Model coloured noise
• Spatial correlations
Fine Tuning

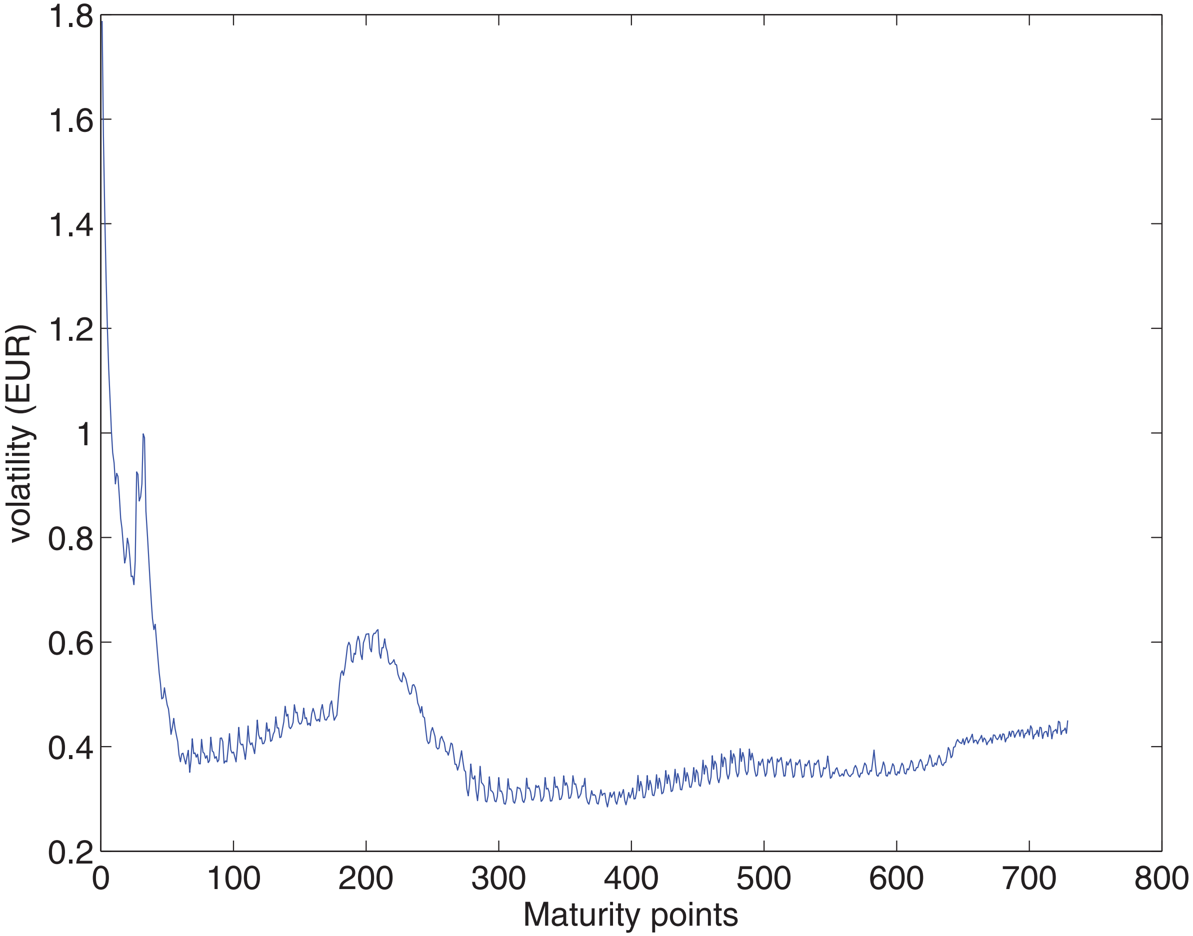
Risk premia

- Short-term: it oscillates around zero and has higher volatility (similar in *Pietz (2009)*, *Paraschiv et al. (2015)*)
- Long-term: is becomes negative and has more constant volatility (*Burger et al. (2007)*): In the long-run power generators accept lower futures prices, as they need to make sure that their investment costs are covered.



