

Institute for Operations Research
and Computational Finance



University of St. Gallen

Modeling the Rigidity of the Client Rate of Non-maturing Accounts

Computational Management Science Conference,
Vienna, July 2010

Florentina Paraschiv



Agenda

- Problem statement: Non-maturing Accounts (NMA)
- Motivation
- Client rate model specification
- Case study
- Results
- Conclusion

Simplified bank balance and variable products

A	L	
Variable positions: • overdraft facilities • non-fixed mortgages • other variable loans	Variable positions: • savings accounts • sight deposits	
	Fixed-income positions: • credits • fix-rate mortgages • bonds	Fixed-income positions: • savings certificates • issued bonds
	Other assets	Other liabilities
		<i>Equity capital</i>

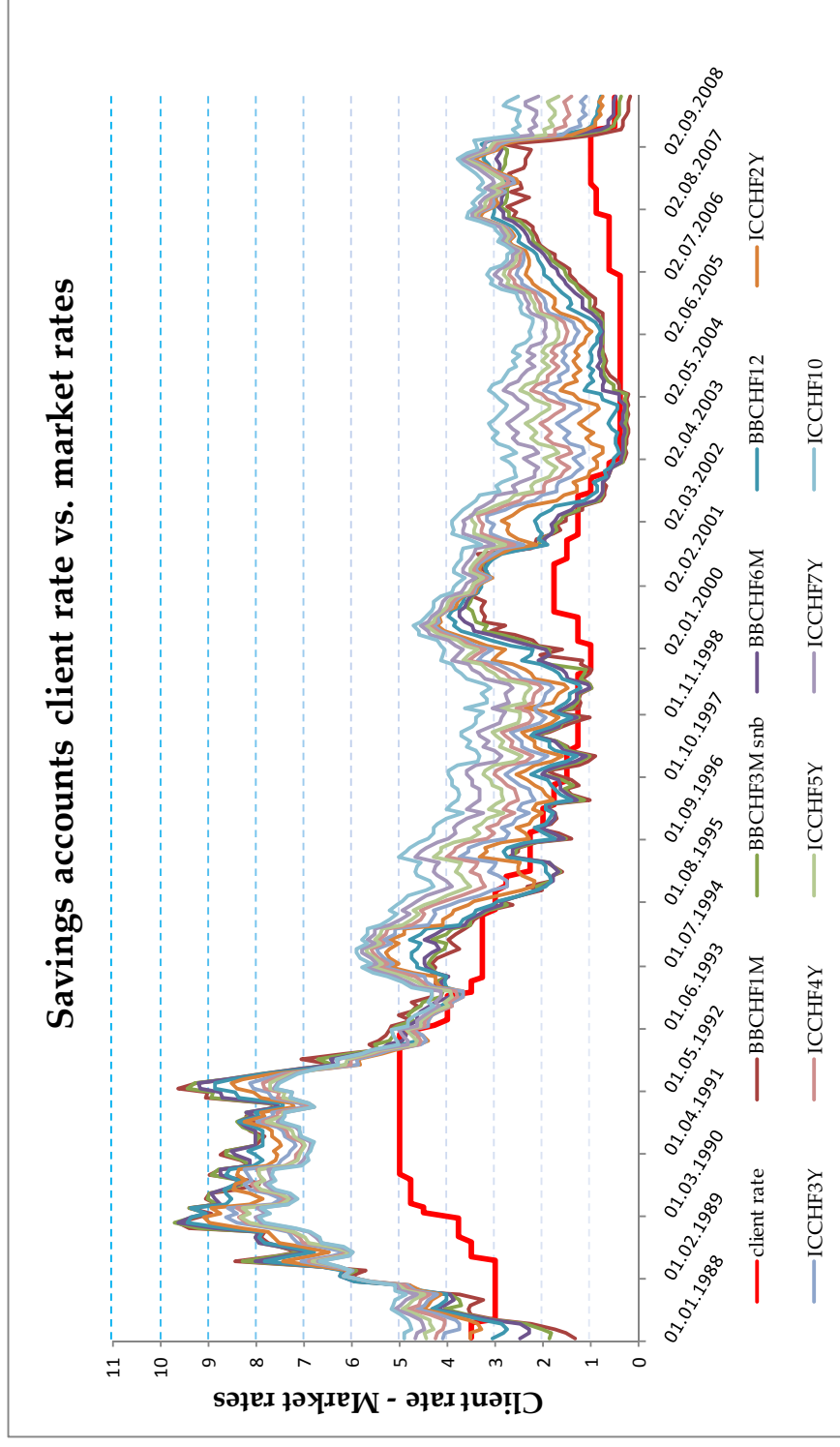
Variable banking products:

