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FRAGILITY OF MONEY MARKETS

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WORKING PAPERS ON FINANCE NO. 2016/01

SWISS INSTITUTE OF BANKING AND FINANCE (S/BF – HSG)

JANUARY 2016
THIS VERSION: APRIL 2016

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April 28, 2016

*We thank Marc Arnold, Martin Brown, Markus Brunnermeier, Antoine Martin, Cyril Monnet, Erwan Morellec, Kjell Nyborg, and Paul Söderlind for helpful comments. This work was supported by the Sinergia grant 154445 from the Swiss National Science Foundation.

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We provide the first comprehensive theoretical model for money markets encompassing unsecured and secured funding, asset markets, and central bank policy. In our model, leveraged banks invest in assets and raise short-term funds by borrowing in the unsecured and secured money markets. We derive how funding liquidity across money markets is related, explain how a shock to asset values can lead to mutually reinforcing liquidity spirals in both money markets, and show how borrowers' flight-to-safety and risk-seeking behavior impacts their liability structure. We derive the socially optimal leverage ratio and funding structure, and show which combination of conventional and unconventional monetary policies and regulatory measures can reduce money market fragility.

KEYWORDS: Money markets, unsecured and secured funding liquidity, liquidity spirals, monetary policy, regulation

JEL CODES: E43, E58, G01, G12, G21, G28

Being a major source of funding for financial intermediaries, money markets are at the heart of the financial system. Well-functioning money markets are crucial for financial stability and disruptions can have severe consequences even for the real economy. In times of immediate liquidity needs, financial institutions borrow in the unsecured money market, obtain funding in the secured (or "repo") market[1], or liquidate assets (Freixas, Laeven, and Peydró, 2015). If liquidity from all of these sources dries up simultaneously, bank failures can occur, which cause contagion and spillover effects throughout the financial system and urge central banks to intervene as the lender of last resort.

This paper provides the first comprehensive theoretical model that includes all major sources of short-term liquidity jointly. Our model provides a unified framework that explains (i) the intricate dynamics and contagion channels between unsecured and secured funding and security markets, (ii) cross-sectional differences and time-series variation in banks' unsecured and secured money market funding volumes and costs, and (iii) how central bank and regulatory policy impact money markets.

A comprehensive model of money markets is important for at least two reasons. First, we gain a thorough understanding of banks' funding risks only with an integrated view and joint modeling approach of all short-term liquidity sources. Since money market liquidity is determined by both secured and unsecured funding, fragility crucially depends on the interrelation between money markets. Second, the fragility of money markets contributed significantly to the global financial crisis (see, e.g., French et al., 2010). In its aftermath, central banks introduced various (unconventional) policies to alleviate funding strains and liquidity risk is at the center of regulators' efforts to reform the financial system. To perform such important tasks, policy makers need a comprehensive model to assess the immediate impact of new policies on different money market segments and, more generally, to understand the policy effects on future fragility of funding and asset markets.

The key feature of our model is that banks can borrow in the secured *and* unsecured money market.

[1] A repurchase agreement or "repo" is essentially a collateralized loan based on a simultaneous sale and forward agreement to repurchase securities at the maturity date. Throughout this paper, we use the terms secured funding, collateralized funding, and repo interchangeably.

In the secured market, borrowing is subject to margins (haircuts to the value of the collateral securities), and in the unsecured market the interest rate increases with the borrower's credit risk. Furthermore, our model includes a central bank conducting conventional and unconventional policy. In equilibrium, money market rates are linked to the central bank's benchmark rate and the spread between unsecured and secured interest rates is a risk premium which increases with borrowers' leverage.

We investigate money market fragility, in the sense that a small shock to the fundamental value of an asset can lead to a large, discontinuous drop in its market price with adverse feedback effects on funding markets. Such shocks occurred, for instance, in mortgage-backed securities during the subprime crisis in the United States or in sovereign bonds during the European sovereign debt crisis. If borrowers hold similar distressed assets, fragility can have market-wide effects.

We show that markets are not fragile, if borrowing in one of the funding markets is unconstrained. In this case, a loss of funding liquidity in the constrained market can be substituted in the unconstrained market. Thus, it is not sufficient to analyze funding sources in isolation as is done in prior research. In contrast, markets can be fragile and adverse liquidity spirals arise, when borrowers face funding problems in both the secured and the unsecured money market at the same time. We identify two new liquidity spirals, namely an interest rate and a loss spiral for unsecured funding. The former describes the increase in the interest rate due to higher leverage and counterparty credit risk, making it more costly to roll over existing positions. The loss spiral reinforces the interest rate spiral and describes the eroding effect on capital following an asset price shock, enforcing a deleveraging process and further downward pressure on prices.

We show that the liquidity spirals in the unsecured market arise simultaneously with funding problems in the secured market, where margins tend to increase after initial losses, which may trigger secured liquidity spirals (Gromb and Vayanos, 2002; Brunnermeier and Pedersen, 2009). Moreover, unsecured spirals and secured spirals mutually reinforce each other, and induce commonality in funding illiquidity

across money markets. For instance, when initial losses increase market illiquidity and margins, and exert further downward price pressure (loss and margin spirals), this increases leverage and thus unsecured interest rates rise (interest rate spiral). Hence, it is necessary to analyze unsecured and secured funding jointly to fully understand the intricate dynamics and contagion channels in money markets. These mechanisms and interrelations in our model are summarized in Figure 1, which shows the interest rate spiral in the unsecured money market, the margin spiral in the secured money market, and the combined loss spiral, including feedback effects from both money markets.

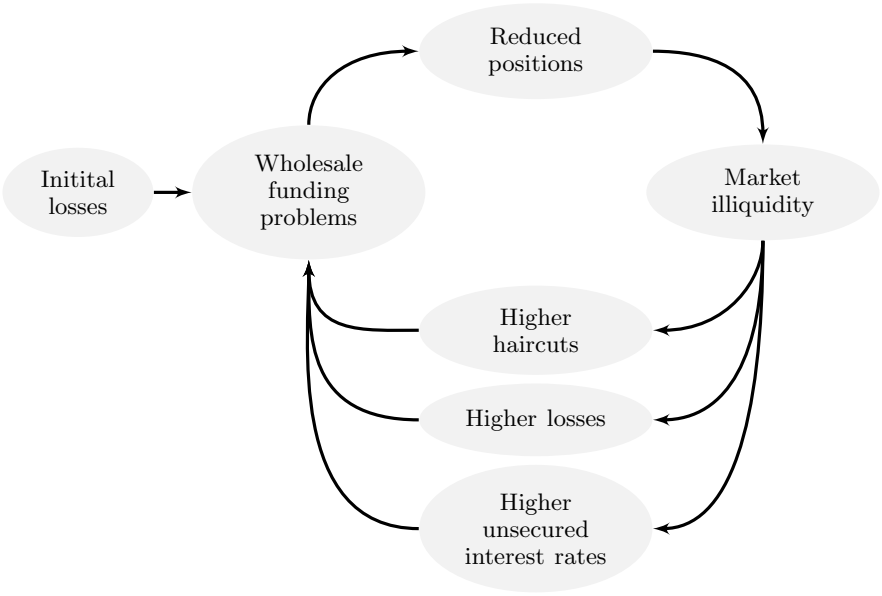

Figure 1. Liquidity Spirals in Money Markets
The figure shows the interest-rate spiral in the unsecured money market (outer circle), the margin spiral in the secured money market (Brunnermeier and Pedersen (2009), inner circle), and the combined loss spiral, including feedback effects from both money markets. Spirals start when initial losses lead to funding problems, i.e., borrowers' funding constraints are binding in both the unsecured and the secured money market at the same time.

In equilibrium, these liquidity spiral dynamics lead to a re-allocation from secured to unsecured funding. Since banks are capital-constrained, spirals continue until deleveraging relaxes the (endogenous) financial constraint in the unsecured market sufficiently to allow for a substitution of liquidity. Our model brings to light a dual role of haircuts for money market fragility. While higher margins reduce the funding volume in the secured money market, they relax the funding constraint in the unsecured money

market by lowering leverage and thus counterparty credit risk. By alleviating the interest rate spiral, higher margins eventually cause an increase in the equilibrium share of unsecured funding.

Indeed, we show that the substitutability of funding liquidity critically depends on borrowers' initial leverage. The higher is borrowers' leverage at the time of the shock, the more difficult it is to substitute a loss of secured funding liquidity in the unsecured market. This happens because the shadow costs of capital in the secured market represent the marginal funding costs in the unsecured market when the capital constraint is binding. As banks can borrow less in the secured market after margins increased, high initial leverage might even cause capital to erode before the bank is able to tap unsecured funding at sufficiently low costs. In this extreme situation, the unsecured market freezes and banks default.

Moreover, our model highlights the connection between a bank's funding structure and the liquidity risk of its asset portfolio. Due to relatively higher funding costs, assets with higher liquidity risk are funded more in the unsecured market, whereas low liquidity-risk assets are funded predominantly in the secured market. Hence, when market participants re-allocate their asset positions towards safer assets, the corresponding effect on the liability side is a flight-to-secured-funding. In turn, risk-seeking behavior of borrowers trying to exploit the opportunity to profit from illiquidity and higher expected returns on distressed assets is mirrored on the liability side by an increase in the share of unsecured funding.

Having established the market dynamics, we derive the socially optimal leverage ratio and funding structure, which are characterized by equal shadow costs of capital in the secured and unsecured market. Equal shadow costs mitigate the risk of liquidity spirals and facilitate liquidity substitution. We show that central bank monetary policy can restore the socially optimal level of funding liquidity by an efficient combination of conventional policy (i.e., interest-rate policy), and unconventional measures, namely purchasing assets (quantitative easing) or changing the haircuts for collateralized lending by the central bank. For instance, lower interest rates loosen the funding constraint in the unsecured market, whereas offering refinancing facilities with lower haircuts than in the private market eases funding constraints in

the secured market. Such haircut policy allows banks to hold on to these assets and can thus prevent fire-sales, liquidity spirals, and fragility. However, it does not address banks' excess leverage and may actually create future fragility, in the sense that the shadow cost differential between the unsecured and secured market widens, which makes it more difficult for banks to flexibly adjust their funding structure.

In addition to our analysis of monetary policy, we analyze the effects of the main, recently proposed regulatory measures (e.g., Dodd-Frank Act and Basel III), namely countercyclical capital buffers, leverage ratios, and liquidity coverage ratios. Our model delivers two important results for policy makers and regulators: First, policy should aim to prevent that financial institutions are constrained in both the secured and unsecured funding market at the same time. In fact, as long as banks are able to borrow in one of the two money market segments, they can substitute secured and unsecured funds, thereby preventing money market fragility. Second, static measures such as constant maximum leverage ratios may worsen money market fragility. As leverage increases after an asset shock, a constant maximum leverage cap inhibits banks from efficiently substituting liquidity, causing asset sales and market illiquidity. In contrast, countercyclical measures can preempt the adverse consequences of leverage cycles, i.e., excess leverage and funding liquidity in "good" times and excess deleveraging and illiquidity in "bad" times. As implied by our model, countercyclical measures inversely related to haircuts would reduce this procyclicality and money market fragility.

Our model is consistent with well-known stylized facts and provides several new testable implications, including that (i) an increase in margins leads to a higher share of unsecured funding, (ii) flight-to-safety induces flight-to-secured-funding, (iii) higher initial leverage causes a stronger reduction in total asset holdings after a shock, and (iv) central bank liquidity provision counteracts asset sales. We perform a simple empirical analysis using bank-level data and the European sovereign debt crisis as an example of an asset shock. Despite the limited number of observations, the empirical results support the main predictions of our model.

