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Medium-term planning for thermal electricity production

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Outlook

- We aim at a simplified model for **mid-term planning for thermal electricity production** that can be used for repetitive calculation
- **Optimization model:**
 - Costs: fuel, fixed and variable operating costs
 - Different fuels are bought at the spot market and stored to produce electricity
 - We allow for trading at CO2 spot market (emission certificates)
 - Production is sold at the spot market
 - **Maximization of the asset value** (cash + value of stored fuels) at the end of the planning horizon

Production

- Consider **time periods** $t \in 0, 1, \dots, T$ with length Δ_t .
- We model **thermal generators** i which may use different **fuels** j to produce energy $x_{t,i,j}$ and are characterized by **efficiencies** $\eta_{i,j}$ and **maximum power** β_i , in particular
- We consider Δ_t as **weeks**. If Δ_t smaller, integer decisions related to switching, ramping, minimum power production constraints etc. become relevant
- A **cost model** for the generators:
 - **Fuel costs** (spot markets) are given by $P_{t+1,j}^f(\omega) \cdot x_{t,i,j} / \eta_{i,j}$.
 - **Variable operating costs** are estimated by $\gamma_i \cdot \sum_{j=1}^J x_{t,i,j} / (\beta_i \Delta_t)$
 - In addition we consider **fixed operating costs** κ_i per time unit.

Storage

- We model **storage** s_t , cumulated CO_2 -**emissions** e_t , **cumulated** CO_2 -**certificates** a_t and a **cash position** w_t .
- With $f_{t,j}$ denoting the amount of fuel j bought at time t storage develops as

$$s_{0,j} = s_j^0 \quad (1)$$

$$s_{t,j} = s_{t-1,j} - \sum_{i=1}^I \frac{x_{t-1,i,j}}{\eta_{i,j}} + f_{t,j} \quad \forall t > 0, j \quad (2)$$

$$0 \leq s_{t,j} \leq \bar{s}_j \quad \forall t, j, \quad (3)$$

and production is restricted by

$$\sum_{i=1}^I \frac{x_{t,i,j}}{\eta_{i,j}} \leq s_{t,j} \quad \forall t, j. \quad (4)$$

CO_2 -accounting

- If ε_{ij} denotes the CO_2 -**emissions** (t per MWh) of fuel j if burned by generator i , the amount e_t of CO_2 emitted is

$$e_0 = e^0. \quad (5)$$

$$e_t = e_{t-1} + \sum_{j=1}^J \sum_{i=1}^I \frac{\varepsilon_{ij}}{\eta_{i,j}} \cdot x_{t-1,i,j} \quad \forall t > 0.$$

- At each time it is possible to buy ($c_t \geq 0$) or sell ($c_t < 0$) certificates at the market for CO_2 **allowances** at prices P_t^c . Hence the accumulated amount of pollution covered by certificates is

$$a_0 = a^0$$
$$a_t = a_{t-1} + c_t \quad \forall t > 0.$$

Cash accounting

- The cash position starts with $w_0 = w^0 - \sum_{j=1}^J P_{0,j}^f f_{0,j}$.

and develops by

$$\begin{aligned} w_t &= (1 + \rho_l)w_{t-1}^+ - (1 + \rho_b)w_{t-1}^- \\ &+ P_t^x \cdot \sum_{i=1}^I \sum_{j=1}^J x_{t-1,i,j} \\ &- \sum_{j=1}^J P_{t,j}^f \sum_{i=1}^I f_{t,j} \\ &- P_t^c c_t \\ &- \sum_{j=1}^J \zeta_j \frac{(s_{t,j} + s_{t-1,j})}{2} \\ &- \sum_{i=1}^I \frac{\gamma_i}{\beta_i} \cdot \sum_{j=1}^J x_{t-1,i,j} - \kappa_i \cdot \Delta_{t-1} \quad 0 < t < T \end{aligned}$$

- At time T no fuel is bought anymore, but a penalty has to be paid if certificates are not sufficient: $(\theta + P_T^c)(e_T - a_T)^+$

Optimization problem: Objective

- The producer aims at the asset value (excluding the value of generating units) at the end of the planning horizon

$$v_T = w_T + \sum_{j=1}^J s_{T,j} \cdot P_{T,j}^f. \quad (6)$$

- All prices are stochastic processes. Decisions at time t have to be taken with information available at time t . Hence the decision variables are also stochastic. The equations and inequalities have to be understood as “holds almost surely”.
- Our objective is a mixture of expectation and $AV@R$ with a mixing factor $0 \leq \lambda \leq 1$

$$\begin{aligned} \max_{x,f,c,(s,w,v,a,e)} \quad & (1 - \lambda) \cdot \mathbb{E}[v_T] + \lambda \cdot AV@R_\alpha(v_T) \\ \text{s.t.} \quad & \text{all constraints} \\ & x, f, c \triangleleft \Sigma \\ & s, w, v, a, e \triangleleft \Sigma \end{aligned} \quad (7)$$

Modeling the risk factors

- We look at daily European commodity prices:
 - Gas prices: Gaspool (GPL), April 2007-December 2011
 - Crude oil prices: Brent Crude oil, May 2003-December 2011
 - EUA: April 2008-December 2011
 - Coal: North West Europe(NWE) steam coal marker, December 2005-May 2012
 - Electricity prices: EEX Phelix, September 2008-December 2011
- We employ a common model for simulating commodity prices: gas, oil, coal and emissions allowances (EUA)
 - Similar patterns among commodity prices: leptokurtic distribution, negatively skewed returns, non-stationary variation are described by **Geometric Brownian Motion with Jump Process (GBMPJ)/Merton model**
- Spot electricity prices behave considerably different from other commodities and need a separate modeling approach: **Regime Switching Model**