Term structure volatility

- We observe Samuelson effect: overall higher volatility for shorter time to maturity
- Volatility bumps (front month; second/third quarters) explained by increased volume of trades
- Jigsaw pattern: weekend effect; volatility smaller in the weekend versus working days



Explaining volatility bumps

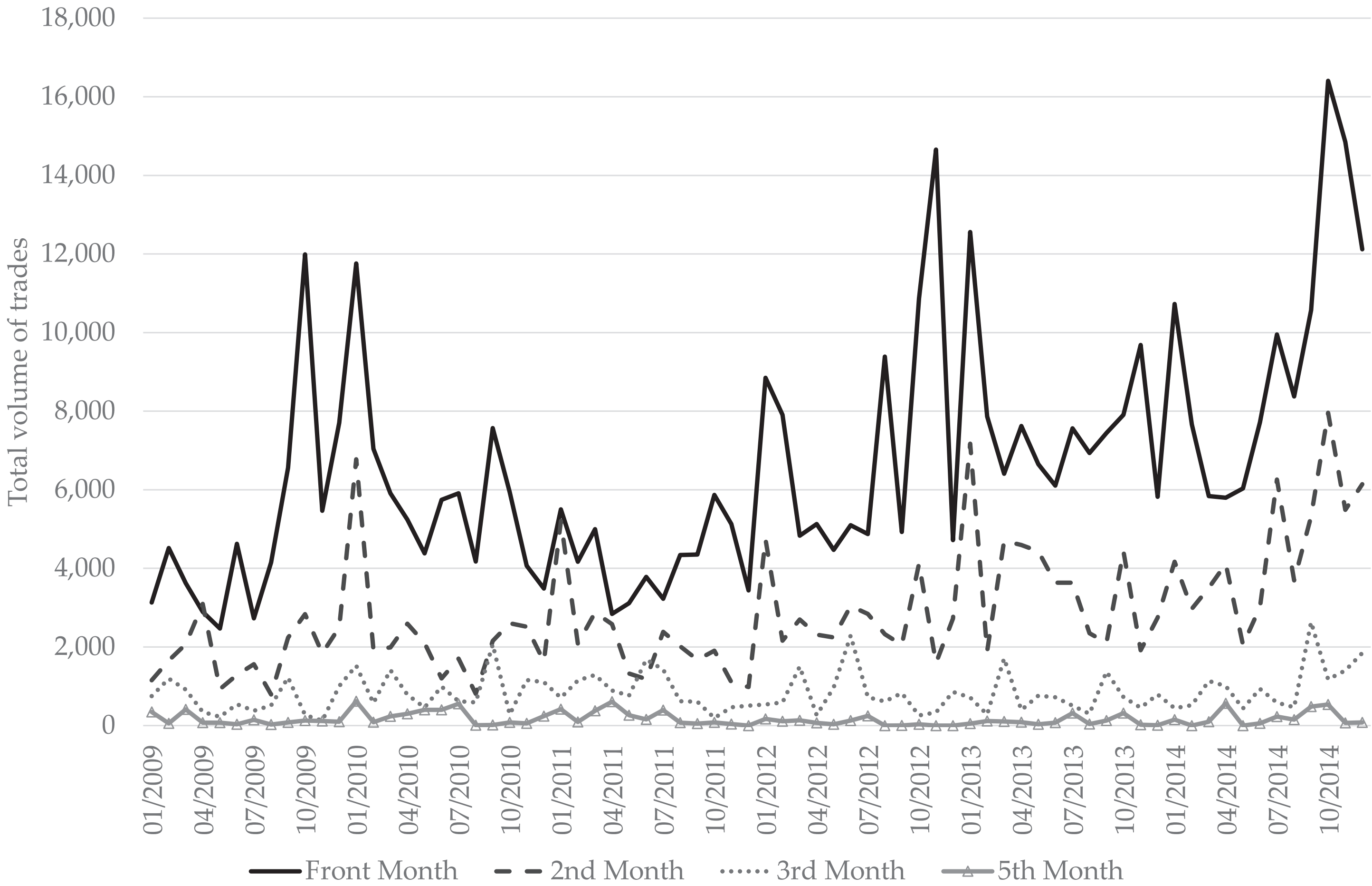


Figure 2: The sum of traded contracts for the monthly futures, evidence from EPEX, own calculations (source of data ems.eex.com).