Our paper is related to two streams of the literature. First, prior theoretical research proposes models for secured funding, e.g., Acharya, Gale, and Yorulmazer (2011), and Martin, Skeie, and von Thadden (2014). Similar to the models of Grossman and Miller (1988), Gromb and Vayanos (2002), and Brunnermeier and Pedersen (2009) in this stream, our model contains a link between market liquidity and funding liquidity. A common feature of our model and Ashcraft, Gârleanu, and Pedersen (2011) and Koulischer and Struyven (2014) is that it includes the relation between secured funding and central bank liquidity. The second stream of the literature focuses on unsecured money markets, e.g., Acharya and Skeie (2011), Acharya and Viswanathan (2011), and Heider, Hoerova, and Holthausen (2015). The connection between unsecured funding and central bank liquidity is modeled in, e.g., Allen, Carletti, and Gale (2009), Freixas, Martin, and Skeie (2011), and Acharya and Tuckman (2014).[2] None of the previous papers proposes a unified theory encompassing unsecured and secured money markets, asset markets, and central bank policy as we do in this paper.

1. The Model

1.1. Assets

There are J assets traded at times $t = 0, 1, 2, 3$. Each asset $j \in J$ pays a final cash flow at time $T = 3$ equal to the sum of income ω_t^j generated in each period, $\sum_{t=0}^{T} \omega_t^j$, evolving as

$$\omega_t^j = \omega^j + \Theta_t^j, \tag{1}$$

where $\omega^j > 0$ is a random variable defined on a probability space $(\Omega, \mathcal{F}, \mathbb{P})$. Shock $\Theta_t^j = \sigma_t^j \varepsilon_t^j$ is normally distributed with volatility σ_t^j and $\varepsilon_t^j \overset{iid}{\sim} \mathcal{N}(0,1)$.[3] Assets are in zero aggregate supply and the risk-free

[2]Starting with Poole (1968), a number of papers model banks' central bank reserves management. Recent contributions linking central bank policy and the interbank market are, e.g., Afonso and Lagos (2015) and Bech and Monnet (2016).

[3]This notation reflects *dirty* prices of coupon bonds, zero-coupon bonds, or any other income-paying security (e.g., Gromb and Vayanos, 2002). All ε_t^j have a standard normal cumulative distribution function Φ and are uncorrelated across time and assets.

rate is normalized to zero. Hence, the fundamental value of each asset is the conditional expected value of the final payoff, $\nu_t^j = \mathbb{E}_t\left[\sum_{\tau=0}^{T}\omega_\tau^j\right]$, and evolves according to

$$\nu_{t+1}^j = \mathbb{E}_{t+1}\left[\sum_{\tau=0}^{T}\omega_\tau^j\right] = \nu_t^j + \omega^j + \Theta_{t+1}^j. \tag{2}$$

Consistent with financial data, fundamental volatility σ_t^j has autoregressive conditional heteroskedasticity (ARCH) dynamics

$$\sigma_{t+1}^j = \underline{\sigma}^j + \delta^j|\Theta_t^j|, \tag{3}$$

with $\underline{\sigma}^j > 0$, and $0 < \delta^j < 1$ implying that future volatility increases after a shock to the fundamental value and the process is stationary.[4]

Akin to the commonly used measure of transaction costs called effective spread (e.g., Roll, 1984), we let market illiquidity be the absolute deviation of an asset's market price p_t^j from its fundamental value, $|\Lambda_t^j| = |p_t^j - \nu_t^j|$, and denote $r_t^j = \frac{\Delta p_t^j}{p_{t-1}^j}$, such that an asset's expected market return ϕ_t^j in the next period is given by $\phi_t^j = \mathbb{E}_t\left[r_{t+1}^j\right]$.

1.2. Agents

There are three types of agents in the economy, namely customers, banks with liquidity surplus ("lenders"), and banks with liquidity deficit ("borrowers").[5]

Customers. There are three risk-averse customers $k = 0, 1, 2$ in the economy, with initial cash $W_0^k > 0$, asset holdings $\mathbf{y}_t^k = 0$ before arriving in the market, and risk aversion coefficient $\gamma > 0$. At time 0, each customer learns about a random endowment shock of $\mathbf{z}^k = \{z^{1,k}, \ldots, z^{J,k}\}$ shares of an asset that

[4]The model can easily by generalized using other (G)ARCH dynamics. Without loss of generality, we choose the dynamics in Equation (3), in line with Brunnermeier and Pedersen (2009).

[5]In Gromb and Vayanos (2002), "arbitrageurs" are borrowers and customers are called investors. In Brunnermeier and Pedersen (2009), lenders and borrowers are referred to as "financiers" and "speculators", respectively. We use the terms "borrowers" and "lenders" to highlight the generality of our model.

he will face at time 3. These shocks represent binding orders which the customers must execute in the market until time T and are tied to the payout of ω_t^j. Since aggregate supply is zero, all shocks across customers aggregate to zero, $\sum_{k=0}^{2} z^{j,k} = 0$.

With probability $(1-a)$, all customers arrive in the market at time 0, and with probability a, customer $k = 1$ arrives at time $t = 1$ and customer 2 at time 2. Without loss of generality, we assume that when there is sequential arrival, aggregate order imbalances at dates $\tau = 0, 1$ are such that $\mathbf{Z}_\tau := \sum_{k=0}^{\tau} \mathbf{z}^k > 0$, and customer 2 takes the opposite position at time 2 and $Z_2 = 0$. Customers have exponential utility $U(W_T^k) = -\exp(-\gamma W_T^k)$ and choose their optimal positions $\mathbf{y}_t$ by maximizing utility over final wealth W_T^k, i.e.,

$$\max_{\mathbf{y}_t} -\mathbb{E}_t \left[e^{-\gamma W_T^k} \right], \tag{4}$$

subject to their budget constraint

$$W_{t+1}^k = W_t^k + (\Delta \mathbf{p}_{t+1} - \boldsymbol{\omega}_t)'(\mathbf{y}_t^k + \mathbf{z}^k). \tag{5}$$

Intuitively, a customer's wealth declines when $y_t^k + z^k > 0$ and the market is illiquid such that $\Delta p_t < \omega_t$. Conversely, customers make a profit if they sell assets at a higher price than the required payout. Finally, we consider securities for which aggregate order imbalances are fairly rarely observed, meaning that $a \to 0$.[6]

Lenders. Banks are risk-neutral and differ with respect to their cash endowment. Lenders (indexed "l" where necessary to distinguish them from borrowers) have capital $W_0^l > 0$ and face a positive liquidity shock that translates into surplus funds $D_0 > 0$. Lenders have to pay liquidity costs for these funds, which are equal to the central bank's refinancing interest rate, i.e., the benchmark rate in the economy, $c_t^l = i_{cb}$. In reality, D_0 could represent a sudden inflow of deposits for which lenders have to pay retail

[6]The case of $a > 0$ is shown in Brunnermeier and Pedersen (2009) and holds equivalently in our model.

deposit rates that are related to central bank rates (see, e.g., Borio, Gambacorta, and Hofmann, 2015).

We assume lenders maximize expected wealth by allocating D_0 into a pool of liquid investments for one period.[7] Such investments include secured and unsecured money market loans $M_t = M_t^s + M_t^u$, "near-money" securities b, or storage at a central bank's deposit facility. For these central bank deposits, banks receive the risk-free rate i_d, which we normalize to zero. Near-money assets exhibit the lowest risk among all assets J, $\sigma_t^b = \inf\left\{\sigma_t^j : j \in J\right\}$, and are traded in fully liquid markets, allowing lenders to readily convert them into cash.[8]

Interbank loans are either collateralized or unsecured, with corresponding interest rates i_t^s and i_t^u, and expose the lender to risk. Unsecured loans expose lenders to counterparty credit risk, whereas secured loans expose lenders to the risk of the collateral security. Banks in our model have limited liability and are bankrupt when capital becomes zero. We denote the probability of a counterparty's default as $\theta \in [0, 1]$, which is a continuously increasing function of borrowers' leverage ratio L_t, i.e., $\frac{\partial \theta}{\partial L_t} > 0$. This specification is in line with structural models of firm default (starting with Merton, 1974) and empirical evidence that default risk increases with leverage (e.g., Collin-Dufresne, Goldstein, and Martin, 2001; Subrahmanyam, Tang, and Wang, 2014). Leverage is computed as borrowers' total assets A_t over capital W_t, $L_t = \frac{A_t}{W_t}$.

For secured loans, lenders protect themselves against the risk of the security by subtracting a margin $0 < m_t^j \leq p_t^j$ from the collateral's market value to determine the size of the loan. Lenders are uninformed about fundamental values and have information set $\mathcal{F}_t = \sigma\{\mathbf{p}_0, \ldots, \mathbf{p}_t, \boldsymbol{\omega}\}$, and set margins based on observed prices and volatility. As $a \to 0$, margins are set to cover the collateral's π-value-at-risk,

$$\pi = \Pr\left(-(\Delta p_{t+1}^j - \omega^j) > m_t^j | \mathcal{F}_t\right), \tag{6}$$

[7] For example, in the spirit of Allen and Gale (2000) or because lenders cannot verify the payoff of riskier and potentially more illiquid projects.

[8] Equivalently, "near-money" can mean eligible for refinancing involving almost no capital requirement.

such that margins are

$$m_t^j = \overline{\sigma}^j + \overline{\delta}^j |\Delta p_t^j - \omega^j| = \overline{\sigma}^j + \overline{\delta}^j |\Theta_t^j + \Delta\Lambda_t^j|, \tag{7}$$

where we define

$$\overline{\sigma}^j = \underline{\sigma}^j \Phi^{-1}(1-\pi),$$

$$\overline{\delta}^j = \delta^j \Phi^{-1}(1-\pi).$$

As in the real world, margins reflect an asset's past volatility and unexpected price movements. The intuition is that lenders expect $\Delta p_t = \omega$ in every period, because they do not observe fundamental dynamics ω_t^j. When price changes deviate from ω, lenders assume that this is due to a change in fundamentals, and increase margins to protect against higher risk. In relative terms, margins are defined as haircuts, $h_t^j \equiv \frac{m_t^j}{p_t^j}$, and since $0 < h_t^j \leq 1$, shocks to the fundamental value or market illiquidity increase the over-collateralization of secured loans and the protection against risk.

Overall, lenders' wealth evolves as

$$W_{t+1}^l = W_t^l + (1-\theta)[i_t^s M_t^s + i_t^u M_t^u] - \theta M_t^u + \phi_t^b (D_0 - M_t) - i_{cb} D_0. \tag{8}$$

Equation (8) states that lenders earn the interest on their money market loans if the borrower survives. We assume that in case of counterparty default, the lender does not receive the interest payment on the secured loan, but gets back the principle M_t^s. Since π is low, margins effectively internalize the costs of liquidating collateral assets in distressed markets. In contrast, unsecured loans to defaulted borrowers have a zero value to the lender in the short-run (Acharya and Skeie, 2011).[9] Thus, the lenders' optimization

[9]Note that our results hold irrespective of the exact value lenders receive from selling collateral in the market as long as secured creditors can expect higher reimbursement than unsecured creditors in case of counterparty default. For instance, when secured loans are traded, e.g., via a central clearing house (CCP), lenders are protected by several layers of defense undertaken by the clearing party to guarantee repayment (for a detailed description see Mancini, Ranaldo, and Wrampelmeyer, 2016). In contrast, unsecured loans are unavailable in the short-run until the counterparty's bankruptcy case is filed and claims are negotiated.

problem is given by

$$\max_{M_t^s, M_t^u} \mathbb{E}_t \left[W_{t+1}^l \right], \tag{9}$$

subject to the wealth dynamics in Equation (8).

