

Quantifying the Extensive Margins of Trade and Production

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- International trade has experienced major disruptions in recent years
 - US-China, Brexit, COVID-19 pandemic, Russian invasion of Ukraine

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- “Industry” examples: batteries, motorcycles, wire of stainless steel
- RQ: When key partner countries drop out of the system, . . .
 - . . . how can they be replaced?
 - . . . what are the implications for welfare?

Procedure

- Build GE framework capturing (bilateral) industry-level zeros
 - Standard trade models: export to & import from every country in all industries
 - Based on multi-industry [Eaton and Kortum \(2002\)](#) (**EK**), nested as special case

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- Build GE framework capturing (bilateral) industry-level zeros
 - Standard trade models: export to & import from every country in all industries
 - Based on multi-industry [Eaton and Kortum \(2002\)](#) (**EK**), nested as special case
- Key ingredients:
 - Non-homothetic final-goods-assembly function
 - Upper-bounded productivity distribution

Preview of Results

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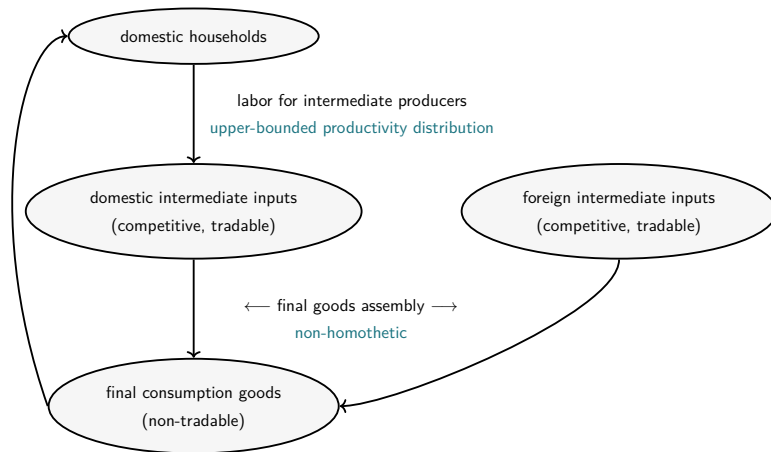
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- Welfare impact of trade-cost shocks typically amplified with zeros
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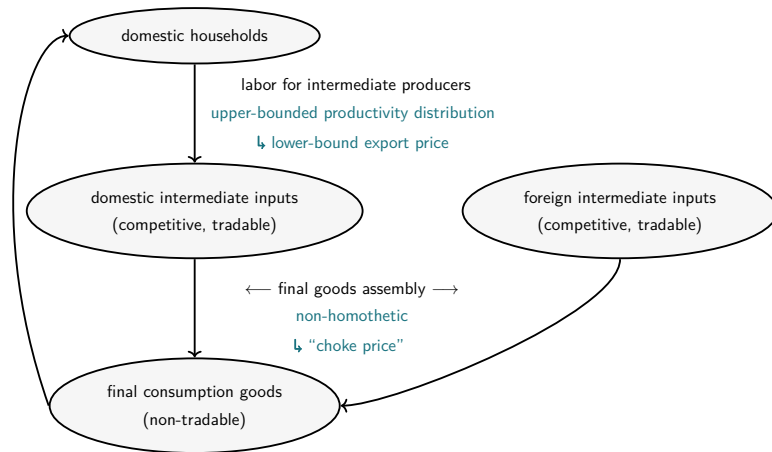
Related Literature

- Quantitative analyses of trade policy (cf. [Caliendo and Parro \(2022\)](#) for review)
 - Non-homothetic preferences and trade (e.g., [Fieler \(2011\)](#); [Caron et al. \(2014\)](#); [Fajgelbaum and Khandelwal \(2016\)](#); [Foellmi et al. \(2018\)](#))
 - Export and production diversification (e.g., [Imbs and Wacziarg \(2003\)](#); [Hidalgo et al. \(2007\)](#); [Cadot et al. \(2011\)](#))
 - Extensive margin at country-level: [Helpman et al. \(2008\)](#); [Chor \(2010\)](#); [Eaton et al. \(2013\)](#); [Eaton and Fieler \(2019\)](#)
 - Need (i) industry-level production data and/or (ii) expensive simulation of model
- ⇒ [This paper](#): Quantification with closed-form using only readily available data