Regime switching model for electricity prices

$$MCP_t := \begin{cases} f_t^L - Spike_t^- & \text{with } p_t^- \\ f_t \cdot \exp(r_t) & \text{with } 1 - p_t^- - p_t^+ \\ f_t^U + Spike_t^+ & \text{with } p_t^+ \end{cases}$$

with

$$Spike_t^+ \sim \text{Exp}(1/\lambda_t^+)$$

$$Spike_t^- \sim \text{Exp}(1/\lambda_t^-)$$

$$r_t \sim N(0, \sigma_t^2)$$

$$f_t^L = f_t * \exp(\alpha_L * \sigma_t)$$

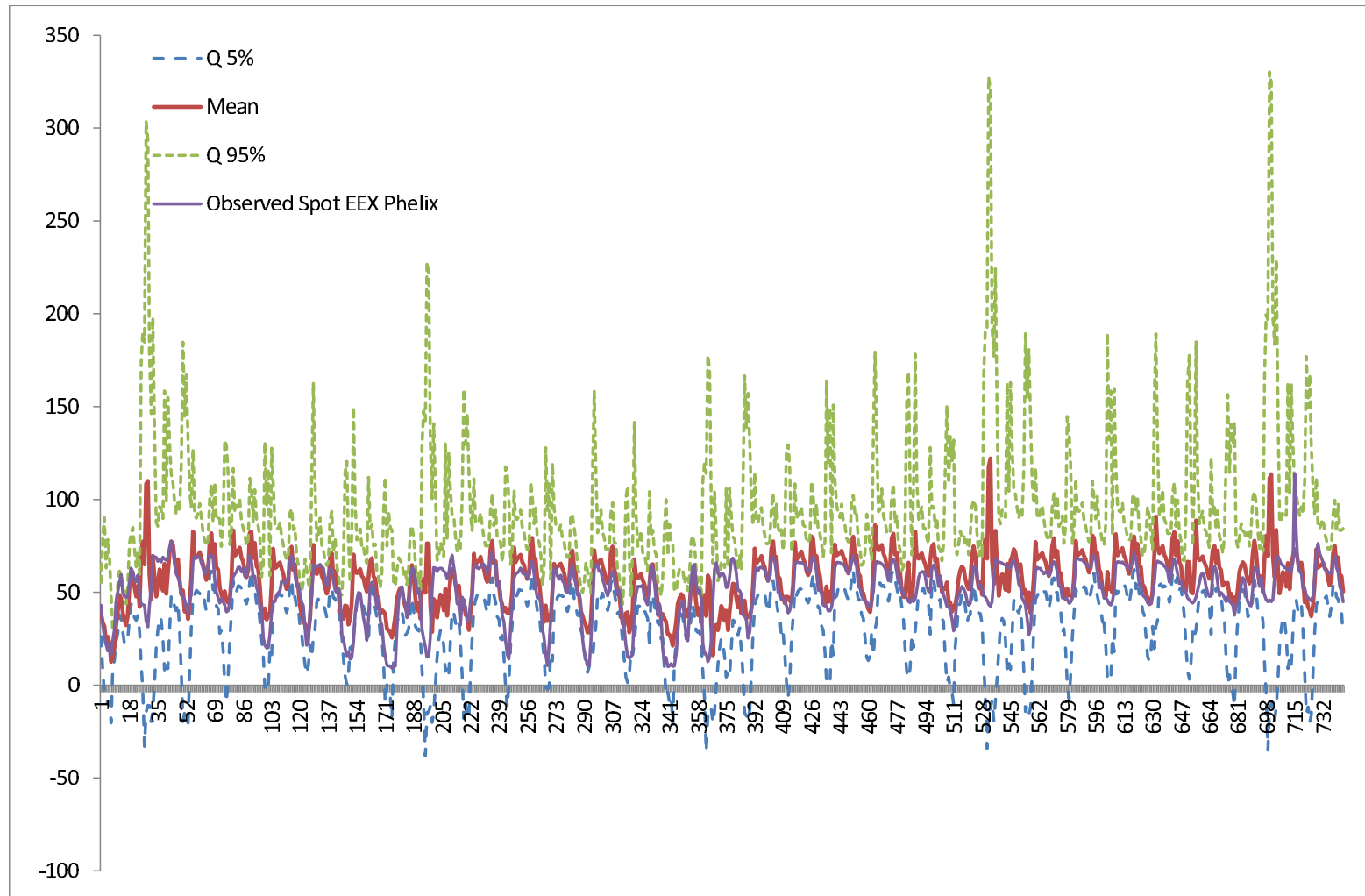
$$f_t^U = f_t * \exp(\alpha_U * \sigma_t)$$

Energy prices: Results

	Sample	Parameter estimation					
		α	σ	λ	μ	δ	ML
Crude oil (monthly)	01.05.2003-01.12.2011	0.325 (0.141)	0.259 (0.013)	80.373 (19.490)	-0.0017 (0.0017)	0.027	-5314.05
	01.05.2003-01.12.2010	0.283 (0.149)	0.271 (0.013)	68.981 (17.156)	-0.0013 (0.0020)	0.028	-4705.27
Heating oil (monthly)	01.05.2003-01.12.2011	0.218 (0.134)	0.245 (0.011)	99.953 (18.398)	-0.0005 (0.0013)	0.028	-5405.37
	01.05.2003-01.12.2010	0.158 (0.149)	0.253 (0.013)	103.751 (20.784)	0.0000 (0.0014)	0.028	-4781.89
EUA (monthly)	01.04.2008-01.12.2011	0.178 (0.202)	0.254 (0.016)	81.165 (18.070)	-0.0051 (0.0029)	0.036	-2152.87
	01.04.2008-01.12.2010	0.327 (0.246)	0.268 (0.020)	78.921 (21.967)	-0.0057 (0.0037)	0.036	-1595.28
Gas (monthly)	01.04.2007-01.12.2011	0.321 (0.281)	0.379 (0.019)	99.790 (14.135)	-0.0006 (0.0038)	0.068	-2015.13
	01.04.2008-01.12.2010	0.316 (0.361)	0.423 (0.025)	105.479 (17.925)	0.0003 (0.0045)	0.071	-1514.65
Coal (weekly)	09.12.2005-01.12.2011	0.308 (0.117)	0.170 (0.020)	21.749 (7.506)	-0.0082 (0.0068)	0.053	-552.264
	09.12.2005-01.12.2010	0.437 (0.140)	0.172 (0.024)	25.860 (10.157)	-0.0098 (0.0071)	0.052	-450.484