Borrowers. Each borrower has initial capital $W_0 > 0$ and zero positions before entering the market at time 0. Borrowers act as financial intermediary and trade assets with the customers. They finance their positions by borrowing from lenders in the money market. Hence, the liquidity surplus D_0 is redistributed in the economy after endowment shocks $\mathbf{z}^k$ have realized at time 0 and trades are settled.[10]

Borrowers can fund their asset holdings either in the unsecured money market or in the secured money market. Positions funded in the secured and unsecured market are denoted by $x_t^{j,s}$ and $x_t^{j,u}$, respectively, and the total holding of a security j is $x_t^j = x_t^{j,s} + x_t^{j,u}$. Positions funded in the secured market are constrained by the amount of capital, W_t, that borrowers have available to satisfy the margin requirements set by the lenders:

$$\sum_j x_t^{j,s} m_t^j \leq W_t. \tag{10}$$

The secured funding volume totals $M_t^s = \sum_j (1 - h_t^j) x_t^{j,s} p_t^j$. The funding costs for each asset funded in the secured market are given by the haircut-weighted average of the interest rate i_t^s and equity costs e:

$$c_t^{j,s} = (1 - h_t^j) i_t^s + h_t^j e. \tag{11}$$

For $e > i_t^s$, we have that higher haircuts imply higher funding costs, $\frac{\partial c_t^{j,s}}{\partial h_t^j} > 0$.

In the unsecured market, the funding volume is given by $M_t^u = \sum_j x_t^{j,u} p_t^j$, and funding costs are equal to the interest rate, $c_t^{j,u} = i_t^u$. Combining secured and unsecured positions, borrowers' have wealth

[10] Thus, borrowers can be seen as speculators/arbitrageurs with trading capital W_t in the spirit of Gromb and Vayanos (2002) and Brunnermeier and Pedersen (2009), or, alternatively, as (commercial) banks managing their liquidity needs after facing a negative liquidity shock at time 0 (e.g., Martin, Skeie, and von Thadden, 2014). In line with the empirical evidence of Mitchell, Pedersen, and Pulvino (2007), we assume that borrowers cannot raise new capital in the short-run.

dynamics

$$W_t = W_{t-1} + (\mathbf{r}_t - \mathbf{c}^s_{t-1})'(\mathbf{x}^s_{t-1} \circ \mathbf{p}_{t-1}) + (\mathbf{r}_t - \mathbf{c}^u_{t-1})'(\mathbf{x}^u_{t-1} \circ \mathbf{p}_{t-1}) + \eta_t, \tag{12}$$

where η_t is an independent wealth shock, e.g., from other business units. Borrowers maximize expected wealth by choosing the optimal security positions $x^{j,s}_t$ and $x^{j,u}_t$,

$$\max_{x^s_t, x^u_t} \mathbb{E}_t \left[W_{t+1} \right], \tag{13}$$

subject to the capital constraint (10) and wealth dynamics (12). To summarize, an equilibrium in the economy is defined as follows.

Definition 1. *A competitive equilibrium consists of a price process $\boldsymbol{p}_t$ and interest rates i^s_t and i^u_t, such that (i) $\boldsymbol{x}_t$ maximizes the borrowers' expected final wealth, subject to the capital constraint, (ii) $\boldsymbol{y}^k_t$ maximizes customer k's expected utility after arrival in the market place and is zero beforehand, (iii) money market loan volumes M^s_t and M^u_t maximize lenders' expected wealth as given by Equation* (8)*, (iv) the probability of counterparty default is an increasing function of borrowers' leverage, $\theta(L_t)$, (v) margins are set according to the VaR specification in Equations* (6) *and* (7)*, and (vi) markets clear, $\boldsymbol{x}_t + \sum_{k=0}^{2} \boldsymbol{y}^k_t = 0$.*

The dynamics comprise price processes in the asset market as well as in the money markets, and the market clearing condition ensures that all markets are in equilibrium simultaneously. In the next section, we analyze the equilibrium outcome of the economy.

2. Equilibrium

There are two simultaneous decisions at each time t, namely, lenders interact with borrowers in the money market, and borrowers trade assets with customers.

2.1. Money Market

We begin with the lenders' maximization problem as it is identical for all time periods. Solving Equation (9) with respect to M_t^s and M_t^u, the equilibrium condition for short-term interest rates yields

$$(1-\theta)i_t^s = \phi_t^b + i_{cb} = (1-\theta)i_t^u - \theta. \tag{14}$$

Thus, the spread between money market interest rates is determined by the borrowers' default probability $\theta(L_t)$, which is a function of borrowers' leverage, i.e.,

$$i_t^u = i_t^s + \frac{\theta}{1-\theta} = i_t^s + \mu(L_t). \tag{15}$$

Equation (15) has interesting implications, which we summarize in Lemma 1. Proofs to all lemmas and propositions are shown in the appendix.

Lemma 1. *The equilibrium dynamics of money market interest rates are such that*

(i) *the spread between i_t^u and i_t^s is a credit risk premium, which increases with leverage, i.e., $\mu(L_t) = i_t^u - i_t^s > 0$ with $\frac{\partial \mu}{\partial L_t} > 0$*

(ii) *money market interest rates are positively affected by the central bank's refinancing rate i_{cb}, and*

(iii) *the spread of i_t^s to the expected return of the near-money asset, ϕ_t^b, moves with i_{cb}, reflecting the opportunity costs of holding money.*

All these patterns find empirical support. First, the spread between unsecured and secured rates is commonly used to proxy money market risk premiums and it is well-known that it tends to increase in times of crisis. Second, (conventional) monetary policy steers interest rates in the economy by influencing banks' costs of refinancing, which is one of the reasons i_{cb} is referred to as the benchmark rate. Another

reason is that i_{cb} reflects the opportunity costs of holding money, which are high when investment opportunities are promising. Empirically, the spread between the secured rate and the expected return on near-money assets, $i_t^s - \phi_t^b$, co-moves with the unsecured rate in the United States (Nagel, 2016). Extending the finding of Nagel (2016), Equation (14) suggests that the spread includes the term θi_t^s, which is time-varying and increasing in bank leverage, i.e., credit risk.

2.2. Asset Market

We derive the optimal values for $\mathbf{x}_t$ and $\mathbf{y}_t$ by backward induction, starting with optimal asset holdings at time $t = 2$.

Time 2. Let Γ be a customer's value function, then his optimization problem at $t = 2$ becomes

$$\Gamma_2(W_2^k, \mathbf{p}_2, \boldsymbol{\nu}_2) = \max_{y_2^k} -\mathbb{E}_2[e^{-\gamma W_3^k}]. \tag{16}$$

Knowing that assets pay off at date 3, the solution to this problem is

$$y_2^{j,k} = \frac{\nu_2^j - p_2^j}{\gamma(\sigma_3^j)^2} - z^{j,k}. \tag{17}$$

Since all customers are in the market at time 2, the aggregate endowment shock is $\mathbf{Z}_2 := \sum_k y_2^{j,k} = 0$, equilibrium prices equal fundamental values, $\mathbf{p}_2 = \boldsymbol{\nu}_2$. and the customer's value function is $\Gamma_2(W_2^k, \mathbf{p}_2 = \boldsymbol{\nu}_2, \boldsymbol{\nu}_2) = -e^{-\gamma W_2^k}$. As markets must clear, borrowers' demand is zero and the their value function is $\Psi_2(W_2, \mathbf{p}_2 = \boldsymbol{\nu}_2, \boldsymbol{\nu}_2) = W_2$.

Time 0 and 1. If all customers arrive at time 0, $\mathbf{Z}_t = 0$ for all t. Hence, we concentrate on the case with potential trade at times $\tau = 0, 1$ when $\mathbf{Z}_\tau > 0$. Customers $k = 0, 1$ optimally choose $y_1^{j,k}$ according

to

$$y_1^{j,k} = \frac{\nu_1^j - p_1^j}{\gamma(\sigma_2^j)^2} - z^{j,k}, \tag{18}$$

and the aggregate endowment shock is $\mathbf{Z}_1$. Note that Equation (18) resembles Equation (17), because prices at time 2 are equal to fundamental values. At time 0, customer $k = 0$ enters the market and maximizes his expected wealth at time 1, $\mathbb{E}_0[\Gamma_1(W_1^k, \mathbf{p}_1, \boldsymbol{\nu}_1)]$, subject to the value function

$$\Gamma_1(W_1^k, \mathbf{p}_1, \boldsymbol{\nu}_1) = -\exp\left\{-\gamma\left[W_1^k + \sum_j \frac{(\nu_1^j - p_1^j)^2}{2\gamma(\sigma_2^j)^2}\right]\right\}. \tag{19}$$

For borrowers, who maximize their expected wealth as given by Equation (13), the value function at time 1 is

$$\Psi_1(W_1, \mathbf{p}_1, \boldsymbol{\nu}_1, \mathbf{c}_1) = \max_{\mathbf{x}_1^s, \mathbf{x}_1^u} \mathbb{E}_1[\Psi_2(W_2, \mathbf{p}_2 = \boldsymbol{\nu}_2, \boldsymbol{\nu}_2, \mathbf{c}_2)]. \tag{20}$$

Maximizing expected wealth with respect to x_1^s, results in a corner solution: Borrowers invest in security j for which the expected net return is maximized, $\max_j \{\phi_1^j - c_1^{j,s}\}$, until the capital constraint binds,

$$x_1^s = \frac{1}{m_1} W_1. \tag{21}$$

Fully leveraging W_1 in the secured market is optimal as neither holding capital nor directly purchasing assets provides a higher expected net return. If the maximum spread $\phi_1^j - c_1^{j,s}$ is equal to zero, borrowers are indifferent between any position up to the constraint.

Next, we maximize borrowers' expected wealth with respect to x_1^u and denote $\phi_1 = \max_j \left\{\phi_1^j\right\}$:

$$x_1^u = \frac{\phi_1 - i_1^u}{\frac{\partial \mu}{\partial L_1} p_1} W_1. \tag{22}$$

From Equation (22), a larger net return increases demand for unsecured borrowing, while higher leverage

reduces it. In other words, unsecured funding is optimal when the marginal return of investing in an additional asset equals marginal funding costs, i.e.,

$$\phi_1 - i_1^u = \frac{M_1^u}{W_1} \frac{\partial \mu}{\partial L_1}. \tag{23}$$

Equation (23) establishes an endogenous financial constraint for unsecured funding. That is, if $M_1 < D_0$, borrowers' leverage is constrained by marginal profitability and leverage is optimal when Equation (23) holds. Leverage is higher when assets yield higher returns, providing a link to macroeconomic conditions and pro-cyclicality of bank leverage (Adrian and Shin, 2010). If M_1 exceeded D_0, the lending volume would be limited to D_0, and Equation (23) would hold with "$\geq$". Alternatively, credit rationing could also occur if D_1 was subject to, e.g., contagion risk of deposit withdrawals at time 1 (e.g., Diamond and Dybvig, 1983; Shin, 2009). In our model, credit is rationed endogenously by lenders demanding a higher risk premium μ which increases the unsecured interest rate when leverage, and thus borrowers' default probability, increases (e.g., Stiglitz and Weiss, 1981; Acharya and Viswanathan, 2011).