Model Structure (i/ii)



Model Structure (ii/ii)



Households (i/ii)

- Countries indexed by $i, j = \{1, \dots, N\}$, final goods (**industry-level**) indexed by $k = \{1, \dots, K\}$
- L_j households, utility from final goods consumption

$$\begin{aligned} \max_{\{q_j^k\}} U_j &= \frac{\sigma}{\sigma - 1} \sum_{k=1}^K \left(\xi_j^k \right)^{\frac{1}{\sigma}} \left(q_j^k \right)^{\frac{\sigma-1}{\sigma}} \\ \text{s.t. } E_j &\geq \sum_{k=1}^K P_j^k q_j^k \end{aligned}$$

Households (ii/ii)

- Non-tradable final good k assembled using k -specific intermediate varieties

$$q_j^k = \int_0^1 q_j^k(\omega^k) d\omega^k,$$

where $q_j^k(\omega^k) \in \{0, 1\}$

- Note: intermediate inputs substitutable along *extensive* margin
⇒ Generates “choke price,” $\bar{p}_j^k$

Production of Tradable Intermediate Inputs (i/iii)

- Perfect competition, labor is only input
- Unit cost of variety ω^k produced in country i and shipped to j :

$$c_{ij}^k(\omega^k) \equiv \frac{w_i \tau_{ij}^k}{\varphi_i(\omega^k)}$$

- w_i wage rate, τ_{ij}^k iceberg trade costs, $\varphi_i(\omega^k)$ labor productivity
- Price country i offers to j : $p_{ij}^k(\omega^k) = c_{ij}^k(\omega^k)$

Production of Tradable Intermediate Inputs (ii/iii)

- Assume $\varphi_i(\omega^k)$ independently drawn from truncated Fréchet distribution

$$F_i^k(\varphi; \bar{\varphi}_i^k) = \frac{\exp(-T_i^k \varphi^{-\theta})}{\exp(-T_i^k (\bar{\varphi}_i^k)^{-\theta})}, \quad \text{for } \varphi \in (0, \bar{\varphi}_i^k]$$

- T_i^k scale parameter, θ shape parameter,
 $\bar{\varphi}_i^k$ productivity upper bound (=“technological frontier”)

Production of Tradable Intermediate Inputs (iii/iii)

- Let $\underline{p}_{ij}^k \equiv w_i \tau_{ij}^k / \bar{\varphi}_i^k$
- For a given destination j , sort exporters such that

$$\underline{p}_{1j}^k \leq \underline{p}_{2j}^k \leq \cdots \leq \underline{p}_{Nj}^k$$

⇒ Intuition zeros: $\underline{p}_{ij}^k \geq \bar{p}_j^k$

► Price Distribution

Trade Flows

- Country j expenditure on intermediates from i in industry k [Details](#)

$$X_{ij}^k = \sum_{n=i}^N \frac{T_i^k (w_i \tau_{ij}^k)^{-\theta}}{\Phi_j^{k,1:n}} \tilde{\Gamma} \left(\min\{\underline{p}_{nj}^k, \bar{p}_j^k\}, \min\{\underline{p}_{(n+1)j}^k, \bar{p}_j^k\} \right) \tilde{\gamma}_j^{k,1:n}$$

- $\bar{p}_j^k$ is the choke price
- $\tilde{\Gamma}(p, p) = 0$
- Note: trade flows in levels

Trade Flows

- Country j expenditure on intermediates from i in industry k [Details](#)

$$X_{ij}^k = \sum_{n=i}^N \frac{T_i^k (w_i \tau_{ij}^k)^{-\theta}}{\underbrace{\Phi_j^{k,1:n}}_{\text{"intensive margin" (EK)}}} \tilde{\Gamma} \left(\min\{\underline{p}_{nj}^k, \bar{p}_j^k\}, \min\{\underline{p}_{(n+1)j}^k, \bar{p}_j^k\} \right) \tilde{\gamma}_j^{k,1:n}$$