Table 1: *ML Estimation results of the GMBJ model for oil, EUA, gas and coal spot prices. Standard errors are in paranthesis.*

Electricity prices: Out of sample results



Scenario trees

- The above introduced optimization problem (??) cannot be solved directly
- It is common in stochastic programming to approximate the relevant stochastic processes by *scenario trees* (which represent the filtration of the process)
- To keep the error of this approximation small, the tree should be “close” to an original process which can be measured by an appropriate distance $\mathbf{dl}$
- It can be shown that, under certain conditions, it holds for the difference between the original and the approximated problem

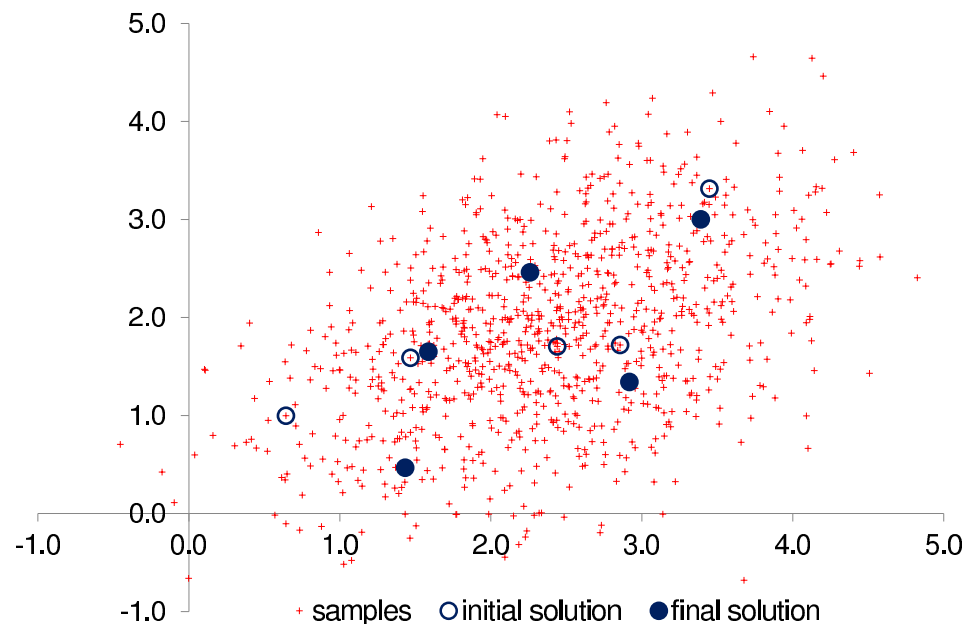
$$\left| v(\mathbb{P}) - v(\tilde{\mathbb{P}}) \right| \leq L \cdot \mathbf{dl}(\mathbb{P}, \tilde{\mathbb{P}}), \quad (8)$$

where $\mathbb{P}$ represents the original process and $\tilde{\mathbb{P}}$ its approximation

- Thus, by keeping the distance $\mathbf{dl}$ between the processes as small as possible, we minimize the error on the left-hand-side of (??)

Minimizing the distance for distributions

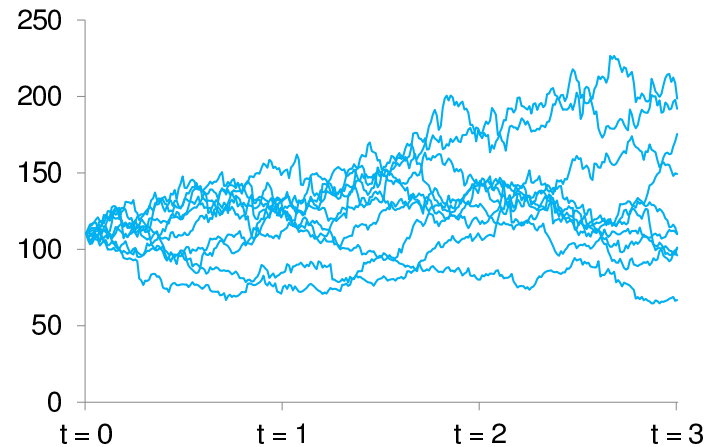
- Consider a random variable G that is either continuous or discrete (with a very large number of mass points)
- We want to approximate G by a simpler random variable $\tilde{G}$
- The distance between G and $\tilde{G}$ can be measured by the Wasserstein distance $d(G, \tilde{G})$
- It is known that the Wasserstein distance is related to a transportation problem
- The problem of minimizing the distance $d(G, \tilde{G})$ is solved by assigning data points to a few “clusters” (which represent the approximate distribution $\tilde{G}$)



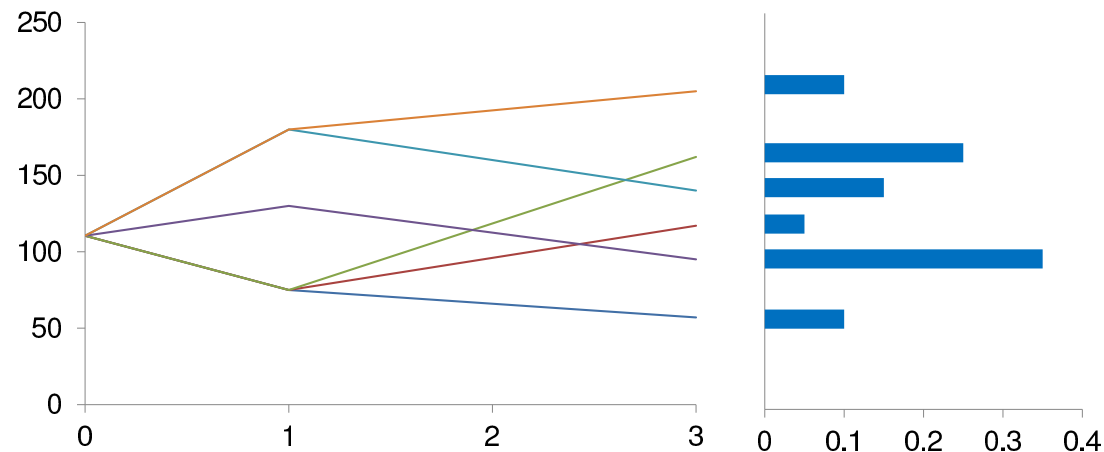
$$\mu = \begin{pmatrix} 2.37 \\ 1.83 \end{pmatrix}$$
$$\Sigma = \begin{pmatrix} 0.71 & 0.33 \\ 0.33 & 0.91 \end{pmatrix}$$