Finally, the shadow costs of capital, denoted by φ_1^u for the unsecured market, are the net return on borrowers' wealth:

$$\varphi_1^u = \phi_1 - i_1^u. \tag{24}$$

When Equation (23) holds, the funding constraint is binding and the price for relaxing the constraint is given by the shadow cost. Similarly, the shadow costs of capital in the secured market are given by $\varphi_1^s = \phi_1 - c_1^s$, and reflect the net return on borrowers' wealth when the capital constraint becomes binding. Thus, borrowers' value function at time 1 is given by $\Psi_1(W_1, \mathbf{p}_1, \boldsymbol{\nu}_1, \mathbf{c}_1) = W_1 + x_1^s p_1 \varphi_1^s + x_1^u p_1 \varphi_1^u$. At time 0, each borrower maximizes $\mathbb{E}_0[\Psi_1(W_1, \mathbf{p}_1, \boldsymbol{\nu}_1, \mathbf{c}_1)]$, subject to the financial constraints. Since banks in our model have no disutility from negative wealth, the analysis is similar to time 1.

In sum, equilibrium interest rates are given by Equation (15) and borrowers' positions are shown in

Equations (21) and (22). We discuss the properties of asset prices at times 0 and 1 and the implications for money market fragility in the next section.

3. Money Market Fragility

In this section, we show that fragility can arise due to binding funding constraints, even if borrowers have access to *both* the secured and unsecured money market. In line with Brunnermeier and Pedersen (2009), markets are fragile when the equilibrium price $p_1(\eta_1, \Theta_1)$ cannot be chosen to be continuous in the exogenous shocks η_1 and Θ_1. Fragility occurs when a small fundamental shock $\Theta_1 < 0$ is followed by a (much) larger drop in the market price, i.e., $\Delta p_1 < \Delta \nu_1$.[11] From the equilibrium derivations, we know that all trades are reversed at time 2 when the complementary customer enters the market, so we focus on the dynamics of funding liquidity at time 1 conditional on borrowers' security holdings at time 0.

If either the secured or unsecured market is unconstrained, markets are not fragile and asset markets remain liquid. For instance, assume that borrowers fund assets in the secured market until their capital constraint binds, but borrowing in the unsecured market is unconstrained. In this scenario, a fundamental shock increases margins, but the borrower would absorb the reduction in secured liquidity simply by raising the corresponding funds in the unsecured market.[12] As a result, the market price of the shocked asset would not drop below the new fundamental value and $p_1 = \nu_1$.

Only if borrowers are constrained in *both* funding markets simultaneously, markets might be fragile with equilibrium prices subject to market illiquidity. Under the assumption that borrowers do not have access to the unsecured money market, funding liquidity in the secured market has been shown to be fragile in Brunnermeier and Pedersen (2009). Lemma 2 shows that money market liquidity can be fragile even when borrowers have access to secured and unsecured funding markets. Namely, funding constraints

[11] We show our main results for a single asset $J = 1$ at time 1. We discuss portfolio dynamics in Section 4.3.

[12] Theoretically, a substitution from unsecured to secured liquidity could be possible as well if only unsecured funding was constrained. However, capital costs incentivize borrowers to fully leverage their wealth for financing margins in the secured market, so that the more flexible source of funding is the unsecured market.

can become binding in both funding markets at the same time, causing excess demand $x_1 + \sum_{k=0}^{1} y_1^k \geq 0$ to be decreasing for lower prices.

Lemma 2. *There exists $\underline{x}$ such that the market is fragile if borrowers' position at time 0 is larger than $\underline{x}$, i.e., $x_0 > \underline{x}$, where $x_0 = x_0^s + x_0^u$, and customers' demand shock is in the same direction, i.e., $Z_1 > 0$.*

As markets become fragile, liquidity spirals do not only arise in the secured market, but also in the unsecured market. The spirals interact *across* funding markets and mutually reinforce each other. To fully understand the dynamics after a fundamental shock, Proposition 1 summarizes *common* money market fragility.

Proposition 1. *When the funding constraint is slack in at least one funding market, the price change is equal to the change in the fundamental value, $\Delta p_1 = \Delta \nu_1$, and the market is in a liquid equilibrium.*
In a stable illiquid equilibrium with selling pressure from customers, $Z_1 > 0$, and exposure $x_0 > 0$,

(i) *the price sensitivity to a fundamental shock $\Theta_1 < 0$ is given by*

$$\frac{\partial p_1}{\partial \Theta_1} = \frac{m_1 \left(\frac{2\gamma\sigma_2^2 - 4\gamma\sigma_2(\nu_1 - p_1)\frac{\partial \sigma_2}{\partial \Theta_1}}{(\gamma(\sigma_2)^2)^2} \right) - \frac{\partial m_1}{\partial \Theta_1} x_1^s}{\frac{\partial m_1}{\partial p_1} x_1^s + m_1 \frac{2}{\gamma(\sigma_2)^2} - \frac{\partial W_1}{\partial p_1}}, \tag{25}$$

for assets funded in the secured money market, and by

$$\frac{\partial p_1}{\partial \Theta_1} = \frac{\frac{4\gamma\sigma_2(\nu_1 - p_1)\frac{\partial \sigma_2}{\partial \Theta_1} - 2\gamma\sigma_2^2}{(\gamma(\sigma_2)^2)^2} - W_1 \frac{1}{\frac{\partial \mu}{\partial L_1}(p_1)^2}}{\frac{\frac{\partial \mu}{\partial L_1} p_1 \left[\left(\frac{\partial \phi_1}{\partial p_1} - \frac{\partial i_1^u}{\partial p_1} \right) W_1 + (\phi_1 - i_1^u)\frac{\partial W_1}{\partial p_1} \right] - (\phi_1 - i_1^u) W_1 \frac{\partial \mu}{\partial L_1}}{\left(\frac{\partial \mu}{\partial L_1} p_1 \right)^2} - \frac{2}{\gamma(\sigma_2)^2}} \tag{26}$$

for assets funded in the unsecured money market.

(ii) *there exist two mutually reinforcing liquidity spirals for the unsecured money market, namely, an interest rate spiral, $\frac{\partial i_1^u}{\partial p_1} < 0$, and a loss spiral, $\frac{\partial W_1}{\partial p_1} > 0$.*

(iii) ***(Money Market Fragility)*** *margin and loss spirals for the secured market,* $\frac{\partial m_1}{\partial p_1} < 0$ *and* $\frac{\partial W_1}{\partial p_1} > 0$*, are integrated with liquidity spirals in the unsecured market by mutually exerting downward pressure on prices and deteriorating borrowers' capital.*

This result is intuitive as funding problems arise from a reduction in borrowers' capital, both directly and indirectly. The direct effect comes from the deteriorating effect of lower prices on mark-to-market capital, i.e., the loss spiral. In turn, this reduction causes two indirect, market-specific effects, namely the interest rate and margin spirals. In the unsecured market, the interest rate increases with leverage (Equation (15)). Thus, if an asset shock decreases capital W_1, this increases leverage for existing positions $x_0^u > 0$, which in turn increases the unsecured interest rate. As borrowers' positions in the unsecured market depend on marginal profits, a higher interest rate i_1^u, *ceteris paribus*, increases the marginal costs of funding and enforces asset sales. In the secured market, margins increase in response to lower prices and force borrowers to sell assets as the capital constraint becomes binding. Additionally, less capital reinforces selling pressure as now fewer of the higher margins (per unit of security) can be paid.

By the downward pressure on the market price, loss, interest rate, and margin spirals mutually reinforce each other, which ultimately leads to fragility of common money market liquidity.[13] Although lower prices raise the expected return and demand for the asset, higher interest rates and higher margins in combination with customers' selling pressure ($Z_1 > 0$) can prevent borrowers from investing into the asset until marginal returns equal marginal costs again (Equation (22)). Consequently, asset demand declines as p_1 decreases due to higher i_1^u, and increases again only when expected returns ϕ_1 are sufficiently high. Overall, Proposition 1 shows that all three sources of immediate funding for financial institutions, namely borrowing in the secured market, borrowing in the unsecured money market, and selling assets are interconnected and may become illiquid at the same time.

[13]Empirical support for the link between unsecured funding and asset markets is provided, e.g., by Nyborg and Östberg (2014). Gorton and Metrick (2012) show that high funding cost in the unsecured market (as measured by the LIBOR-OIS spread) and increased haircuts in the secured market occur at the same time.

4. Secured and Unsecured Funding Liquidity

Having established that common money market liquidity can be fragile, in this section we analyze in more detail how secured and unsecured funding are related in equilibrium. That is, we investigate the relative funding dynamics after a negative price shock and how the time-0 position x_0 and leverage L_0, affect these dynamics.

4.1. Liquidity Dynamics

Due to liquidity spirals, a drop in the market price p_1 affects the borrowers' security positions x_1^s and x_1^u. Formally, the price sensitivity of demand for assets funded in the secured market is given by

$$\frac{\partial x_1^s}{\partial p_1} = \frac{m_1 \frac{\partial W_1}{\partial p_1} - W_1 \frac{\partial m_1}{\partial p_1}}{m_1^2} > 0. \tag{27}$$

As prices decrease and the capital constraint becomes binding, the margin and loss spirals force borrowers to deleverage. Through the increase in haircuts, the secured funding volume M_1^s decreases. Since total leverage is the sum of secured and unsecured leverage, $L_1 = \frac{(x_1^s + x_1^u)p_1}{W_1} = L_1^s + L_1^u$, a price shock affecting secured leverage L_1^s also changes total leverage by

$$\left.\frac{\partial L_1^s}{\partial p_1}\right|_{\Theta_1 < 0} = \frac{m_1 - p_1 \frac{\partial m_1}{\partial p_1}}{m_1^2} > 0. \tag{28}$$

As a fundamental shock increases margins initially, secured leverage decreases since $L_1^s = h_1^{-1}$. In an illiquid equilibrium, margins increase further, forcing borrowers to deleverage more.

In the unsecured market, the equilibrium effect of a price shock on x_1^u can be positive or negative. However, unsecured leverage,

$$L_1^u = \frac{\phi_1 - i_1^u}{\frac{\partial \mu}{\partial L_1}}, \tag{29}$$

unambiguously increases after a shock. That is, if total leverage has decreased in equilibrium, the risk premium and interest rate i_1^u have done so as well, and L_1^u has increased less than L_1^s has decreased in total. Conversely, if L_1 has increased, unsecured leverage must have increased as well. Thus, the numerator of Equation (29), which represents the shadow cost of capital in the unsecured market φ_1^u, always increases following an asset shock. As secured and unsecured leverage represent the relative share of total assets funded in each market, the opposing effects of an asset shock on L_1^s and L_1^u implies a reallocation of funding from the secured to the unsecured money market after a shock.

Proposition 2. *In equilibrium, a fundamental shock leads to a re-allocation of money market liquidity from secured to unsecured funding. While higher margins reduce the funding volume in the secured money market, they*

(i) ***(Substitution)*** *relax the funding constraint in the unsecured money market through lower secured leverage, such that*

$$\Delta \left.\frac{M_1^u}{M_1}\right|_{\Theta_1<0} > 0 \quad \textit{and} \quad \Delta \left.\frac{M_1^s}{M_1}\right|_{\Theta_1<0} < 0.$$

(ii) ***(Commonality)*** *Funding illiquidity, as measured by the borrowers' shadow costs of capital, φ_1^s and φ_1^u, co-moves, i.e.,*

$$Cov_0(\varphi_1^s, \varphi_1^u) > 0, \tag{30}$$

implying common illiquidity, if shadow costs increase, and perfect substitution from secured to unsecured funding, if shadow costs decrease.