- $\Phi_j^{k,1:n} := \sum_{i=1}^n T_i^k (w_i \tau_{ij}^k)^{-\theta}$
- Trade flows determined by relative technology & trade costs

Trade Flows

- Country j expenditure on intermediates from i in industry k [▶ Details](#)

$$X_{ij}^k = \sum_{n=i}^N \frac{T_i^k (w_i \tau_{ij}^k)^{-\theta}}{\Phi_j^{k,1:n}} \underbrace{\tilde{\Gamma} \left(\min\{\underline{p}_{nj}^k, \bar{p}_j^k\}, \min\{\underline{p}_{(n+1)j}^k, \bar{p}_j^k\} \right)}_{\text{"extensive margin"}} \tilde{\gamma}_j^{k,1:n}$$

- Exporter i cannot compete when $p < \underline{p}_{ij}^k$
- Recall: $\underline{p}_{ij}^k$ increasing in index i

Benchmark: Standard Gravity

- If we let $\bar{\varphi}_i^k \rightarrow \infty \forall i$, we obtain EK trade shares

$$\frac{X_{ij}^k}{X_j^k} = \frac{T_i^k \left(w_i \tau_{ij}^k \right)^{-\theta}}{\sum_{i'=1}^N T_{i'}^k \left(w_{i'} \tau_{i'j}^k \right)^{-\theta}},$$

where $X_j^k = \sum_{i=1}^N X_{ij}^k$

- **Note:** holds irrespective of $\bar{p}_j^k$

► Elasticity

► Piecewise Gravity

Equilibrium Conditions

- Implicit expression for $\bar{p}_j^k$ [► Details](#)

$$\frac{w_j L_j}{G_j^k(\bar{p}_j^k)} = \left(\xi_j^k\right)^{-1} \left(\bar{p}_j^k\right)^\sigma \left[\sum_{k'=1}^K \xi_j^{k'} \left(\bar{p}_j^{k'}\right)^{-\sigma} \frac{X_j^{k'}}{G_j^{k'}(\bar{p}_j^{k'})} \right]$$

- Balanced trade pins down w_j

$$\sum_{i \neq j}^N \sum_{k=1}^K X_{ij}^k - X_{ji}^k = 0$$

Calibration (i/ii)

- Solve and calibrate model using equilibrium conditions
 - Set $T_i^k, \bar{\varphi}_i^k, \tau_{ij}^k$ to minimize deviations betw. empirical and model trade flows, letting $\tau_{ij}^k = \tau_{ij} \tau_j^k$
 - Benchmark model: $\bar{\varphi} \rightarrow \infty$
- Data (80 countries, 1,240 industries):
 - Bilateral 4-digit HS-industry-level trade flows ([Atlas of Economic Complexity](#))
 - Aggregate production ([ITPD-E](#))
 - Population ([WDI](#))

Calibration (ii/ii)

- Set:
 - $\sigma = 2, \theta = 4.1$
 - ξ_j^k = product k 's trade share
 - $w_j = X_j/L_j$, with $\sum_{j=1}^N w_j = 1$
- Moments (min. rel. deviations betw. data and model):
 - $\bar{\varphi}_i^k$: no. of served destinations in i - k cell
 - T_i^k : total exports in i - k cell
 - τ_j^k : total imports in j - k cell
 - τ_{ij} : total trade in i - j cell
- ⇒ Solve for $\bar{p}_j^k$ after every iteration
- ⇒ $\approx 300k$ parameters, 7.8M observations

Counterfactual Exercises

- What happens when global trade barriers, τ , change?
 - ⇒ Bilateral zeros change by approx. half the shock size [▶ Table](#)
 - ⇒ Welfare changes after trade-cost shocks typically amplified by extensive margin
 - ▶ Welfare: $\tau \times 0.9$
 - ▶ Welfare: $\tau \times 1.1$
 - Primarily true for low- to medium-income countries

Conclusion

- To create industry-level zeros in Ricardian model, allow for...
 - ...finite productivity levels of most productive firms
 - ...intermediate inputs to be substitutable along extensive margin