The scenario generation problem

- In a multistage problem, decisions are taken at discrete time steps
- The evolution of the data is described by a stochastic process:

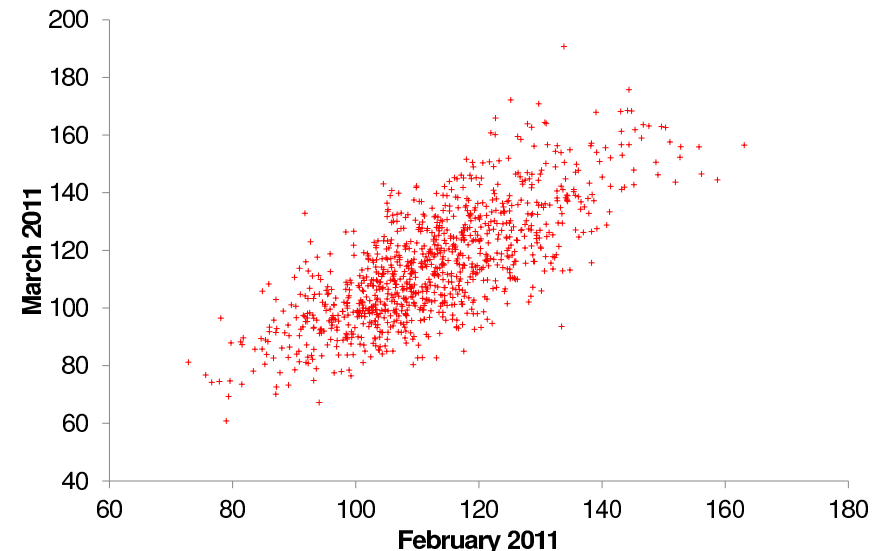
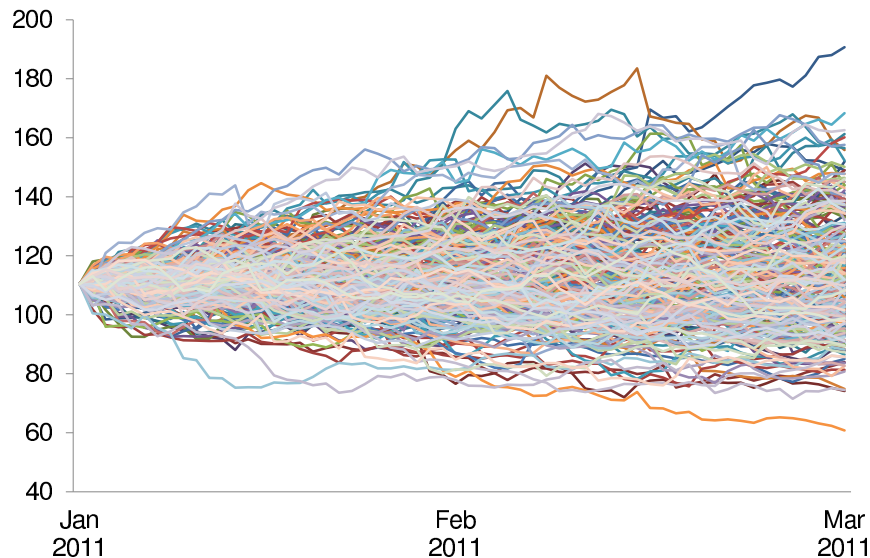


- This process is approximated by a *scenario tree* with corresponding path probabilities:



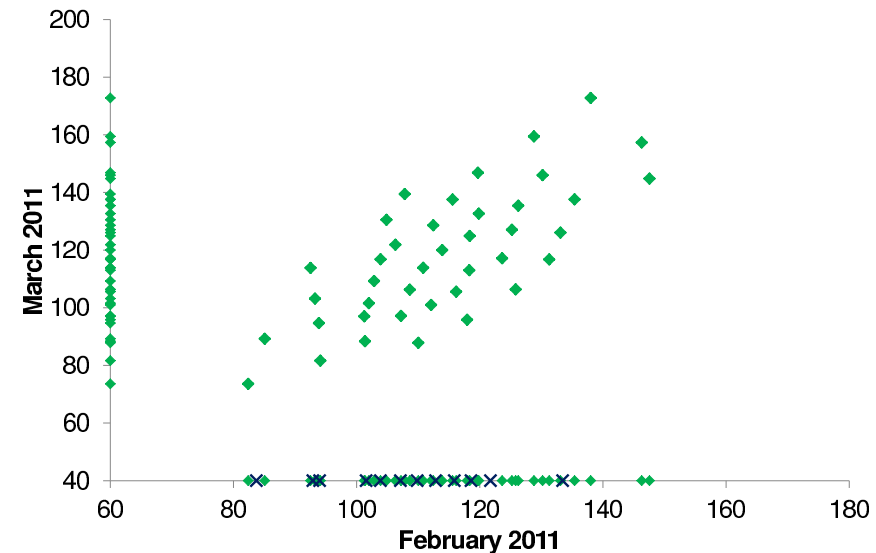
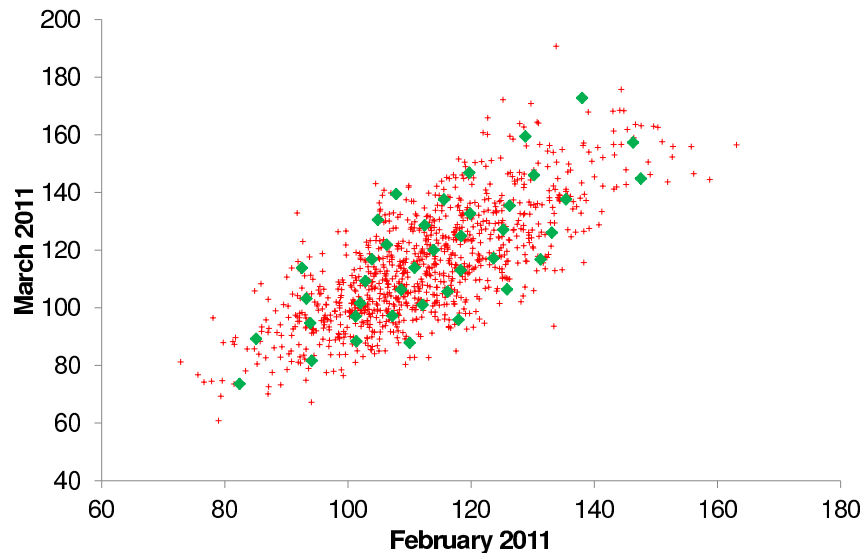
Facility location applied to processes (1)