The economic interpretation for Proposition 2 follows from the shadow costs of capital. After a shock, haircuts increase the funding costs in the secured market, and higher leverage increases i_1^u in the unsecured market. If the funding constraint in the unsecured market is binding, the market price must fall as shown in Proposition 1. This leads to a (non-linear) increase in the expected return ϕ_1, so that a higher shadow cost φ_1^u reflects unsecured funding illiquidity, which co-moves with funding illiquidity in the

secured market. As borrowers reduce positions, the shadow costs φ_1^s and φ_1^u jointly increase, indicating that liquidity spirals in the money market are mutually reinforcing as shown in Proposition 1. Moreover, the allocation of funding liquidity changes towards more unsecured funding. Intuitively, this happens because the funding constraint in the secured market is exogenously imposed by capital W_1, whereas the funding constraint in the unsecured market is endogenously determined by marginal profits. Furthermore, lenders in the secured market know the collateral risk and reduce funding volume M_1^s, so that borrowers must replace the reduction in secured liquidity by funding a larger share of the security in the unsecured market. If the leverage constraint is slack, there is perfect substitution of funding liquidity in the sense that the reduction of secured funding after an asset shock is compensated by an equivalent increase in unsecured funding, and shadow costs decrease. In contrast, if the leverage constraint is binding after a shock, borrowers are forced to reduce their balance sheet until marginal profits are non-negative, which increases the shadow costs of capital and the share of unsecured funding.

In fact, the commonality of funding liquidity in Proposition 2, has important implications for the extent of fragility, which we analyze in the next subsection.

4.2. Extent of Fragility

Fragility at time 1 arises from a large enough previous position x_0, leading to capital losses and a reduction in asset demand. This happens because deleveraging leads to drops in asset prices until $\varphi_1^u = \phi_1 - i_1^u$ is sufficiently large to allow borrowers to raise enough funds to clear the market. Since more x_0 increases i_0^u through higher leverage, borrowers' ability to substitute funding liquidity at time 1 depends on their shadow costs of capital at time 0.

Lemma 3. *The funding constraint in the unsecured money market at time 1 is slack for $\varphi_0^u \geq \varphi_0^s$, and binding for $\varphi_0^u < \varphi_0^s$.*

Intuitively, Lemma 3 shows that large holdings x_0 imply sufficient access to funding liquidity at time

0. In this case, shadow costs in the unsecured market are relatively small due to a high interest rate i_0^u, and shadow costs of capital in the secured market, $\varphi_0^s = \phi_0 - c_0^s$, are relatively large for low haircuts and/or high returns. Since borrowers encumber all their capital in the secured market at time 0, $\varphi_0^u < \varphi_0^s$ leads to funding problems at time 1 if borrowers face a shock. As the capital constraint becomes binding after a shock, low shadow costs in the unsecured market at time 0 exert selling pressure at time 1, because borrowers' leverage constraint becomes binding from too high leverage L_0. As shown in Proposition 2, this leads to asset sales until prices are low enough and φ_1^u sufficiently high to clear the market. In contrast, $\varphi_0^u \geq \varphi_0^s$ allows borrowers to substitute funding liquidity by raising the required amount of unsecured debt to compensate for the reduction in secured funding from an increase in haircuts. Since φ_0^u is linked to the marginal funding costs in the unsecured market, as given by Equation (23), Proposition 3 relates the feasibility of liquidity substitution to the shadow costs of capital.

Proposition 3. *Common funding liquidity at time 1 depends on the relative shadow costs of capital at time 0:*

(i) ***(Shadow illiquidity)*** *The shadow costs of capital in the secured market represent the shadow marginal funding costs in the unsecured market, and constitute shadow illiquidity if*

$$\frac{\partial i_0^u}{\partial L_0} L_0^u \leq \varphi_0^u < \varphi_0^s. \tag{31}$$

In contrast, for $\varphi_0^s \leq \frac{\partial i_0^u}{\partial L_0} L_0^u$, the market at time 1 is liquid and $p_1 = \nu_1$.

(ii) ***(Liquidity dry-up)*** *When there is shadow illiquidity at time 0, leverage L_0 is so high that it impairs the substitution of funding liquidity at time 1 in case of a fundamental shock, $\Theta_1 < 0$, and selling pressure in the market, $Z_1 > 0$. In extremis, the unsecured market is "frozen" and borrowers default.*

Shadow illiquidity exists if the shadow costs of capital in the secured market exceed the marginal funding costs in the unsecured market. This is intuitive because the shadow costs of capital represent the

expected loss in case the constraint becomes binding. That is, when borrowers face initial losses at time 1 or haircuts increase, the capital constraint becomes binding and positions funded in the secured market must be reduced. Consequently, borrowers need to fund these assets in the unsecured market, which is feasible if the leverage constraint is slack. Hence, φ_0^s represents the marginal return required to substitute secured funding liquidity in the unsecured market. In other words, if Equation (31) holds, there exists shadow illiquidity at time 0, because the marginal funding costs in the unsecured market will exceed the marginal return in case the capital constraint becomes binding at time 1. Clearly, it is *shadow* illiquidity since there is no problem if time-1 fundamentals remain solid.

Moreover, shadow illiquidity implies that leverage L_0 is so high and φ_0^u so low that substitution of liquidity at time 1 is impaired. This happens because high initial leverage increases further after capital losses and induces assets sales until the market price drop creates sufficient marginal return for borrowers to stop reducing positions. Figure 2 illustrates borrowers' demand as a function of shadow costs in the unsecured market. Borrowers want to deleverage following a shock when $\varphi_0^u < \varphi_0^s$, i.e., $\Delta x_1 < 0$. The lower φ_0^u, the higher was initial leverage L_0 and the larger $\Delta x_1 < 0$ (in absolute value). For sufficiently small φ_0^u, demand stabilizes as borrowers profit from buying back assets. In contrast, the leverage constraint is slack when $\varphi_0^u \geq \varphi_0^s$, implying that borrowers have access to unsecured funding and $\Delta x_1 > 0$. The larger φ_0^u, the more borrowers want to take advantage of profitable investment opportunities.

Figure 2 restates the leverage cycle, which describes that excess leverage is followed by excessive deleveraging (Geanakoplos, 2010) and links it to developments in money markets. When leverage is low, borrowers raise additional funding to benefit from profitable opportunities ($\Delta x_1 > 0$) and deleverage in a crisis when markets are illiquid ($\Delta x_1 < 0$). For very high leverage, the corresponding deleveraging is due to illiquidity in the unsecured market, which can be considered stressed until volumes have decreased sufficiently. This result is consistent with empirical evidence from the 2008 financial crisis (Afonso, Kovner, and Schoar, 2011). When borrowers cannot raise unsecured debt to fund their assets at affordable costs,

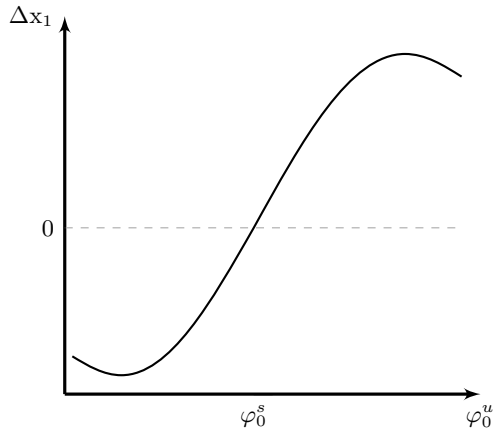

Figure 2. Leverage Cycle
The solid line represents the change in borrowers' demand Δx_1 as a function of the relation between the shadow costs of capital at time 0.

the unsecured market is "frozen", i.e., closed for borrowers with too high counterparty credit risk.

Our results show that fragility arises because high leverage impairs the flexible re-allocation of liquidity from secured to unsecured funding after a fundamental shock. In the next section, we analyze the relation between secured and unsecured funding liquidity when borrowers anticipate liquidity risk, i.e., we analyze price dynamics at time 0.

4.3. Liquidity Risk

Since the market is perfectly liquid at time 2, borrowers face no liquidity risk at time 1. Hence, we analyze borrowers' portfolio decision at time 0, when they build expectations about future illiquidity. This allows us to determine the role of liquidity risk for the use of funding markets, and the impact on time-0 shadow costs of capital.

At time 0, borrowers maximize their expected wealth $\mathbb{E}_0[W_1(1+\varphi_1)]$, which depends on time-1 illiquidity, measured by the weighted shadow costs of capital, φ_1. Solving for x_0, we obtain the time-0 price

$$p_0 = \frac{\mathbb{E}_0[p_1]}{1+c_0} + \frac{\mathrm{Cov}_0[p_1,\varphi_1]}{(1+c_0)\mathbb{E}_0[1+\varphi_1]}, \tag{32}$$

where the covariance term captures the notion of liquidity risk (Brunnermeier and Pedersen, 2009). A negative covariance means that the market price decreases when funding illiquidity is high, exposing borrowers to potential fragility. Thus, the higher liquidity risk, the lower is the time-1 market price, implying a liquidity risk premium. Moreover, as p_0 decreases, volatility and margins increase, and unsecured demand increases. Proposition 4 highlights how liquidity risk affects money market funding liquidity.

Proposition 4. *Liquidity risk affects money market funding liquidity as follows:*

(i) ***(Flight-to-safety)*** *Securities with low liquidity risk, i.e., higher time-0 price p_0, are funded more in the secured money market,*

$$\frac{\partial M_0^s}{\partial p_0} > 0,$$

due to lower funding costs c_0^s, and higher secured leverage.

(ii) ***(Risk-seeking)*** *Securities with high liquidity risk, i.e., $Cov_0(p_1, \varphi_1) < 0$, are funded more in the unsecured money market. As p_0 decreases,*

$$\frac{\partial M_0^u}{\partial p_0} < 0,$$

unsecured funding increases due to higher expected marginal returns and unsecured leverage.

These cross-sectional findings relate to aggregate patterns observed in money markets before and during the recent crisis. As a "flight to quality" or "flight to liquidity" set in amidst the subprime crisis, banks shifted their portfolio holdings to safer, less volatile assets. This change on the asset side corresponds to a shift to secured funding on the liability side, which has been shown empirically for the European money market (Mancini, Ranaldo, and Wrampelmeyer, 2016). The reason is that securities that are expected to be little affected or unaffected by common funding illiquidity carry a low return and low haircuts in the next period. This implies that secured funding costs are low and secured leverage is

high, which crowds out unsecured funding volumes and appears as a "flight to secured funding".

In contrast, a portfolio of securities carrying a liquidity risk premium is funded predominantly in the unsecured market due to higher expected returns and haircuts. Hence, a lower time-0 price causes volatility and higher margins, so that risk-seeking borrowers fund their portfolio in the unsecured market. Related to this pattern, Acharya and Steffen (2015) provide empirical support for risk-seeking behavior during the recent European sovereign debt crisis. Banks invested in riskier government bonds, expecting prices to go up as common funding conditions would improve. As these assets carry high haircuts or are even ineligible as collateral in the secured market, risk-seeking behavior corresponds to an increase in unsecured funding.[14]

Overall, both flight-to-safety and risk seeking behavior can increase the shadow cost differential, $\varphi_1^u < \varphi_1^s$, and make markets more susceptible to future shocks. The vulnerability of borrowers stems from high leverage, which impairs the substitution of funding liquidity and leads to fragility. In the next section, we turn to the social optimum to assess monetary and regulatory policy measures with regard to their effectiveness in preventing fragility.