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 - Especially for small- to medium-sized economies

Conclusion

- To create industry-level zeros in Ricardian model, allow for...
 - ...finite productivity levels of most productive firms
 - ...intermediate inputs to be substitutable along extensive margin
- Accounting for extensive margin seems important for welfare analyses
 - Especially for small- to medium-sized economies
- Many industry-level trade relations seem relatively weak

References (i/ii)

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Appendix: Distribution of Prices

- Price distribution in country j :

$$G_j^k(p) = \begin{cases} 1 - \gamma_j^{k,1:n} \exp\left(-p^\theta \Phi_j^{k,1:n}\right) & \text{for } p \in \left[\underline{p}_{nj}^k, \underline{p}_{(n+1)j}^k\right) \\ 0 & \text{otherwise,} \end{cases}$$

where

$$\gamma_j^{k,1:n} := \exp\left(\sum_{i=1}^n T_i^k (\bar{\varphi}_i^k)^{-\theta}\right)$$

$$\Phi_j^{k,1:n} := \sum_{i=1}^n T_i^k (w_i \tau_{ij}^k)^{-\theta}$$

► Main text

Appendix: Detailed Derivations (i/iii)

- Mass of varieties sourced from i

$$\begin{aligned}
 m_{ij}^k &\equiv \mathbb{1} \left\{ \bar{p}_j^k \geq \underline{p}_{ij}^k \right\} \times \int_{\underline{p}_{ij}^k}^{\bar{p}_j^k} \prod_{i' \neq i} [1 - G_{i'j}^k(p)] g_{ij}^k(p) dp \\
 &= T_i^k (w_i \tau_{ij}^k)^{-\theta} \\
 &\quad \times \sum_{n=i}^N \frac{\gamma_j^{k,1:n}}{\Phi_j^{k,1:n}} \left[\exp \left(- \left[\min \{ \underline{p}_{nj}^k, \bar{p}_j^k \} \right]^\theta \Phi_j^{k,1:n} \right) - \exp \left(- \left[\min \{ \underline{p}_{(n+1)j}^k, \bar{p}_j^k \} \right]^\theta \Phi_j^{k,1:n} \right) \right] \\
 m_j^k &\equiv G_j^k(\bar{p}_j^k)
 \end{aligned}$$

► Main text

Appendix: Detailed Derivations (ii/iii)

- Expected price conditional on i being lowest-cost provider

$$\begin{aligned} & \frac{1}{m_{ij}^k + \mathbb{1}\{m_{ij}^k = 0\}} \sum_{n=i}^N \gamma_j^{k,1:n} \frac{T_i^k (w_i \tau_{ij}^k)^{-\theta}}{\Phi_j^{k,1:n}} \int_{\min\{\underline{p}_{nj}^k, \bar{p}_j^k\}}^{\min\{\underline{p}_{(n+1)j}^k, \bar{p}_j^k\}} p \Phi_j^{k,1:n} \exp(-p^\theta \Phi_j^{k,1:n}) \theta p^{\theta-1} dp \\ &= \frac{1}{m_{ij}^k + \mathbb{1}\{m_{ij}^k = 0\}} \sum_{n=i}^N \frac{T_i^k (w_i \tau_{ij}^k)^{-\theta}}{\Phi_j^{k,1:n}} \gamma_j^{k,1:n} \left(\Phi_j^{k,1:n} \right)^{-1/\theta} \tilde{\Gamma} \left(\min\{\underline{p}_{nj}^k, \bar{p}_j^k\}, \min\{\underline{p}_{(n+1)j}^k, \bar{p}_j^k\} \right) \end{aligned}$$

⇒ Expression above multiplied by m_{ij}^k yields X_{ij}^k

► Main text

Appendix: Detailed Derivations (iii/iii)

- $\tilde{\Gamma}(\underline{x}, \bar{x})$ is defined as

$$\tilde{\Gamma}(\underline{x}, \bar{x}) \equiv \int_{\underline{x}^{\theta}\Phi}^{\bar{x}^{\theta}\Phi} x^{1/\theta} \exp(-x) dx$$