- Contractual maturity not specified
- Two options:
 - Clients: option to withdraw liquidity or prepay credits/mortgages
 - Bank: option to adjust client rate
- Future cash flows are uncertain

Modeling NMA

- Dynamic modeling and optimization of NMA
- Valuation models
- Balance sheet management
- Multistage stochastic programming model that determines an optimal replicating portfolio from scenarios for future outcomes of the relevant risk factors:
 - market rates
 - client rates
 - volumes
- Realistic client rate models are required

Deposit rate individual banks (source SNB)



Client rate characteristics:

- asymmetric adjustment to the market rates
- **rigidity** (delay in the client rate's adjustment to market rates)
 - small number of positive changes
 - large number of "no-changes"

Discussion

- Tendency of deposit rate to remain unchanged while exogenous rates fluctuate; therefore, the usual least-square estimator is inappropriate (biased and inconsistent)
 - the client rate must be represented as a limited dependent variable whose value is at times unrelated to exogenous rates
- Modeling client rate rigidity by a **friction model**
 - on **short-term** horizon, the friction models reflect the non-linearity of banks' behavior in adjusting the deposit/mortgages rates to exogenous rates
 - at the same time, the model allows on a **long-term** a flux of deposit/mortgage rate adjustment to market rates

Rigidity in the Literature

- Burger (1998), Leithner/Michaelsen (1993)
 - ✓ model the discrete changes in the mortgage rate (+50bp, +25bp, 0, -25 bp, -50 bp) which depend on the net margin (difference between the mortgage rate and the refinancing rate and the costs of the bank)
 - ✓ an increase/decrease in the mortgage rate occur if the cumulated loss/profit in the margin fall below/exceeds an upper/lower threshold
- Forbes/Mayne (1989):
 - ✓ proves the rigidity of the prime rate by modeling three different scenarios: positive changes, "no changes" and negative changes as functions of the spread between the prime rate and relevant market rates
 - ✓ more flexibility than the Leithner/Michaelsen model (less restrictive assumptions)

Friction model specification

$$\Delta D_t = \alpha_1 - \beta(D_{t-1} - R_t) + \varepsilon_t \quad \text{if } \alpha_1 - \beta(D_{t-1} - R_t) + \varepsilon_t > 0$$

$$\Delta D_t = 0 \quad \text{if } \alpha_1 - \beta(D_{t-1} - R_t) + \varepsilon_t < 0 \quad \text{and } \alpha_2 - \beta(D_{t-1} - R_t) + \varepsilon_t > 0$$