5. Social Optimum and Central Bank Monetary Policy

5.1. The Social Equilibrium

We consider an unconstrained social planner who maximizes total welfare, by choosing the socially optimal demand, x_t^*, and leverage ratio, L_t^*. From all agents' utility functions, the optimal demand is $x_t^* = Z_t$ (Gromb and Vayanos, 2002). The social costs differ from the private funding costs by the externality of leverage on the borrowers' funding constraint in the unsecured market. Hence, the planner derives L_t^* by maximizing borrowers' expected wealth for x_t^s and x_t^u, internalizing that $i_t^u(L_t)$ is a function of total leverage. The planner's optimal choice of x_t^u is equal to the borrowers' solution. For the first-order

[14] We discuss the impact of central bank monetary policy on funding conditions in the next section.

condition with respect to x_t^s, we have

$$\frac{\partial \mathbb{E}_t[W_{t+1}]}{\partial x_t^s} = (\phi_t - c_t^s) - x_t^u \frac{\partial \mu}{\partial L_t} \frac{\partial L_t}{\partial x_t^s}. \tag{33}$$

The first right-hand term of Equation (33) is identical to the borrowers' optimization and yields the net return of an asset funded in the secured market. The second term is neglected by the borrowers and it is given by the shadow costs in the unsecured market (Equation (24)). Thus, the social costs of secured funding amount to the sum of c_t^s and φ_t^u. We arrive at the socially optimal leverage ratio by inserting x_t^u from Equation (22) into Equation (33).

Proposition 5. *The social optimum is determined by the following conditions:*

(i) ***(Liquidity)*** *Market and funding liquidity is optimal when* $x_t^* = Z_t$.

(ii) ***(Leverage)*** *The optimal leverage ratio* L_t^* *is determined by equal shadow costs of capital,*

$$\varphi_t^u = \varphi_t^s, \tag{34}$$

implying equal funding costs, i.e., $L_t = L_t^*$ *when* $c_t^u(L_t^*) = c_t^s$.

The social equilibrium is characterized by (i) perfect liquidity, and (ii) sufficient capital such that $L_t = L_t^*$. As shown for the market equilibrium, the relation of shadow costs indicates whether liquidity can be substituted in the money market. If $\varphi_t^u < \varphi_t^s$, borrowers have excess leverage, i.e., $c_t^u > c_t^s$, which exerts negative externalities on customers due to illiquidity, and on lenders if borrowers go bankrupt. The equality of funding costs indicates that the socially optimal leverage ratio is time-varying, i.e., it is lower when margins are low and higher when volatility is high.

When funding constraints bind, the market allocation fails to achieve the social optimum, which calls for policy intervention. In the next two subsections, we analyze how central bank and regulatory policies

affect the market allocation, and evaluate their effectiveness in preventing fragility by comparing the market allocation with the social optimum.

5.2. Central Bank Monetary Policy

The central bank in our model is balance-sheet unconstrained (Gertler and Karadi, 2011), operates as the lender of last resort (Bagehot, 1873), and sets interest rate i_{cb} and haircut h_{cb} for collateralized lending. We consider lending via a standing facility through which the central bank offers liquidity for a fixed rate i_{cb}. To comply with its role as lender of last resort, we assume that at time 0 the central bank adds an additional buffer to the π-value-at-risk, such that haircuts are higher than in the private market and borrowers face lower costs $c_0^s < c_{cb}$ for interbank liquidity.[15]

To ease funding conditions, central banks usually intervene by conventional interest rate policy and reduce i_{cb}, which affects money market rates according to Equation (15). In particular, the reduction in i_1^u loosens the funding constraint in the unsecured market and increases demand as $\frac{\partial x_1^u}{\partial i_1^u} < 0$.

During the recent crises, central banks around the world have conducted unconventional monetary policies in addition to conventional interest rate policy. For instance, the Federal Reserve and European Central Bank intervened in asset markets by purchasing liquid "near-money" bonds as well as distressed securities. The effect of the former is similar to a further reduction in money market interest rates, which can be seen from Equation (14). As the central bank buys bonds b, the expected return ϕ_1^b decreases due to higher demand and prices, which pushes interbank rates down. On the other hand, purchase programs of distressed assets provide market liquidity, as the central bank directly intervenes in the open market with position x_1^{cb}, which allows banks to shift illiquid securities to the central bank.

Moreover, most central banks have provided emergency lending facilities for financial institutions to obtain funding liquidity at haircuts lower than in the private market. Such haircut policy is particularly effective when assets have become ineligible as collateral or when they are highly capital-intense in the

[15] Alternatively, the central bank haircut is simply $h_{cb} = 1$, and assets are ineligible.

private market.[16] Since costs of secured funding depend positively on the haircut, haircut policy essentially represents a subsidy vis-à-vis the interbank money market (Drechsler et al., 2016). Both asset purchases and haircut policy are associated with financial risk, as central banks provide liquidity against assets or collateral from private agents without knowing the fundamental value.

To be socially efficient, monetary policy interventions must meet the liquidity and leverage conditions of Proposition 5. Trivially, all policy measures increase demand and prevent fragility if $x_1 = Z_1$. Through asset purchases, the central bank reduces supply by $Z_1 - x_1^{cb}$, which alleviates downward price pressure. For interest rate and haircut policy, borrowers' equilibrium demand for secured and unsecured funding increases as given by Equations (21) and (22). Most importantly, the optimal liquidity condition assures the prevention of present fragility, while the optimal leverage condition aims to prevent future fragility.

Proposition 6. *After a fundamental shock, conventional and unconventional monetary policy can prevent present fragility and restore liquidity such that $x_1 = x_1^*$. However, monetary policy measures differ in their effectiveness in achieving the socially optimal leverage ratio and preventing future fragility:*

(i) ***(Interest rate policy)*** *Conventional policy decreases money market interest rates and retains excess leverage at $L_1 > L_1^*$. Lower interest rates facilitate the substitution from secured to unsecured funding and the shadow cost differential $\varphi_1^u < \varphi_1^s$ remains.*

(ii) ***(Haircut policy)*** *Haircut policy retains excess leverage at $L_1 > L_1^*$. If $h_{cb} < h_0$, haircut policy leads to substitution from unsecured to secured funding and $\varphi_1^u < \varphi_1^s$ increases.*

(iii) ***(Security purchases)*** *An asset purchase program can reach $L_1 = L_1^*$ by allowing borrowers to sell illiquid securities to the central bank, such that $\varphi_1^u = \varphi_1^s$.*

As shown in Proposition 6, interest rate and haircut policies provide funding liquidity at conditions

[16] Some examples of unconventional measures in which haircut policies were involved are the ECB extensions of eligible (riskier) assets for its repo loans in March 2009 and January 2011, or the Federal Reserve's Term Auction Facility (TAF), Primary Dealer Credit Facility (PDCF), Term Securities Lending Facility (TSLF), and Term Asset-Backed Securities Loan Facility (TALF).

that prevent borrowers from selling these assets as fragility arises. If haircuts and interest rates reduce after prices have started to decrease, borrowers are able to maintain funding the shocked asset at lower funding costs, especially in the unsecured market, or by means of lower margin requirements for secured funding at the central bank. However, the adverse consequence of such liquidity provision is that borrowers hold on to their excess leverage, which is shown by $\varphi_1^u < \varphi_1^s$ (Acharya and Tuckman, 2014). As shadow costs are lower in the unsecured market, a future shock to borrowers' capital causes unsecured liquidity to dry up until prices decrease sufficiently and marginal returns increase. As shown by the leverage cycle, excess liquidity is followed by excess illiquidity, the higher is borrowers' leverage. Moreover, if haircuts are set lower than they were before the shock, borrowers replace unsecured liquidity by secured liquidity, which increases the shadow cost differential and potential illiquidity in the future.

In contrast, an asset purchase program allows borrowers to reduce illiquid leverage and their exposure to risky securities. Consequently, the shadow costs in the unsecured market increase due to the reduction in leverage, improving borrowers' resilience to future shocks. If liquidity can be substituted in the money market without the need to deleverage assets, future fragility is absent. Nevertheless, slack funding constraints incentivize borrowers to lever up in other assets, which raises the need for regulation to prevent future fragility before a shock occurs.

5.3. Regulatory Policy

In the previous subsection, we show that central bank intervention may have adverse effects on banks' future resilience. Therefore, it is important to understand what policy makers can do to make banks less vulnerable and prevent future fragility. We also show that the key destabilizing factor is that secured and unsecured funding is jointly constrained. Thus, the overall objective for international regulators should be the preservation of slack funding constraints to facilitate the re-allocation between secured and unsecured funding to prevent liquidity spirals and market freezes.

In the remaining part of this section, we discuss three important policy measures, namely enforcing maximum leverage ratios, capital buffers, and liquidity coverage ratios in the context of our model. These measures are part of the Basel III and Dodd-Frank regulatory frameworks and strive to improve banks' resilience to sudden changes in asset values.

5.3.1 Leverage Ratio

Excessive leverage has been identified as one of the key triggers of the recent crises, which led regulators to cap bank leverage at a maximum ratio. A maximum leverage ratio is only effective when it becomes binding, such that it inhibits banks from leveraging up more. It is socially optimal when it prevents excessive leverage, i.e., if it enforces $L \leq L^*$. An important outcome from our model is that funding liquidity follows pro-cyclical patterns. This implies that a dynamic rather than static leverage ratio is more effective in preventing future fragility. Leverage is time-varying and depends on liquidity, meaning that in times of low haircuts and profitable investment opportunities, access to unsecured funding liquidity leads to $\varphi_1^u < \varphi_1^s$. In those times, it is socially optimal to have a binding leverage ratio, which enhances financial stability by creating slack funding constraints. As leverage increases after an asset shock, a constant maximum leverage cap inhibits banks from efficiently substituting liquidity, which might cause asset sales and illiquidity. Moreover, from Proposition 4, banks unexposed to distressed assets are potential providers of market liquidity in times of stress, and may be inhibited from buying assets if $\overline{L} < L^*$. Thus, our model suggests that a dynamic maximum leverage ratio, which takes into account financial cycles would be preferable over a static cap on bank leverage.

5.3.2 Capital Buffers

According to the current regulatory framework, banks are required to hold capital as a percentage of their risk-weighted assets, plus an additional countercyclical capital buffer in good times, if required by national regulators. As holding capital is costly, capital buffers effectively increase banks' funding costs

by an increasing function of the assets' riskiness. As long as risk weights actually mirror the riskiness of assets, this buffer disincentivizes the build-up of risky portfolios and encourages safer investments. In our model, a buffer restrains borrowers from fully encumbering their capital W for margins in the secured market and makes the capital constraint slack. However, such a buffer incentivizes banks to invest in low risk-weight assets. As the capital buffer is low for these assets, leverage and concentration risk in these holdings may become excessive, which leaves banks more exposed to future fragility. Moreover, being based on risk weights, the countercyclical buffer inherits all their drawbacks. For instance, risk weights and thus the buffer are typically set to zero for sovereign bonds, even if they bear some risk as, e.g, during the recent European sovereign debt crisis.

According to Equation (33), the social funding costs in the secured market differ from the private costs by the negative externality on the funding constraint in the unsecured market. In particular, the externality increases with lower haircuts, as higher (secured) leverage implies more fragility in case of a shock. Our model suggests that a straightforward and consistent way to reduce the externality is to tie countercyclical capital buffers to market haircuts rather than risk weights. More specifically, by holding more capital for high-leverage (low-margin) assets, borrowers would internalize the negative externality of leverage on the ability to substitute secured funding in times of increasing volatility and margins. A haircut-based countercyclical buffer could be implemented by having banks regularly report their portfolio holdings to regulatory authorities. Regulators would specify the size of the buffer per unit of haircut and, as long as market haircuts accurately reflect the risk of securities, the haircut-based weighting methodology would consistently adjust capital buffers for market risk.

5.3.3 Liquidity Coverage Ratio

New regulations also comprise a liquidity coverage ratio (LCR), imposing banks to hold high-quality liquid assets (as a fraction of short-term expected liabilities) to better cope with sudden liquidity needs.

In essence, these assets are similar to near-money bonds in our model, as they are required to be readily convertible into cash. In case of an asset shock, these securities can be sold or pledged to repay short-term loans, essentially functioning like a capital buffer. As these assets earn a lower return, they reduce borrowers' profits (Equation (15)). Yet, due to flight-to-safety or central bank asset purchase programs, their demand and prices generally increase in bad times, and thus these bonds provide a hedge to riskier securities.