- $\tilde{\gamma}_j^{k,1:n} \equiv \gamma_j^{k,1:n} \left(\Phi_j^{k,1:n} \right)^{-1/\theta}$

► Main text

Appendix: Trade Elasticity

- Standard gravity (with $\bar{\varphi} \rightarrow \infty$) is log-additively separable

$$\log(X_{ij}^k) = \underbrace{\log(T_i^k w_i^{-\theta})}_{i\text{-specific}} + \underbrace{\log(X_j^k / \Phi_j^{k,1:N})}_{j\text{-specific}} - \theta \log(\tau_{ij}^k)$$

$\Rightarrow$ constant “trade elasticity”

► Main text

Appendix: Gravity Add-On

- Total expenditures on intermediate varieties in industry k

$$X_j^k = \sum_{n=1}^N \underbrace{\tilde{\Gamma} \left(\min\{\underline{p}_{nj}^k, \bar{p}_j^k\}, \min\{\underline{p}_{(n+1)j}^k, \bar{p}_j^k\} \right)}_{:= X_j^{k,(n)}} \tilde{\gamma}_j^{k,1:n}$$

$\Rightarrow$ bilateral trade flows given by

$$X_{ij}^k = \sum_{n=i}^N \frac{T_i^k (w_i \tau_{ij}^k)^{-\theta}}{\sum_{i'=1}^n T_{i'}^k (w_{i'} \tau_{i'j}^k)^{-\theta}} X_j^{k,(n)}$$

► Main text

Appendix: Equilibrium (i/ii)

- UMP: Lagrange function

$$\mathcal{L}(\{\bar{\omega}_j^k\}_k, \lambda_j) = \frac{\sigma}{\sigma-1} \sum_{k=1}^K (\xi_j^k)^{\frac{1}{\sigma}} \left(\int_0^{\bar{\omega}_j^k} d\omega^k \right)^{\frac{\sigma-1}{\sigma}} + \lambda_j \left[E_j - \sum_{k=1}^K \int_0^{\bar{\omega}_j^k} p_j^k(\omega^k) d\omega^k \right]$$

⇒ yields expenditure shares

$$P_j^k q_j^k = E_j \frac{\xi_j^k (\bar{p}_j^k)^{-\sigma} P_j^k}{\sum_{k'=1}^K \xi_j^{k'} (\bar{p}_j^{k'})^{-\sigma} P_j^{k'}},$$

where

$$P_j^k := \frac{1}{q_j^k} \int_0^{\bar{\omega}_j^k} p_j^k(\omega^k) d\omega^k.$$

► Main text

Appendix: Equilibrium (ii/ii)

- In equilibrium, we have $Q_j^k = G_j^k(\bar{p}_j^k)L_j$ and $E_j = w_j$.
- Using $P_j^k = X_j^k/Q_j^k$ yields

$$\frac{w_j L_j}{G_j^k(\bar{p}_j^k)} = (\xi_j^k)^{-1} (\bar{p}_j^k)^\sigma \left[\sum_{k'=1}^K \xi_j^{k'} (\bar{p}_j^{k'})^{-\sigma} \frac{X_j^{k'}}{G_j^{k'}(\bar{p}_j^{k'})} \right]$$

$\Rightarrow$ pins down $\bar{p}_j^k$ for given w_j and parameters.

► Main text

Appendix: Model Fit

The Extensive Margin of Trade – Data vs. Model

Panel A: Data	Min.	Q1	Median	Q3	Max.
Sh. Served Destinations ⁽ⁱ⁾	0.01	0.1	0.3	0.67	1
Sh. Source Countries ⁽ⁱⁱ⁾	0.01	0.2	0.35	0.53	1
No. of Observations	7,836,800				
Panel B: Model	Min.	Q1	Median	Q3	Max.
Sh. Served Destinations ⁽ⁱ⁾	0.01	0.09	0.27	0.65	1
Sh. Source Countries ⁽ⁱⁱ⁾	0.01	0.19	0.37	0.58	1
No. of Observations	7,836,800				

Notes. This table depicts the distribution of industry-level export and import extensive margins. Panel A reports the distribution in the data, Panel B the distribution in the model.