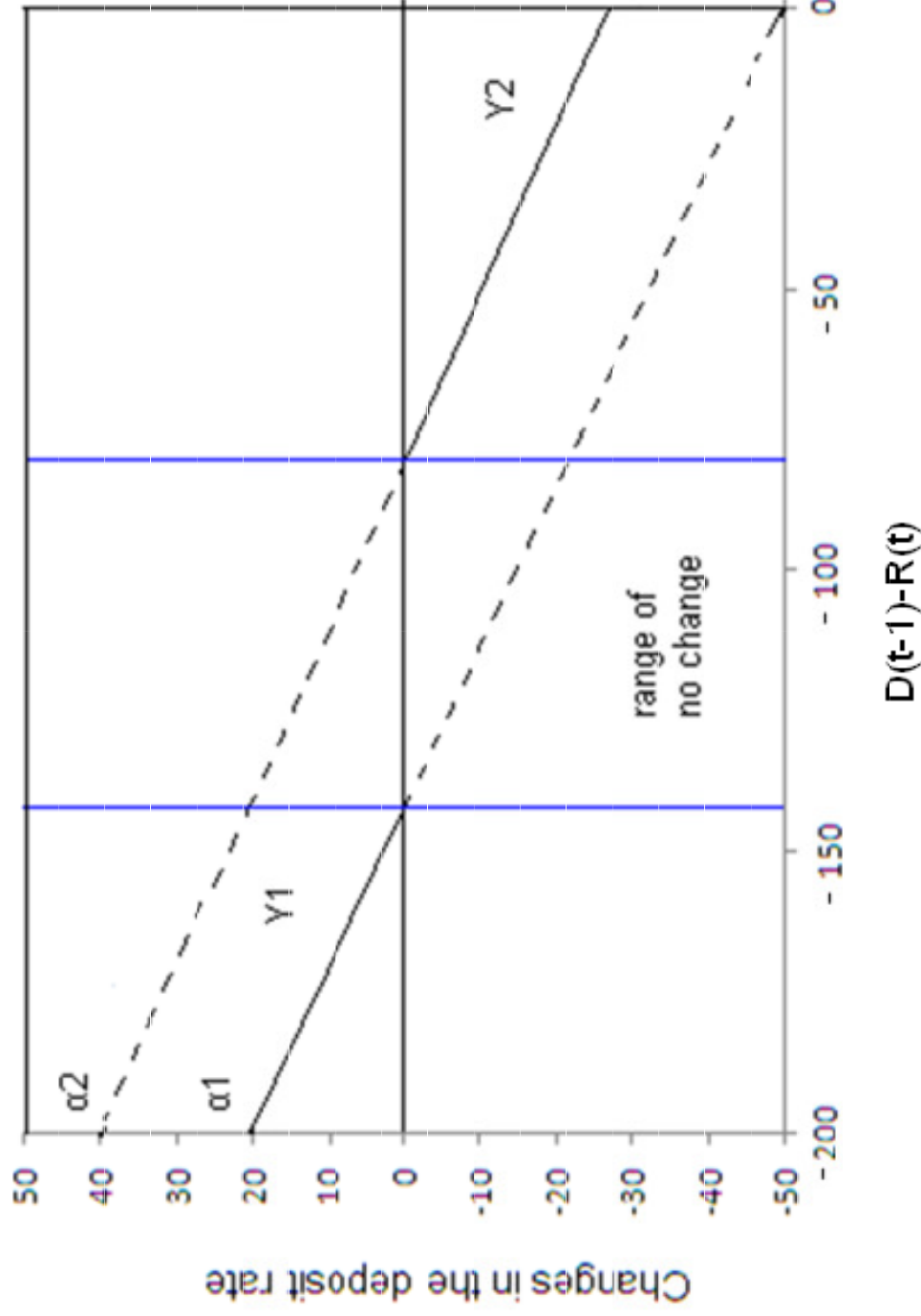
$$\Delta D_t = \alpha_2 - \beta(D_{t-1} - R_t) + \varepsilon_t \quad \text{if } \alpha_2 - \beta(D_{t-1} - R_t) + \varepsilon_t < 0$$

D = deposit rate; R = market rate

Interpretation:

- Changes in the rate are modeled, as this serves to focus attention on the incidence and on the magnitude of that which is noteworthy in the client rate: its relatively **infrequent changes**
- It is assumed that the desirability and the amount of any change are determined by a comparison of the spread between relevant exogenous rates and the previous period deposit rate
 - while the spread is in tolerable bounds, the deposit rate remains unchanged
 - outside the tolerable bounds, the deposit rate is adjusted to restore the "tolerable spread"

Graphical illustration of the rigidity model



Hypothetical relationship between changes in the deposit rate and its spread to the market rate