- Find a tree that is
 - small enough as a numerically tractable approximation
 - large enough to capture important features of the problem
- The scenario tree should be as close as possible to the observed stochastic process
- We apply the concept of the Wasserstein distance again to simulated paths to construct a multistage tree
- The picture on the left shows 1000 simulations of the oil price from January to March 2011
- Assume we generate a tree with two stages ($T = 2$); the right picture shows the facility location view of 1000 paths in $\mathbb{R}^2$



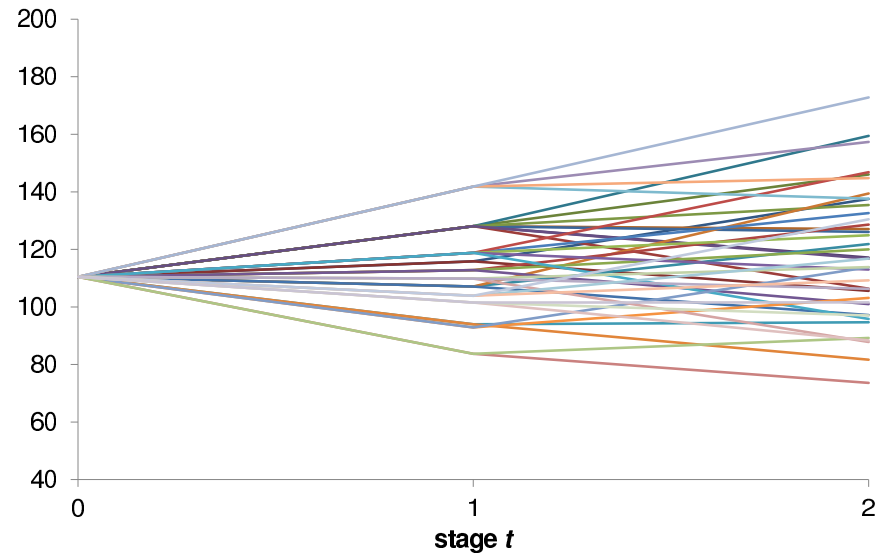
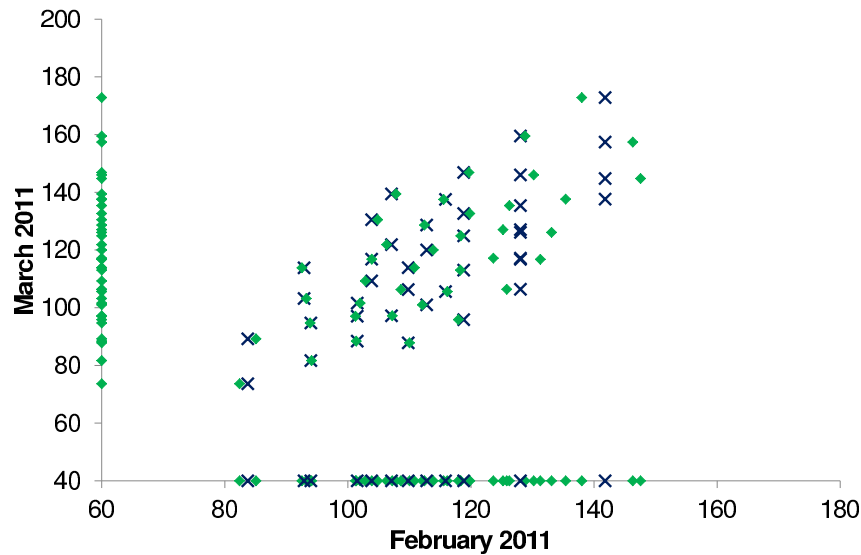
Facility location applied to processes (2)

- The facility location problem is now solved in a backward fashion, given a fixed number of nodes $n_1, \dots, n_T$ per stage
- We start at stage T with n_T clusters in $\mathbb{R}^T$, based on a multidimensional distance $d(\cdot)$
- For the stages $t = T - 1, \dots, 1$, we cluster n_t points from the n_{t+1} points found for the next stage, but using only data from stage 1 to t (i.e., the facility location problem is solved in $\mathbb{R}^t$)