However, the LCR has two main limits: First, it depends on the scenario parameters applied by the policymakers to high-quality liquid assets (HQLA) that might not fully reflect the actual funding and market liquidity risks. Second, a liquidity coverage ratio does not incentivize banks to implement the socially optimal leverage ratio. Given these drawbacks, a LCR combined with the haircut-based capital buffer discussed above should be more effective in averting future fragility and make banks more resilient to adverse market situations.

6. Empirical Application

As a final step, we perform a simple empirical exercise to assess the main mechanisms of our model. To that end, we take the recent European sovereign debt crisis as a real-world example of an asset shock, including significant losses in value of government bonds of Greece, Ireland, Italy, Portugal, and Spain (GIIPS). We analyze changes in banks' funding structure following the shock and highlight the role of margins.

6.1. Data

A complete empirical analysis of our theoretical results would require bank-specific data on assets and liabilities, money market and central bank borrowing volume, margins, and interest rates. These data are not publicly available, unfortunately. Thus, for our empirical investigation, we combine data from the

richest sources available to construct proxies for the various quantities of interest.

First, we use European government bond holdings from stress test data published by the European Banking Association (EBA) since March 2010. From this list, we take all banks that participated in the stress tests for March and December 2010 as well as December 2011. We denote total bond holdings of bank j in year t by $B_{j,t}$.[17] Second, for these banks we collect yearly balance sheet data on money market funding volumes, including secured and unsecured borrowing and lending for 2009 through 2011. We compute each bank's yearly secured (unsecured) *net* borrowing volume as the difference between the secured (unsecured) borrowing and lending volumes. To comply with our theoretical analysis, we consider net borrowers in the money market, for which the sum of secured and unsecured net borrowing is positive. For each bank, we compute the share of unsecured funding, denoted by $S_{j,t}$, as the ratio of unsecured net borrowing over total money market net borrowing. Third, we construct a proxy for the average margin for banks' bond portfolio. We obtain margin span parameters published by LCH.Clearnet, a major clearing house and provider of risk and collateral management services, as a measure of country-specific margins for a range of government bonds, including Germany, France, Italy, Spain, Netherlands, Belgium, and Greece (until end of 2010).[18] We construct bank-specific margins by the average of margin parameters weighted by each bank's exposure to each country from the EBA stress test data. That is, we multiply each margin with a bank's position in that bond and divide by the total position of that portfolio. We denote the weighted average margin by $m_{j,t}$. Lastly, we obtain data on central bank borrowing from Bruegel, which include a breakdown of Eurosystem liquidity across national European central banks. Countries not considered by Bruegel include non-Euro countries such as U.K. and Sweden for which we collect data manually from the respective national central banks and convert them into Euro.[19] As a

[17] We use the March 2010 EBA stress test bond holdings for end-of-year 2009, as stress tests were first conducted in March 2010.

[18] For instance, in March 2010 the spread on German government bonds was 1.49% and on Greek bonds 7.99%, whereas in October 2010 the respective spreads were 1.27% and 17.75%. In 2011, Greek government bonds were ineligible as collateral and thus receive a spread of 100% to compare for changes in margin parameters.

[19] The Dutch central bank does no publish the necessary information, so no data are available for the Netherlands.

proxy for banks' individual central bank exposure we use the share of their GIIPS holdings relative to all their national peers' GIIPS holdings, taking the full list of banks participating in the stress tests.[20] To measure the reliance on central bank funding, we divide each bank's borrowing volume from the central bank by the banks total assets. We denote this variable by $CB_{j,t}$. Merging the different data sources results in a total sample of 26 banks. We provide the list of banks and variables that are included in our sample in Table A.1 in the appendix.

6.2. Regression Analysis

To empirically test the main mechanisms of our model, we perform cross-sectional least squares regressions for changes in variables between two time periods, namely 2009 to 2010 and 2009 to 2011. We use 2009 as the reference date prior to the European sovereign debt crisis and 2010/2011 as time of stress in GIIPS bonds.[21] First, we investigate the relation between margins and the share of unsecured funding. According to the model, we expect a positive relation, as higher margins lead to a substitution from secured to unsecured funding, and thus an increase in $S_{j,t}$. We control for banks' reliance on central bank funding by including the change in $CB_{j,t}$ from 2009 to 2010. In sum, we estimate the following regression:

$$\Delta S_{j,10} = \beta_0 + \beta_1 \Delta m_{j,10} + \beta_2 \Delta CB_{j,10} + \epsilon_j. \tag{35}$$

Second we investigate the relation between banks' deleveraging from 2009 to 2010 and their pre-crisis leverage as well as the change in their reliance on central bank funding. According to our model, banks with higher initial leverage should be affected more by the shock and thus have to deleverage more and

[20] For example, BNP Paribas' share of France's central bank funding volume reported by Bruegel is computed as the ratio of its GIIPS exposure relative to all French banks' GIIPS exposure provided by the EBA.

[21] For the sake of brevity, we report the results for 2009 to 2010 in the paper. The 2009 to 2011 results, which are qualitatively similar, are shown in Table A.2 in the appendix.

rely more on central bank funding. We estimate the following regression model:

$$\Delta B_{j,10} = \beta_0 + \beta_1 \Delta CB_{j,10} + \beta_2 L_{j,09} + \epsilon_j. \tag{36}$$

Table 1 shows the regression results. Despite the limited number of observations and the simplicity of the empirical exercise, the results support the predictions of our theoretical model. An increase of margins and a decrease of reliance on central bank liquidity is associated with an increase in the share of unsecured borrowing. Moreover, higher leverage in 2009 and increases in central bank borrowing are associated with a larger deleveraging.

Table 1
Regression Results for Funding Shares and Bond Holdings

This table shows the results of regressing changes in banks' funding shares (Columns (1) to (3)) and changes in bond holdings (Columns (4) to (6)) on explanatory variables derived from our model. Standard errors are shown in parentheses. The stars ***, **, and * indicate statistical significance at the 1%, 5%, and 10% level, respectively.

	Δ_{09}^{10} Share			Δ_{09}^{10} Bonds		
	(1)	(2)	(3)	(4)	(5)	(6)
Δm	0.081^{**}		0.079^{*}			
	(0.037)		(0.038)			
ΔCB		-0.777	-0.553		145.357^{*}	145.282^{**}
		(1.083)	(1.019)		(76.176)	(63.002)
L_{09}				-1.312^{***}		-1.291^{***}
				(0.386)		(0.363)
const.	-0.103	-0.030	-0.101	10.745	-19.909^{***}	9.887
	(0.063)	(0.061)	(0.067)	(9.747)	(4.377)	(9.112)
R^2	0.17	0.02	0.18	0.32	0.13	0.45
Obs.	26	25	25	26	25	25

7. Conclusion

This paper provides the first comprehensive theoretical model for money markets. It offers a unified framework to analyze the relation between secured and unsecured money market funding liquidity, and the interaction with assets' market liquidity. Additionally, a central bank affects money markets with

conventional and unconventional monetary policies. Our model shows that markets can be fragile and adverse liquidity spirals can arise, when borrowers face funding problems in both the secured and the unsecured money market at the same time. In such a scenario, interest rate and loss spirals in the unsecured market, mutually reinforcing with liquidity spirals in the secured market, exert downward pressure on asset prices and deteriorate borrowers' capital. Moreover, we explain the dynamics of funding liquidity substitution between the secured and unsecured markets and highlight how the extent of money market fragility depends on banks' leverage.

We derive the optimal leverage ratio from a social planner's point of view and analyze central bank and regulatory policy. We show that monetary policies can prevent present fragility and restore liquidity, but with potentially adverse effects on leverage and future fragility. A relaxation of interest-rate (haircut) policy facilitates the substitution from secured (unsecured) to unsecured (secured) funding. However, both interest-rate and haircut policies are ineffective in reducing excess leverage. In contrast, an asset purchase program allows borrowers to reduce leverage and risky securities. Nevertheless, the resulting slackness of borrowers' funding constraints can incentivize the build-up of future leverage and the purchase of other risky assets.

Regarding financial market regulation, our model suggests that the joint funding conditions of unsecured and secured liquidity matter for financial stability. Regulation should strive for slack funding constraints across banks and time to allow banks to re-allocate money market liquidity between secured and unsecured funding and to prevent liquidity spirals and market freezes. It also shows that countercyclical maximum leverage ratios and capital buffers enhance banks' resilience to future shocks more adequately than static measures. A capital buffer restrains borrowers from fully encumbering their capital for margins to obtain secured funding, and relaxes the capital constraint. However, it is not sufficient on its own, as it may induce banks to invest in low-margin assets, resulting in excess leverage and larger vulnerability to an asset shock. Thus, an important policy implication from our model is that the most ef-

fective regulatory measure is a combination of a countercyclical leverage ratio and a haircut-based capital buffer, i.e., capital reserves inversely linked to haircuts. The former would prevent excess leverage while the latter would ease funding strains in the secured market. Jointly, these regulatory measures counteract future fragility and make banks more resilient to adverse market conditions.

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Appendix A: Proofs

Proof of Lemma 1. From Equation (15), the credit risk premium is $\mu(L_t) = \frac{\theta}{1-\theta} > 0$. It increases with leverage, because $\frac{\partial \mu}{\partial L_t} = \frac{1}{(1-\theta)^2}\frac{\partial \theta}{\partial L_t}$ and $\frac{\partial \theta}{\partial L_t} > 0$. Parts (ii) and (iii) of Lemma 1 follow directly from Equation (14).

Proof of Lemma 2. In both the secured and unsecured market, fragility arises if borrowers' funding needs increase the more the market price decreases below its fundamental value. Fragility in the secured market holds as shown in Brunnermeier and Pedersen (2009). Equivalently, from $Z_1 > 0$ it follows that $p_1 \leq \nu_1$ and $x_1^u \geq 0$. From the market clearing condition, we have that $x_1^u = -\sum_{k=0}^{1} y_1^k$. From Equation (22), we get

$$\frac{\phi_1 - i_1^u}{\frac{\partial \mu}{\partial L_1} p_1} W_1 \geq Z_1 + \frac{2(p_1 - \nu_1)}{\gamma(\sigma_2)^2}. \tag{37}$$

We prove non-monotonicity in the borrowers' demand function for an exogenous shock $\eta_1 < 0$. Therefore, we re-arrange Equation (37) for η_1,

$$G(p_1) := \frac{Z_1 + \frac{2(p_1-\nu_1)}{\gamma(\sigma_2)^2}}{\frac{\phi_1 - i_1^u}{\frac{\partial \mu}{\partial L_1} p_1}} - p_1 x_0^u - b_0 \leq \eta_1, \tag{38}$$

where $G(p_1)$ intuitively measures the borrower's funding need in equilibrium as a function of the market price p_1. Thus, fragility occurs if $G'(p_1)$ can be negative:

$$G'(p_1) := \frac{\frac{x_1^u}{W_1}\frac{2}{\gamma(\sigma_2)^2} - \left(Z_1 + \frac{2(p_1-\nu_1)}{\gamma(\sigma_2)^2}\right)\left[\frac{\frac{\partial \mu}{\partial L_1} p_1 \frac{\partial(\phi_1 - i_1^u)}{\partial p_1} - (\phi_1 - i_1^u)\frac{\partial \mu}{\partial L_1}}{(\frac{\partial \mu}{\partial L_1} p_1)^2}\right]}{\left(\frac{\phi_1 - i_1^u}{\frac{\partial \mu}{\partial L_1} p_1}\right)^2} - x_0^u. \tag{39}$$

For the individual terms, we have $\frac{\partial \phi_1}{\partial p_1} < 0$ and $\frac{\partial i_1^u}{\partial p_1} = \frac{\partial \mu}{\partial L_1}\frac{\partial L_1}{\partial p_1} < 0$, because $\frac{\partial L_1}{\partial p_1} < 0$ since $A_0 > W_0$. Since Equation (39) is decreasing in all terms containing x_0, there exists a position $\underline{x}^u$, such that for $x_0 > \underline{x}^u$, the whole expression is decreasing in p_1, and $G'(p_1) < 0$.