Likelihood Function

$$Y_t = \Delta D_t, \quad \phi X_{1t} = Y_{1t}, \quad \phi X_{2t} = Y_{2t}.$$

$$L = \prod_{n=1}^p \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2\sigma^2}(Y_n - \phi X_{1n})^2} \\ \times \prod_{m=1}^q [F(\phi X_{2m}, \sigma^2) - F(\phi X_{1m}, \sigma^2)] \\ \times \prod_{k=1}^r \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2\sigma^2}(Y_k - \phi X_{2k})^2}$$

where

$$F(\phi X_{jt}, \sigma^2) = \int_{-\infty}^{\phi X_{jt}} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2\sigma^2}(\lambda/\sigma)^2} d\lambda.$$

Let $F(\phi X_{jt}, \sigma^2) = F_{jt}$. Then the log-likelihood function is:

$$\log(L) = \sum_{m=1}^q \log(F_{2m} - F_{1m}) - \frac{(p+r)}{2} \log 2\pi\sigma^2 \\ - \left(\frac{1}{2}\sigma^{-2} \right) \sum_{n=1}^p (Y_n - \phi X_{1n})^2 - \left(\frac{1}{2}\sigma^{-2} \right) \sum_{k=1}^r (Y_k - \phi X_{2k})^2.$$

Results

Parameter	1	2	3	4	5	6	7	8
$\alpha 1$ (std error)	-1.176 (0.050)	-2.374 (0.046)	-1.521 (0.003)	-1.953 (0.046)	-0.484 (0.048)	-1.178 (0.24)	-2.135 (0.006)	-2.344 (0.04)
$\alpha 2$	-0.503 (0.011)	-0.09 (0.009)	-0.211 (0.006)	-0.214 (0.009)	-0.045 (0.008)	-0.03 (0.003)	-0.072 (0.014)	-0.329 (0.06)
β	0.298 (0.017)	0.561 (0.051)	0.660 (0.036)	0.562 (0.045)	0.137 (0.037)	0.308 (0.02)	0.531 (0.078)	0.696 (0.033)
σ	0.432 (0.049)	0.547 (0.041)	0.661 (0.033)	0.403 (0.041)	0.185 (0.043)	0.353 (0.014)	0.474 (0.058)	0.452 (0.031)
$ \alpha 1 - \alpha 2 $	0.673	2.284	1.310	1.739	0.439	1.148	2.063	2.015



Interpretation of results

Finding 1: $\text{abs}(\alpha_1) > \text{abs}(\alpha_2)$

- visual inspection:
 - positive changes in the deposit rate occur when its absolute **spread** from the market rates considerably **increases**
 - by contrary ... negative changes occur when the deposit rate is too high (its absolute **spread** from the market rates is too **small**)

Finding 2: the difference between α_1 and α_2 is significantly different from zero, varying between 43 bp and 228 bp between the data sets

- This is consistent with a range of no-change bounded on one end by a smaller negative threshold for increases and on the other by a larger one for decreases

Finding 3: 5 out of the 8 investigated data sets show a high sensitivity of the deposit rate changes to changes in the spread between the deposit rate and the market rate