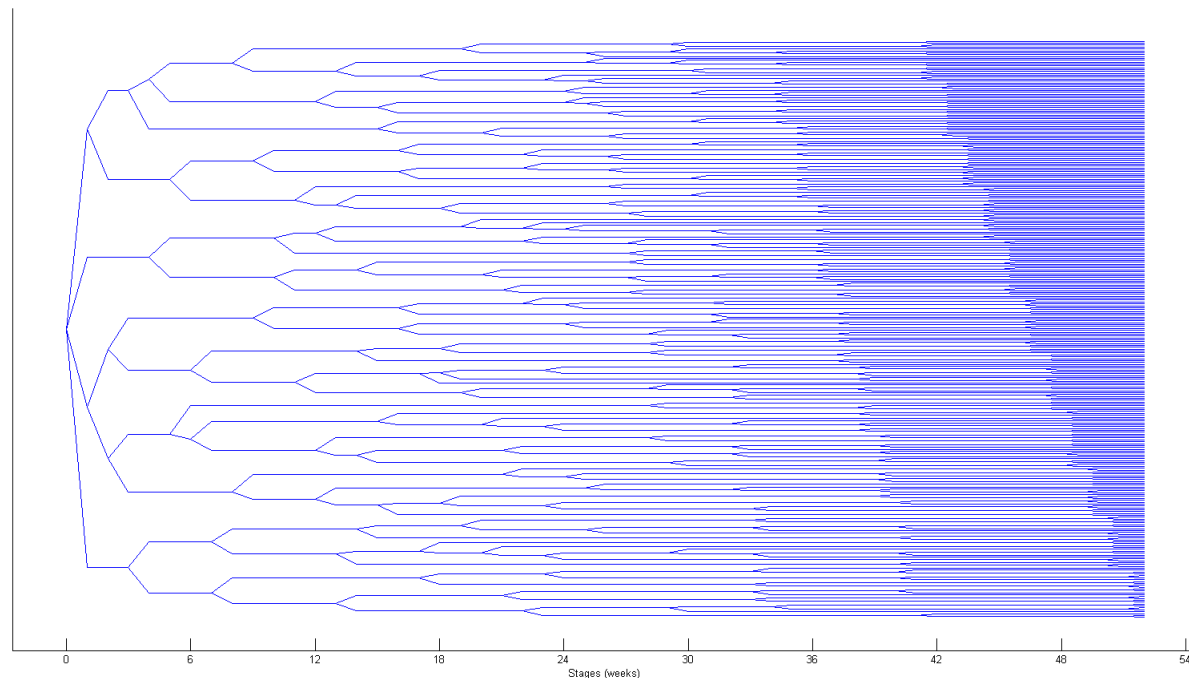
Facility location applied to processes (3)

- We store the allocation of data points to clusters to reconstruct the scenario structure
- The probabilities of the scenario paths are given by the probabilities of the clusters that correspond to the n_T leaves (found in the first step, not shown)



Extension to nested distances

- Pflug/Pichler (2012) introduced and analyzed a generalization of the well known Wasserstein distance
- Kovacevic/Pichler (2012) propose an algorithm for improving the distance between the trees
- This *nested distance* d_I takes the information from the filtration into account (rather than comparing only scenario paths)
- Based on this concept, the tree resulting from the first step is further improved by adjusting the probabilities and values



System specification

- The thermal system consists of:
 - Two combined cycle plants (gas/oil)
 - Three combustion turbines (gas/oil)
 - One steam turbine (coal)
- Premises:
 - We start with a small amount of small fuel
 - Cash position: 1 million EUR
 - Interest on cash: 2.5%; on debt: 12.5%
 - AV@R calculated at level $\alpha = 0.05$
 - Mixture parameter λ is set to 0.5 in the standard case
- Implementation: AIMMS 3.12, solver GUROBI 4.6