Explaining volatility bumps

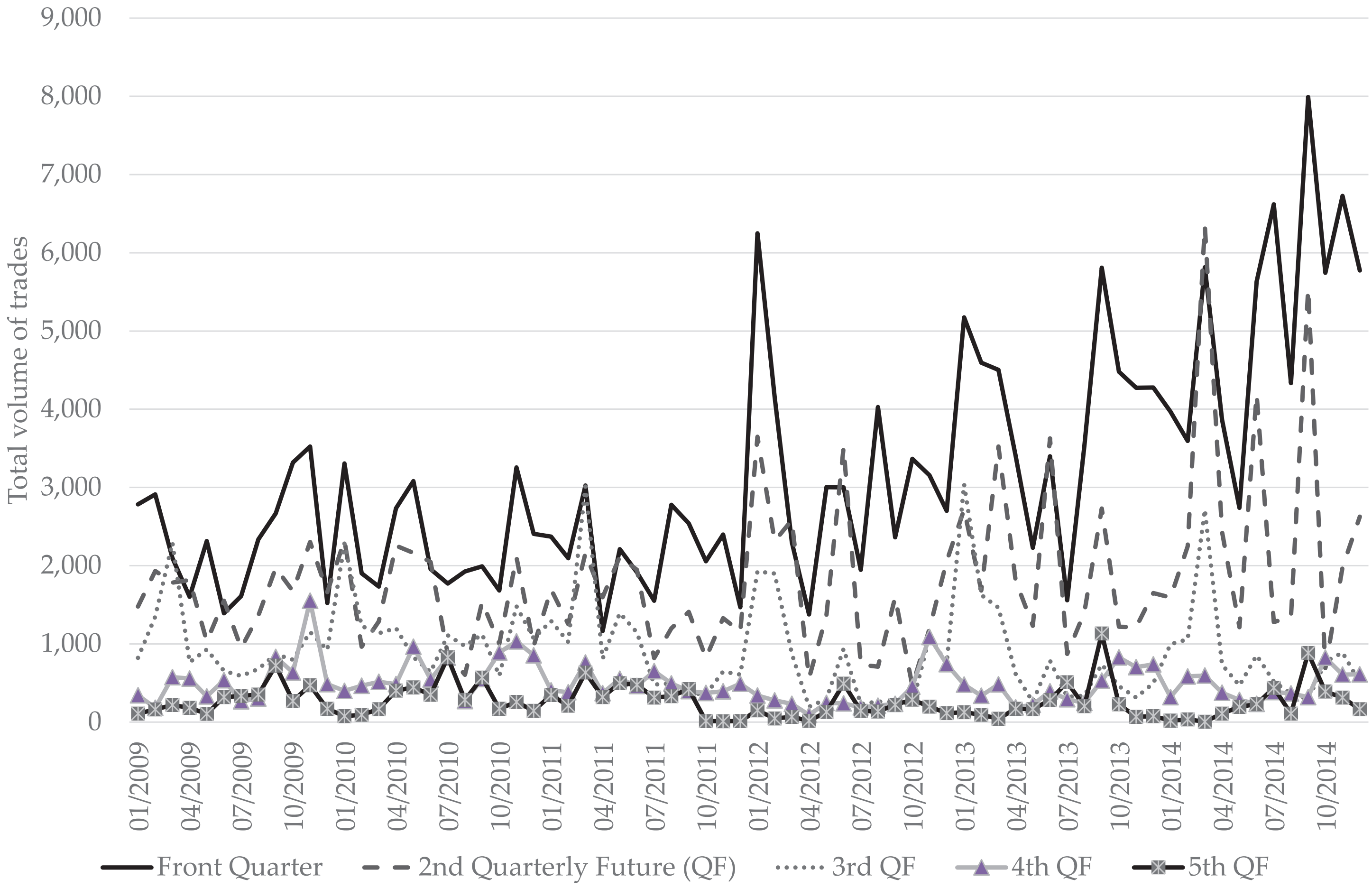


Figure 3: The sum of traded contracts for the quarterly futures, evidence from EPEX, own calculations (source of data ems.eex.com).