Considering both markets jointly, the market is thus commonly fragile if the borrowers' total position $x_0 > \underline{x}$, where $x_0 = x_0^s + x_0^u$, such that the market price cannot be chosen to be continuous after a shock, even if borrowers have access to both secured and unsecured funding liquidity.

Proof of Proposition 1. We show the price sensitivity to a fundamental shock $\Theta_1 < 0$ when funding constraints bind. As in Brunnermeier and Pedersen (2009), using the implicit function theorem to compute the derivatives at the equilibrium position, for the secured market we have

$$m_1 \left(Z_1 - \frac{2(\nu_1 - p_1)}{\gamma(\sigma_2)^2} \right) = b_0 + x_0^s p_1 + \eta_1. \tag{40}$$

Among all parameters, margin m_1, volatility σ_2, and the fundamental value ν_1 are functions of the shock Θ_1. In an illiquid equilibrium, the denominator of Equation (25) is positive. Thus, liquidity spirals occur because $\frac{\partial m_1}{\partial p_1} < 0$ and $\frac{\partial m_1}{\partial \Theta_1} < 0$, representing the margin spiral, and $\frac{\partial W_1}{\partial p_1} > 0$ for $x_0 > 0$ showing the loss spiral.

The equilibrium position in the unsecured market is given by Equation (37). Equation (37), and σ_2, ν_1, and ϕ_1 are functions of Θ_1. As above, the whole term is positive in an illiquid equilibrium, and so the denominator of Equation (26) is negative. Due to the interest rate spiral, $\frac{\partial i_1^u}{\partial p_1} < 0$, and loss spiral, $\frac{\partial W_1}{\partial p_1} > 0$ for $x_0 > 0$, the value of the denominator increases, i.e., becomes less negative, and liquidity spirals exert a negative effect on the price.

Proof of Proposition 2. Part (i) of this proposition follows directly from the way leverage is computed. For the same capital W_1, $L_1 = L_1^s + L_1^u$ is additive, and the only variable by which secured and unsecured leverage differ is the respective position x_1^s and x_1^u. Since L_1^s decreases and L_1^u increases, the share $\Delta \frac{x_1^u}{x_1^s}$ increases, provided that $\Theta_1 < 0$. By the definition of funding volumes, this refers to an

increase in M_1^u relative to M_1^s.

For part (ii) of Proposition 2, the covariance is positive since both shadow costs φ_1^s and φ_1^u depend on the same random variable p_1. When constraints bind and the price decreases, shadow costs increase due to the increase in ϕ_1. In a liquid equilibrium, constraints are slack and the funding costs increase from a small reduction in p_1, such that shadow costs decrease jointly.

Proof of Lemma 3. Since $Z_1 \geq 0$, borrowers' funding constraints are slack when $dx_1 \geq 0$, which happens if wealth $dW_1 \geq 0$. The intertemporal time-1 budget constraint is slack for

$$W_0 + x_0\phi_0 \geq x_0^s c_0^s + x_0^u c_0^u. \tag{41}$$

In the time-0 equilibrium, we have $dW_0 = 0$, and $dx_0 = 0$ implies that $dx_0^s = -dx_0^u$, i.e., less secured funding is perfectly substituted by more unsecured funding, and

$$\phi_0 - c_0^u \geq \phi_0 - c_0^s. \tag{42}$$

If the shadow costs in the unsecured market are larger (or equal) than the shadow costs in the secured market, a binding capital constraint in response to a shock at time 1 allows the borrower to fund $dx_1 \geq 0$ in the unsecured market due to a sufficiently large marginal return. If Equation (42) holds with "<", then a (small) capital loss leads to binding funding constraints in both markets and $dx_1 < 0$.

Proof of Proposition 3. Part (i) of Proposition 3 follows directly from Lemma 3. Since $c_0^u = i_0^u(L_0)$ and for any given expected return ϕ_0, the spread $\phi_0 - i_0^u$ is smaller the larger is L_0. From Proposition 1, for $x_0 > \underline{x}_0$, there is fragility at time 1 if $\Theta_1 < 0$ and $Z_1 > 0$. Consequently, funding illiquidity, as measured by the shadow costs, increases until the market clears. Since p_1 decreases from $dx_1 < 0$, the price drop

is larger the higher was L_0. If $dx_1 < 0$ is so large that $p_1 \leq \overline{p}_1$, where $\overline{p}_1$ is such that $x_0(p_0 - \overline{p}_1) \geq W_0$, funding in the unsecured market is unavailable for a borrower at an interest rate that allows borrowing sufficient funds to clear the market at a price $p_1 > \overline{p}_1$ (part (ii)).

Proof of Proposition 4. We show this proposition for any asset $j \in J$. Since there is commonality in funding liquidity, $Cov_0(\varphi_1^s, \varphi_1^u) > 0$, common money market illiquidity is the volume-weighted average of the shadow costs, i.e., $\varphi_1 = \frac{1}{x_1}(x_1^s \varphi_1^s + x_1^u \varphi_1^u)$. Common funding costs c_0 at time 0 are derived similarly. Hence, the borrower at time 0 solves $\mathbb{E}_0[W_1(1+\varphi_1)]$ for x_0 and gets

$$\mathbb{E}_0\left[[p_1 - p_0(1+c_0)](1+\varphi_1)\right] = 0. \tag{43}$$

Since covariance is bilinear, rearranging yields a time-0 price of

$$p_0 = \frac{\mathbb{E}_0[p_1(1+\varphi_1)]}{(1+c_0)\mathbb{E}_0[1+\varphi_1]} = \frac{\mathbb{E}_0[p_1]}{1+c_0} + \frac{\mathrm{Cov}_0[p_1, \varphi_1]}{(1+c_0)\mathbb{E}_0[1+\varphi_1]}. \tag{44}$$

Accordingly, the time-0 price is lower the higher are funding costs c_0, and if an asset has liquidity risk, $Cov_0[p_1, \varphi_1] < 0$.

Proof of Proposition 6. We show the results of Proposition 6 jointly for all parts (i) to (iii). First, a reduction in i_{cb} reduces money market interest rates according to Equation (14), which relaxes the borrowers' leverage constraint and, ceteris paribus, increases unsecured funding,

$$\frac{\partial M_1^u}{\partial i_{cb}} = \frac{-W_1}{(1-\theta)\mu'(L_1)} < 0. \tag{45}$$

Second, haircut policy relaxes the capital constraint and provides secured funding liquidity from the central bank. We have

$$\frac{\partial M_1^s}{\partial h_{cb}} = \frac{-W_1}{h_{cb}^2} < 0. \tag{46}$$

Third, asset purchases $x_1^{cb} > 0$ decrease the supply of securities to $-(Z_1 - x_1^{cb})$, which leads to market clearing at a higher equilibrium price. Since interest and haircut policy simply reverse the downward pressure on the price, leverage L_1 and thus shadow costs remain unchanged. Finally, asset purchases reduce borrowers' total position x_1 due to positive demand x_1^{cb}, and L_1 decreases.

Appendix B: Additional Tables

Table A.1
Descriptive Statistics

This table provides a list of all banks and variables included in the regression analysis. Descriptive statistics for all variables are given below. All currencies were converted into Euro.

Bank	Country	ΔShare	Δm	ΔCB	L_{09}	ΔBonds
Banca Monte dei Paschi di Siena SpA	IT	-0.048	0.203	0.016	13.090	4.748
Banco Bilbao Vizcaya Argentaria SA	ES	0.148	0.908	-0.012	17.393	-4.095
Banco BPI SA	PT	-0.221	3.131	0.110	21.989	-0.786
Banco Popular Español SA	ES	0.508	0.998	-0.003	15.304	1.287
Banco Santander SA	ES	-0.522	1.013	-0.010	15.033	-14.883
Barclays plc	UK	0.422	0.238	-0.034	23.580	-27.386
BNP Paribas SA	FR	0.050	0.913	-0.003	25.611	-54.812
Caja de Ahorros y Pensiones de Barcelona	ES	0.221	0.905	0.010	12.703	13.527
Commerzbank AG	DE	0.000	0.409	0.008	31.762	-53.481
Dexia SA	BE	-0.046	1.104	-0.041	48.184	-35.496
Erste Group Bank AG	AT	0.001	1.961	-0.063	18.163	-24.898
HSH Nordbank AG	DE	0.356	0.015	-0.001	39.281	-12.635
ING Groep NV	NL	0.020	0.956	-	29.253	-35.658
Intesa Sanpaolo SpA	IT	-0.474	0.313	0.007	11.861	-9.631
Jyske Bank A/S	DK	0.637	7.184	-0.030	17.928	-0.618
KBC Group NV	BE	-0.267	1.433	-0.032	18.876	-42.850
Landesbank Baden-Württemberg	DE	-0.170	0.073	-0.034	39.116	-83.138
Lloyds Banking Group plc	UK	-0.491	-0.141	-0.002	23.290	-9.198
Norddeutsche Landesbank-Girozentrale	DE	-0.130	-0.110	-0.005	41.152	-44.364
Raiffeisen Bank International AG	AT	-0.225	0.410	-0.010	10.896	-11.483
Skandinaviska Enskilda Banken AB	SE	0.000	-0.024	0.000	23.159	-13.011
Société Générale SA	FR	0.000	1.524	0.001	21.856	-24.198
Svenska Handelsbanken AB	SE	0.000	-0.220	0.000	25.549	-7.092
Sydbank A/S	DK	-0.454	-0.220	0.000	17.309	-0.208
Bank of Ireland	IE	-0.114	-0.058	0.239	28.355	4.286
UniCredit SpA	IT	-0.014	0.187	0.008	15.560	-29.929
Mean		-0.031	0.889	0.005	23.317	-19.846
Median		-0.014	0.410	-0.002	21.923	-12.823
Std. dev.		0.291	1.502	0.056	9.858	22.695
Min		-0.522	-0.220	-0.063	10.900	-83.138
Max		0.637	7.184	0.239	48.184	13.527

Table A.2
Regression Results for Funding Shares and Bond Holdings

This table shows the results of regressing changes in banks' funding shares (Columns (1) to (3)) and changes in bond holdings (Columns (4) to (6)) on explanatory variables derived from our model. Banks for which central bank figures are unavailable include ING Groep NV and Dexia SA. Standard errors are shown in parentheses. The stars ***, **, and * indicate statistical significance at the 1%, 5%, and 10% level, respectively.

	Δ_{09}^{11} Share			Δ_{09}^{11} Bonds		
	(1)	(2)	(3)	(4)	(5)	(6)
Δm	0.009		0.009			
	(0.007)		(0.007)			
ΔCB		-2.144^{*}	-2.112^{*}		160.870^{**}	118.933^{*}
		(1.152)	(1.132)		(72.423)	(68.074)
L_{09}				-1.437^{***}		-1.190^{***}
				(0.411)		(0.499)
const.	-0.092	-0.007	-0.045	9.830	-25.676^{***}	1.641
	(0.075)	(0.078)	(0.082)	(10.363)	(4.895)	(12.291)
R^2	0.07	0.14	0.20	0.34	0.18	0.36
Obs.	26	24	24	26	24	24