GUI: input trees



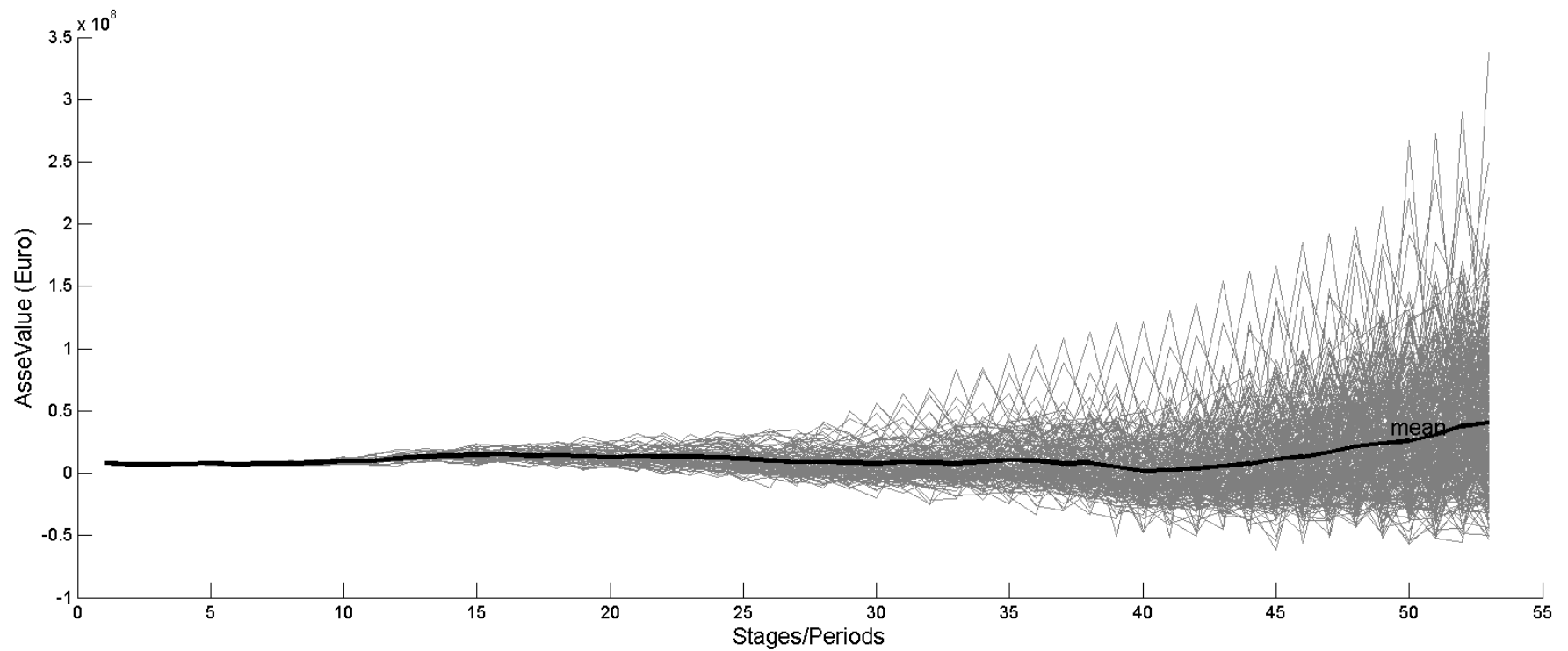
GUI: input trees



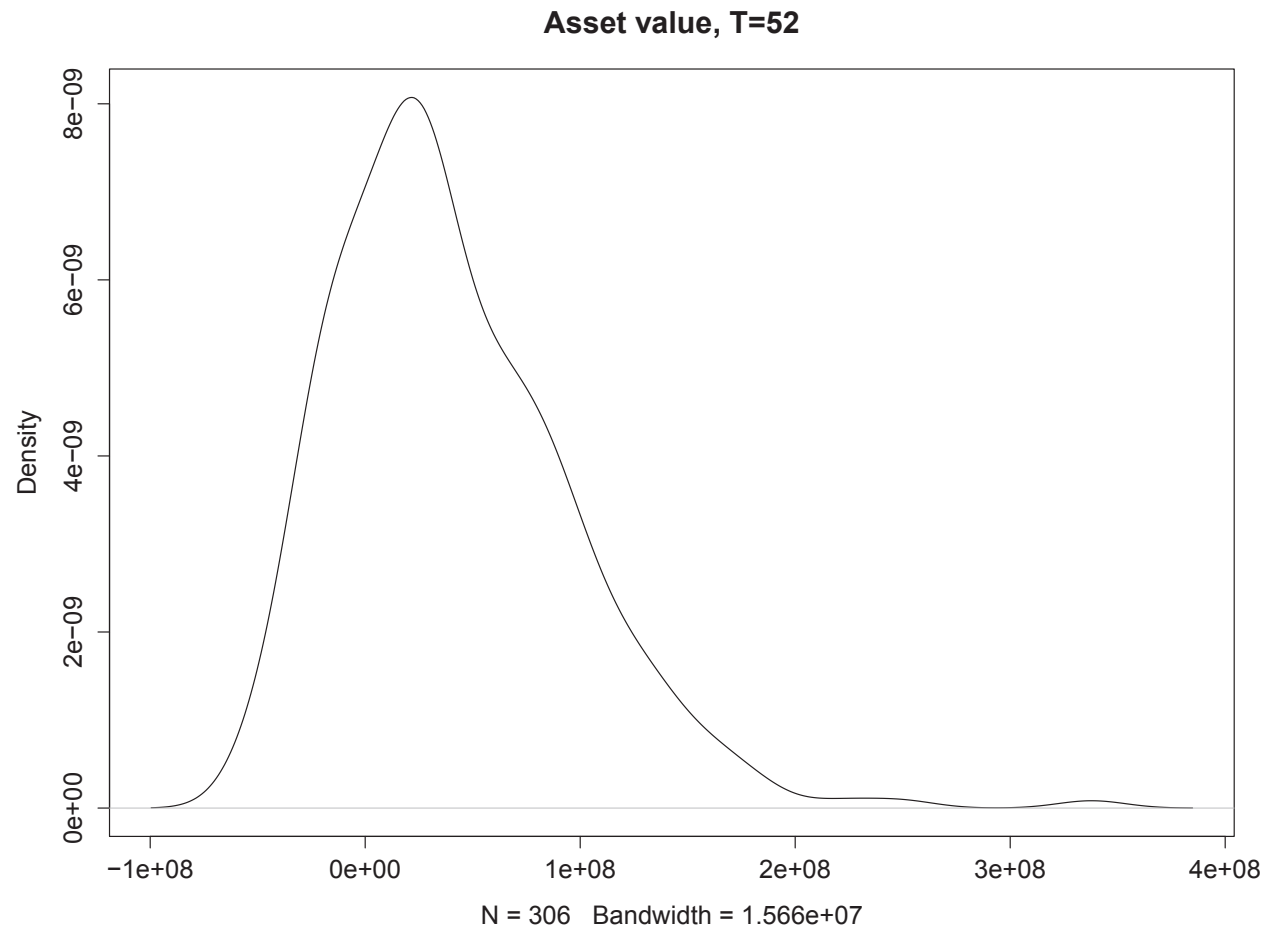
GUI: input trees