Statistical properties of the noise

- We examined the statistical properties of the noise time-series $\tilde{\epsilon}_t(x)$

$$\epsilon_t(x) = \sigma(x)\tilde{\epsilon}_t(x) \tag{12}$$

- We found: Overall we conclude that the model residuals are **coloured noise**, with **heavy tails** (leptokurtic distribution) and with a tendency for **conditional volatility**.

$\tilde{\epsilon}_t(x_k)$	Stationarity	Autocorrelation $\tilde{\epsilon}_t(x_k)$	Autocorrelation $\tilde{\epsilon}_t(x_k)^2$	ARCH/GARCH
	h	h1	h1	h2
Q0	0	1	1	1
Q1	0	0	0	0
Q2	0	1	1	1
Q3	0	0	1	1
Q4	0	1	0	0
Q5	0	1	1	1
Q6	0	1	1	1
Q7	0	1	1	1

Table 1: The time series are selected by quarterly increments (90 days) along the maturity points on one noise curve. Hypotheses tests results, case study 1: $\Delta x = 1day$. In column stationarity, if $h = 0$ we fail to reject the null that series are stationary. For autocorrelation $h1 = 0$ indicates that there is not enough evidence to suggest that noise time series are autocorrelated. In the last column $h2 = 1$ indicates that there are significant ARCH effects in the noise time-series.

Autocorrelation structure of noise time series (squared)