⁽ⁱ⁾ Conditional on exporting in a given industry.

⁽ⁱⁱ⁾ Conditional on importing in a given industry.

► Main text

Appendix: Parameter Correlations (i/ii)

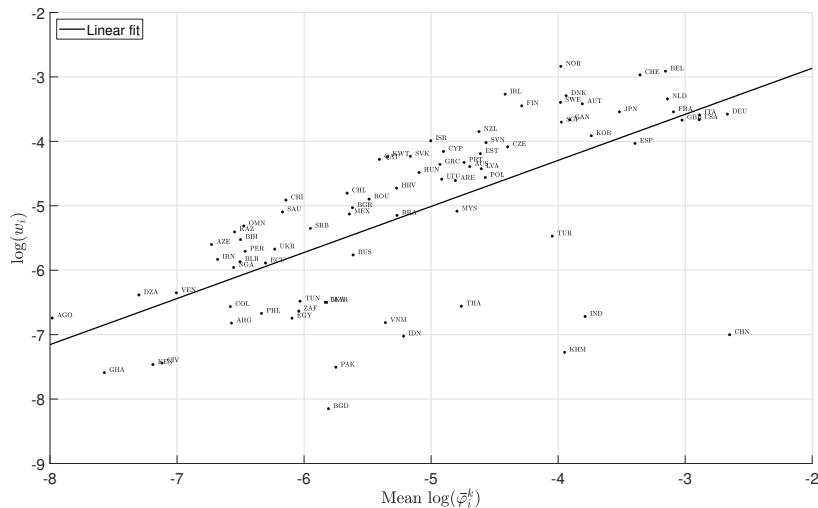
Regression of $-\theta \log(\tau_{ij})$ on Gravity Variables

	Coefficients (SE)	
	(1)	(2)
Log of Distance	-0.642 (0.061)	-0.475 (0.065)
Contiguity	1.468 (0.220)	1.676 (0.217)
Colonial Ties	1.306 (0.302)	0.938 (0.236)
Common Language	0.370 (0.192)	0.633 (0.168)
RTA	0.859 (0.119)	0.356 (0.118)
Imp. & Exp. FE		✓
No. of observations	6,162	6,162

Notes. This table reports regressions of the log of the average τ_{ij}^k on a set of bilateral variables. Robust SEs in parentheses.

► Main text

Appendix: Parameter Correlations (ii/ii)


[► Main text](#)

Appendix: Counterfactuals: Changes in Zeros

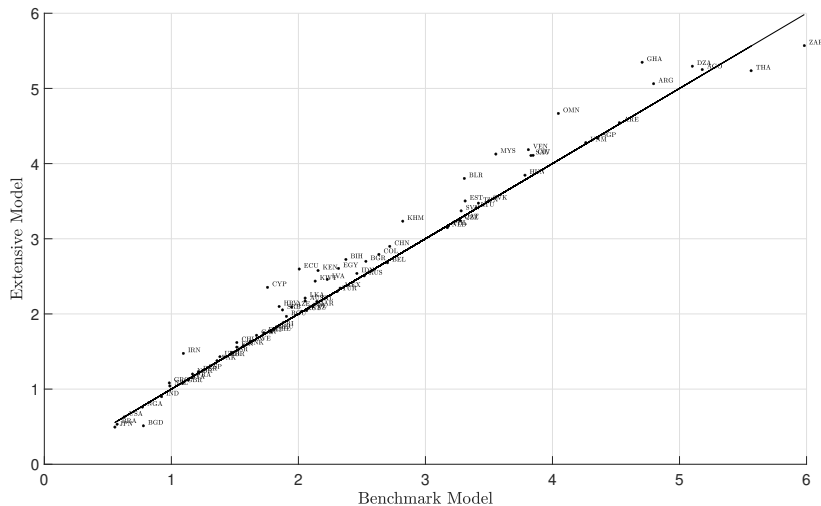
Bilateral Zeros after Changes in Global Trade Costs

%-Change in τ	%-Change in Bil. Zeros Rel. to Baseline	%-Change in Trade (through Zeros)
−5%	−3%	0.09%
−10%	−5.9%	0.35%
−15%	−8.9%	0.8%
+5%	2.9%	−0.09%
+10%	5.4%	−0.26%
+15%	7.7%	−0.48%

Notes. This table reports how bilateral industry-level trade zeros change upon a shock to trade barriers.