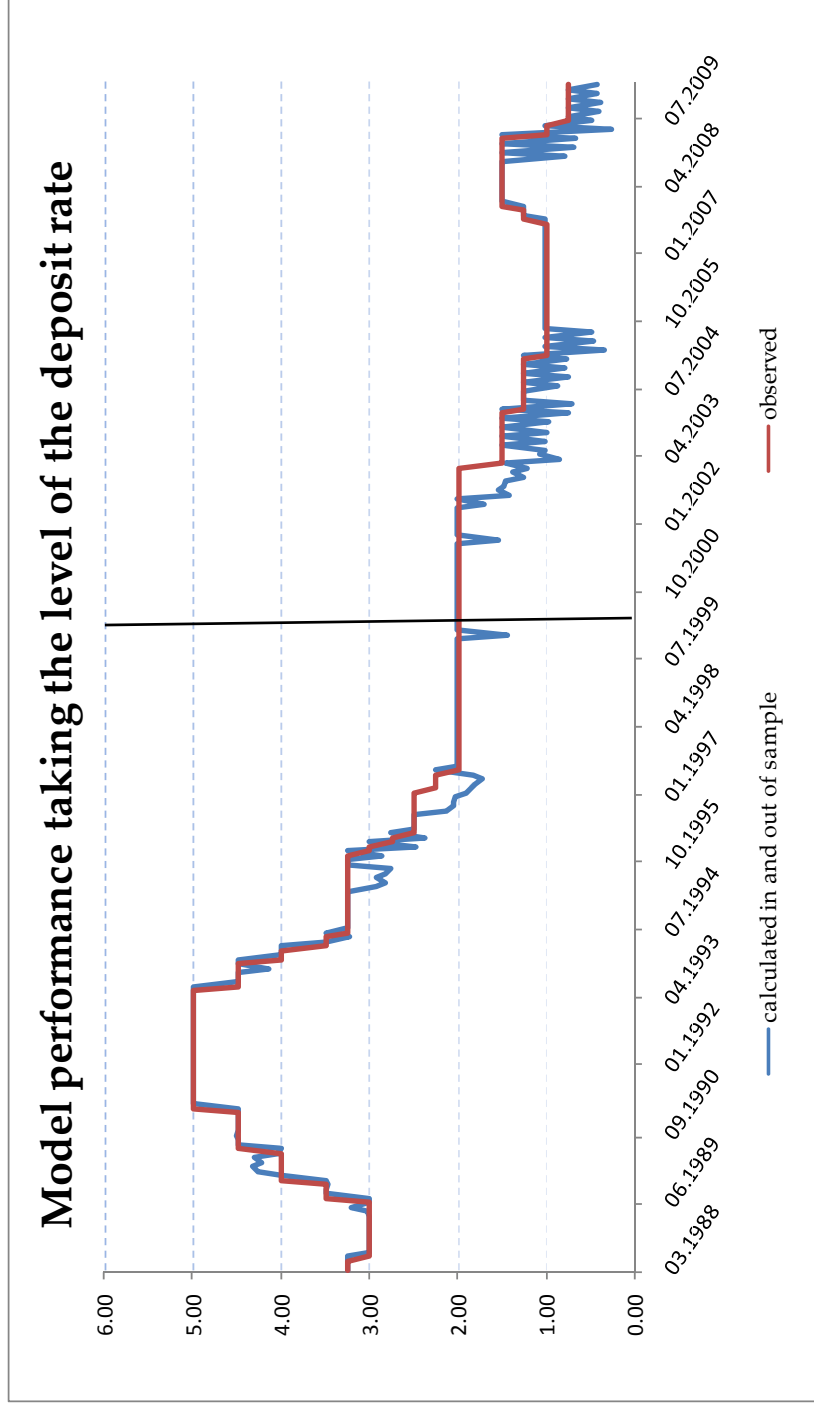
- Overall, for every 100 bp increase in the spread, banks adjust the client rate, on average, with 50 bp to close the gap.

Out of Sample Test – example

We re-estimate the coefficients of the client rate model using data from the beginning of the data sample up to Dec. 1999 and we calculate the out of sample fitted value (for the rest of the data sample)

Data set 3	Over all sample	Jan 1988 – Dec 1999
α_1 (std error)	-1.521 (0.003)	-1.873 (0.002)
α_2	-0.211 (0.006)	-0.258 (0.003)
β	0.660 (0.036)	0.740 (0.024)
σ	0.661 (0.033)	0.679 (0.018)
$ \alpha_1 - \alpha_2 $	1.310	1.615

Model performance



- The client rate rigidity model proves a realistic out-of-sample fit and it can be integrated in the general framework of NMA optimization tools

Conclusion

- The results show that the client rate is adjusted when its spread from the market rate is above some upper threshold (too large) or below some lower threshold (too small)
- While the spread is in tolerable bounds, the deposit rates remain unchanged
- The model reflects on one hand the short term client rate rigidity and on the other hand it reflects the long-term sensitivity to the market rates
- By accounting for rigidity, the friction model offers a good fit to the data in and out of sample, making it useful for the NMA risk management applications