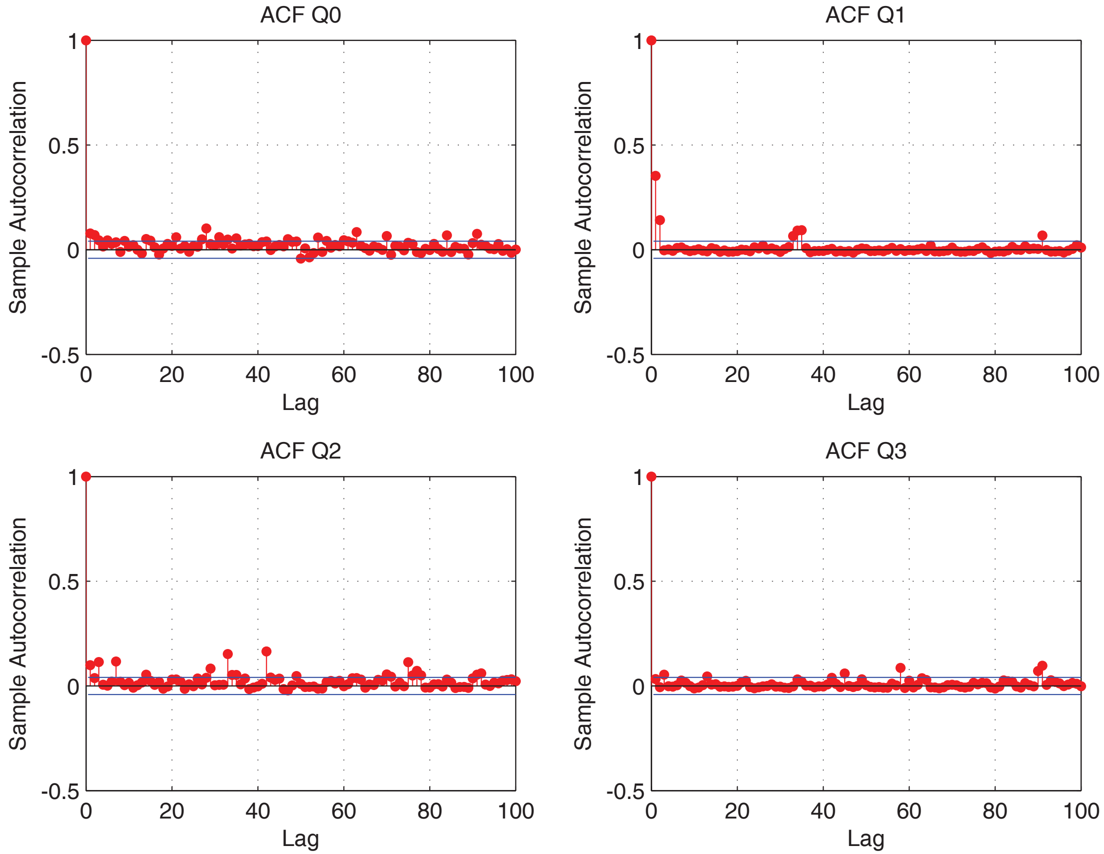
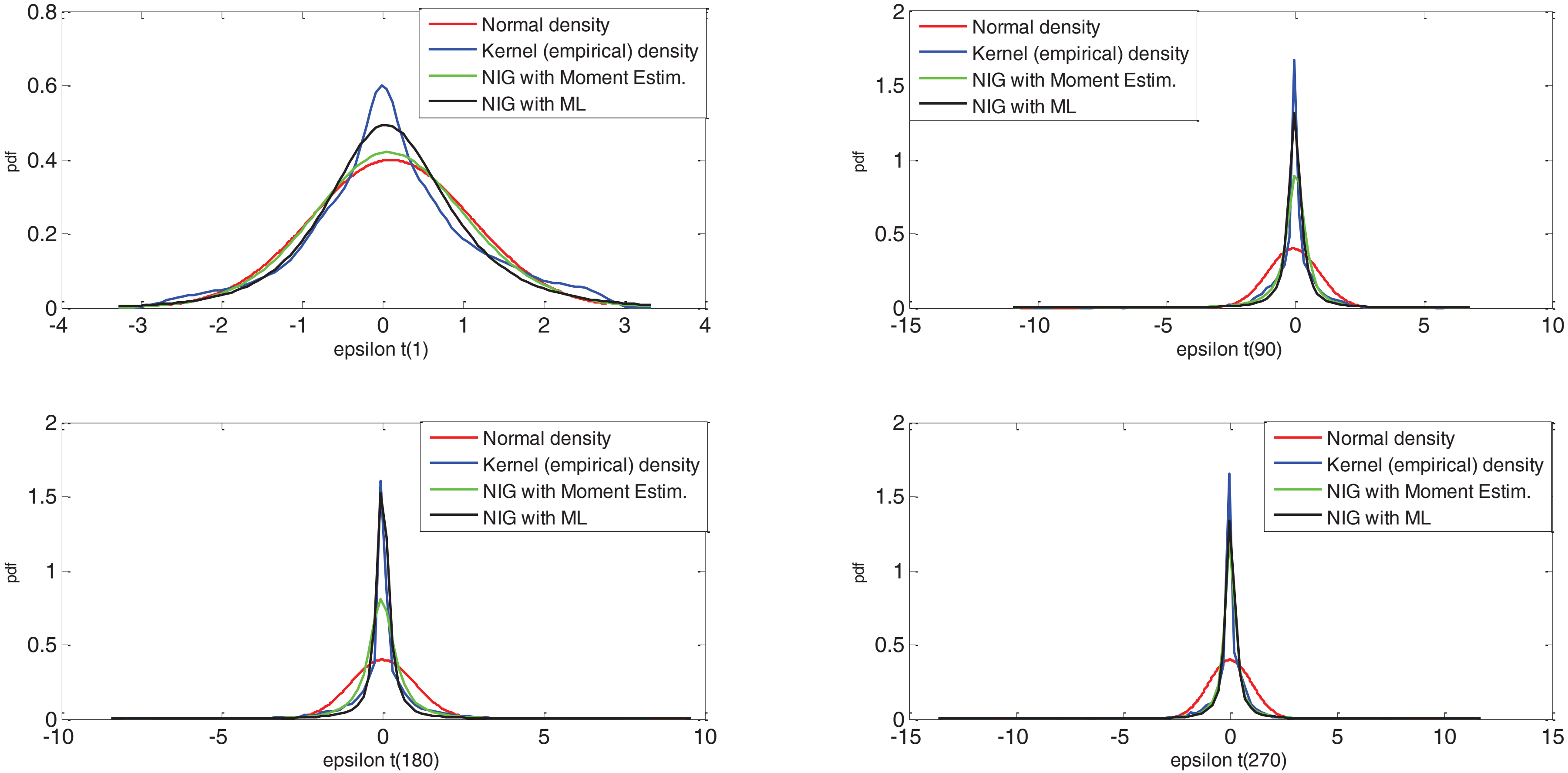