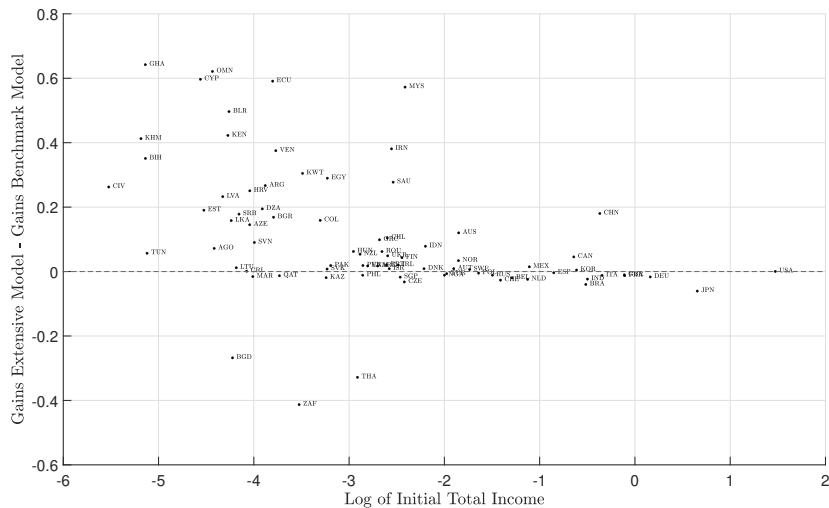
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Appendix: Welfare Changes: $\tau \times 0.9$ (i/iii)

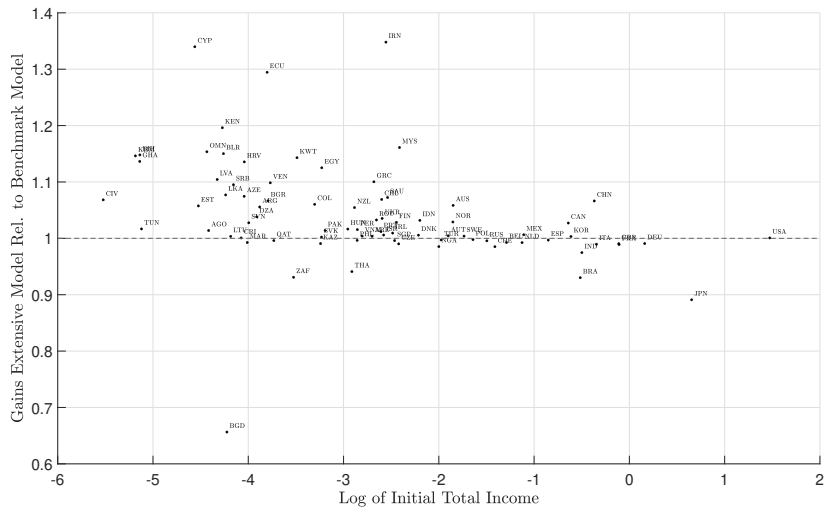


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Appendix: Welfare Changes: $\tau \times 0.9$ (ii/iii)