Figure 4: Autocorrelation function in the squared time series of the noise $\tilde{\epsilon}_t(x_k)^2$, by taking $k \in \{1, 90, 180, 270\}$, case study 1: $\Delta x = 1\,day$.

Normal Inverse Gaussian (NIG) distribution for coloured noise



Spatial dependence structure

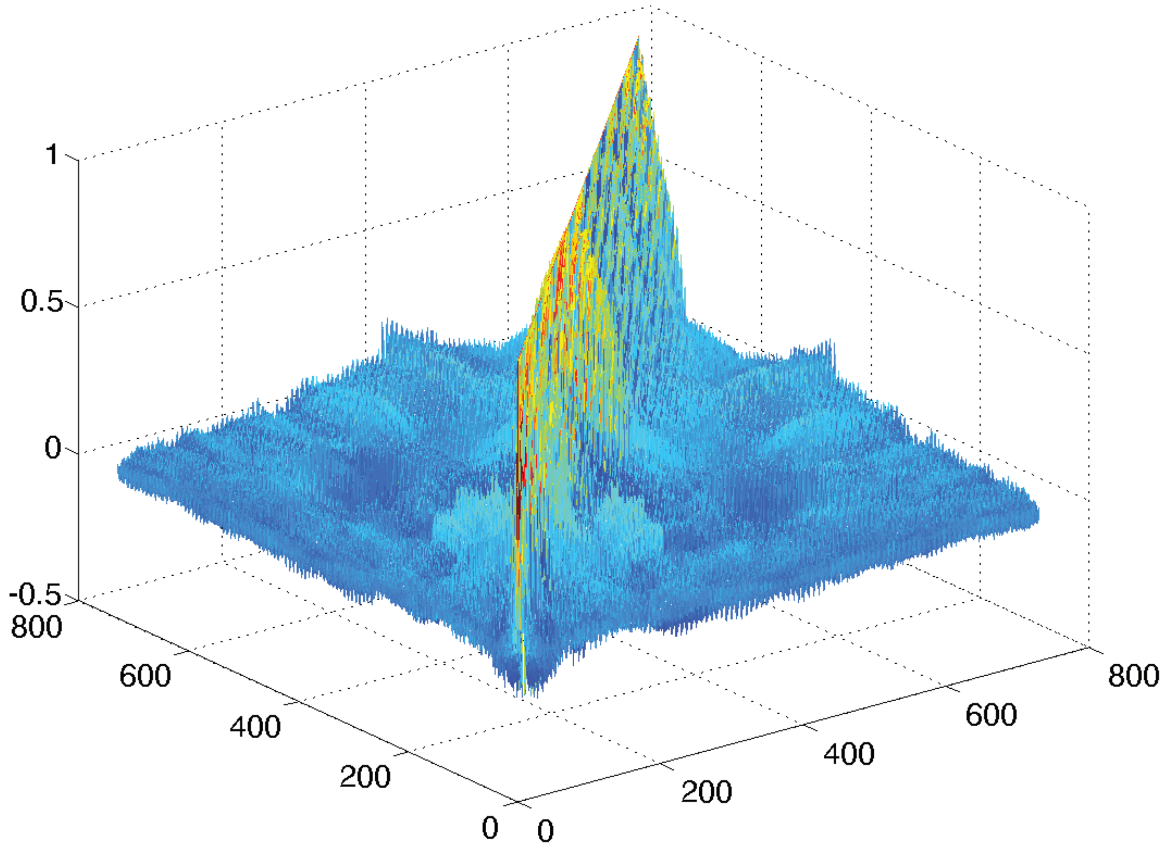


Figure 5: *Correlation matrix with respect to different maturity points along one curve.*

Revisiting the model

- We have analysed empirically the noise residual $dW_t(x)$ expressed as $\epsilon_t(x) = \sigma(x)\widetilde{\epsilon}_t(x)$ in a discrete form
- Recover an infinite dimensional model for $W_t(x)$ based on our findings

$$W_t = \int_0^t \Sigma_s \, dL_s \,, \tag{13}$$

where $s \mapsto \Sigma_s$ is an $L(\mathcal{U}, \mathcal{H})$-valued predictable process and L is a $\mathcal{U}$-valued Lévy process with zero mean and finite variance.

- As a first case, we can choose $\Sigma_s \equiv \Psi$ time-independent:

$$W_{t+\Delta t} - W_t \approx \Psi(L_{t+\Delta t} - L_t) \tag{14}$$

Choose now $\mathcal{U} = L^2(\mathbb{R})$, the space of square integrable functions on the real line equipped with the Lebesgue measure, and assume Ψ is an integral operator on $L^2(\mathbb{R})$

$$\mathbb{R}_+ \ni x \mapsto \Psi(g)(x) = \int_{\mathbb{R}} \widetilde{\sigma}(x,y) g(y) \, dy \tag{15}$$

- we can further make the approximation $\Psi(g)(x) \approx \widetilde{\sigma}(x,x)g(x)$, which gives

$$W_{t+\Delta t}(x) - W_t(x) \approx \widetilde{\sigma}(x,x)(L_{t+\Delta t}(x) - L_t(x)) \,. \tag{16}$$

Revisiting the model (cont)

- Recall the spatial correlation structure of $\widetilde{\epsilon}_t(x)$. This provides the empirical foundation for defining a covariance functional $\mathcal{Q}$ associated with the Lévy process L.
- In general, we know that for any $g, h \in L^2(\mathbb{R})$,

$$\mathbb{E}[(L_t, g)_2(L_t, h)_2] = (\mathcal{Q}g, h)_2$$

where $(\cdot, \cdot)_2$ denotes the inner product in $L^2(\mathbb{R})$

$$\mathcal{Q}g(x) = \int_{\mathbb{R}} q(x, y)g(y)\, dy\,, \tag{17}$$

- If we assume $g \in L^2(\mathbb{R})$ to be close to δ_x, the Dirac δ-function, and likewise, $h \in L^2(\mathbb{R})$ being close to δ_y, $(x, y) \in \mathbb{R}^2$, we find approximately

$$\mathbb{E}[L_t(x)L_t(y)] = q(x, y)$$

- A simple choice resembling to some degree the fast decaying property is $q(|x-y|) = \exp(-\gamma|x-y|)$ for a constant $\gamma > 0$.
- It follows that $t \mapsto (L_t, g)_2$ is a NIG Lévy process on the real line.

Conclusion

- We developed a spatio-temporal dynamical arbitrage free model for electricity forward prices based on the Heath-Jarrow-Morton (HJM) approach under Musiela parametrization
- We examined a unique data set of price forward curves derived each day in the market between 2009–2015
- We examined the spatio-temporal structure of our data set
  - **Risk premia:** higher volatility short-term, oscillating around zero; constant volatility on the long-term, turning into negative
  - **Term structure volatility:** Samuelson effect, volatility bumps explained by increased volume of trades
  - **Coloured (leptokurtic) noise** with evidence of conditional volatility
  - **Spatial correlations structure:** decaying fast for short-term maturities; constant (white noise) afterwards with a bump around 1 year
- After explaining the Samuelson effect in the volatility term structure, the residuals are modeled by an infinite dimensional NIG Lévy process, which allows for a natural formulation of a covariance functional.