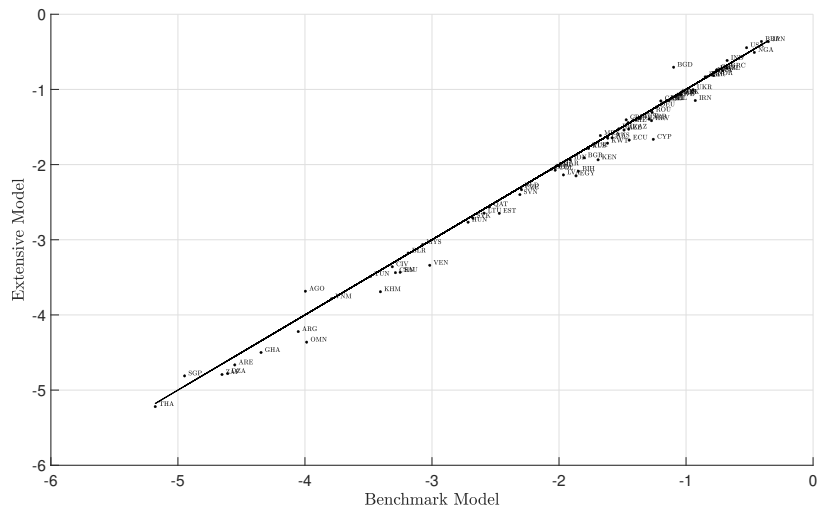

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Appendix: Welfare Changes: $\tau \times 0.9$ (iii/iii)

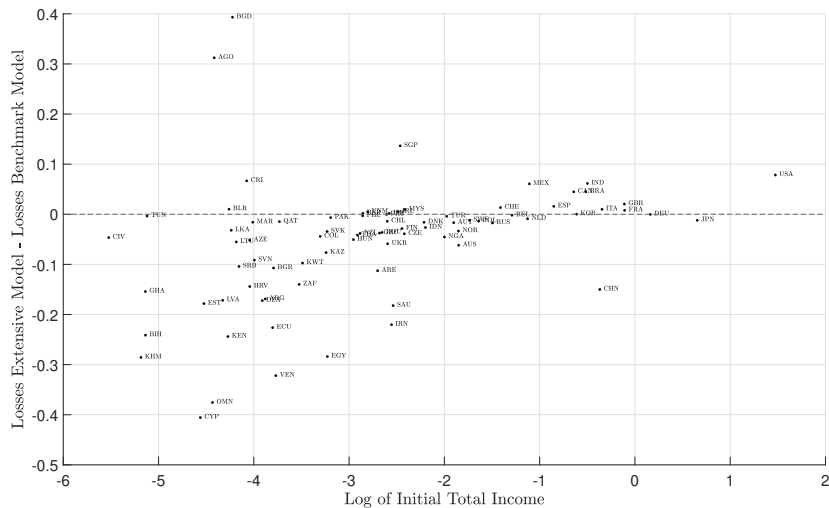


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Appendix: Welfare Changes: $\tau \times 1.1$ (i/iii)

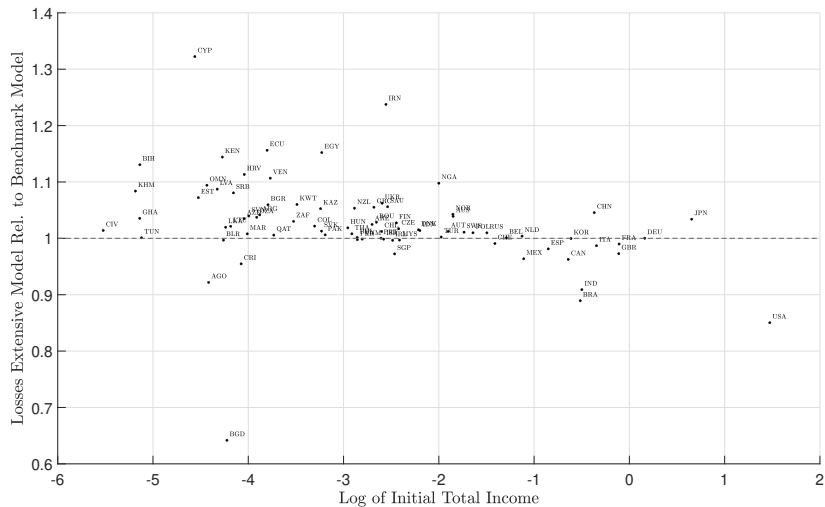

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Appendix: Welfare Changes: $\tau \times 1.1$ (ii/iii)



► Main text

Appendix: Welfare Changes: $\tau \times 1.1$ (iii/iii)



► Main text