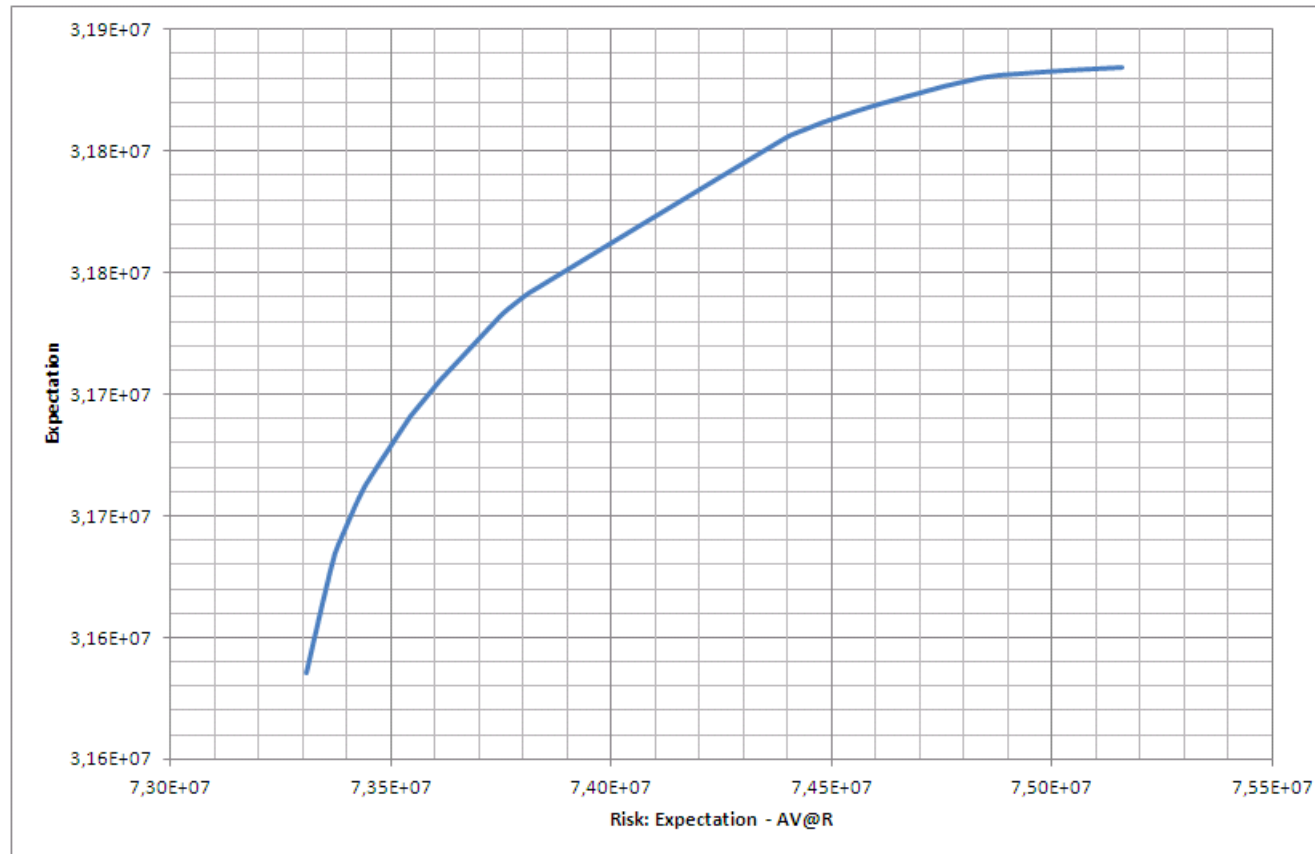
Development of the asset value



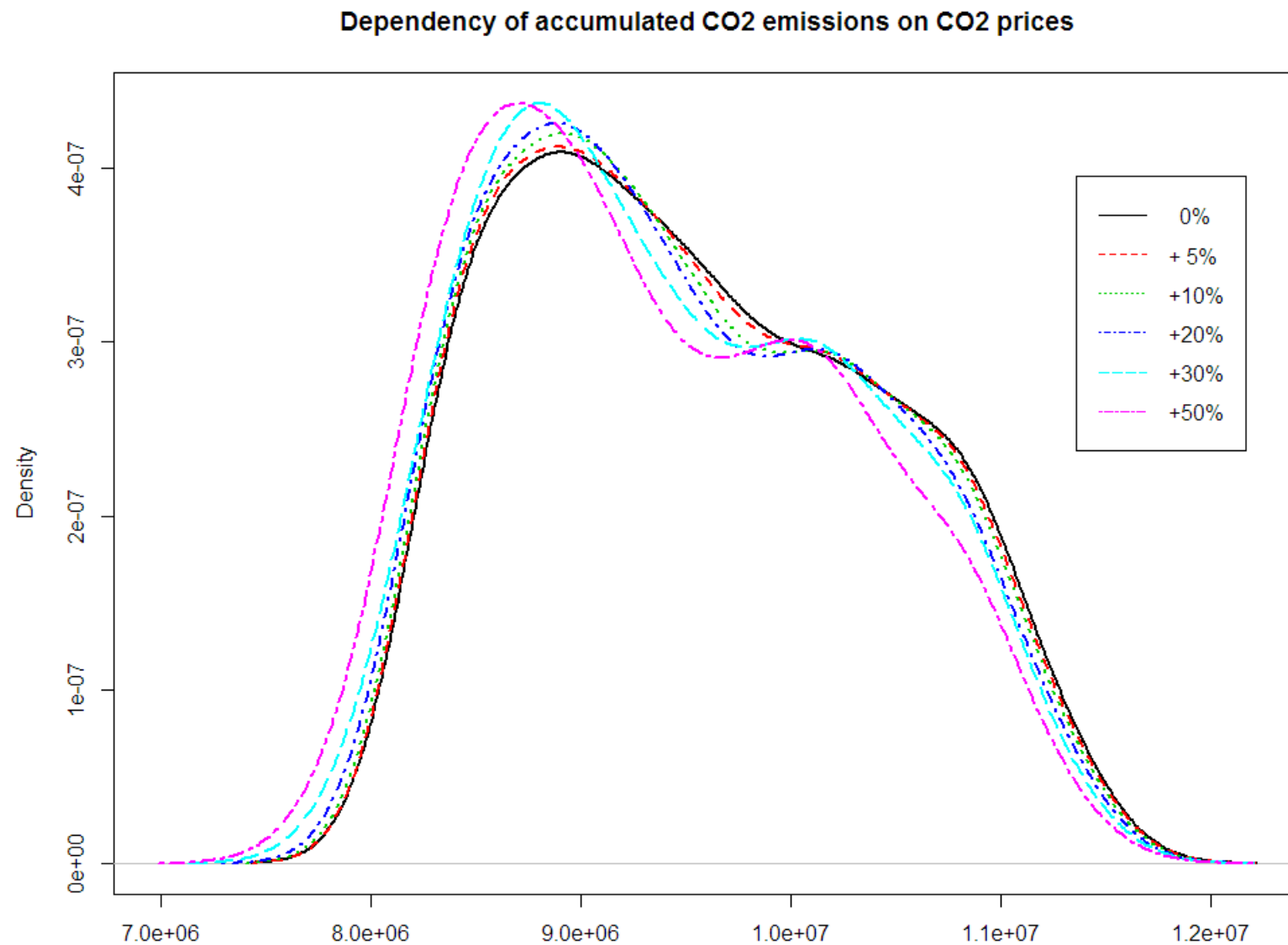
Distribution of the asset value - end of the planning horizon



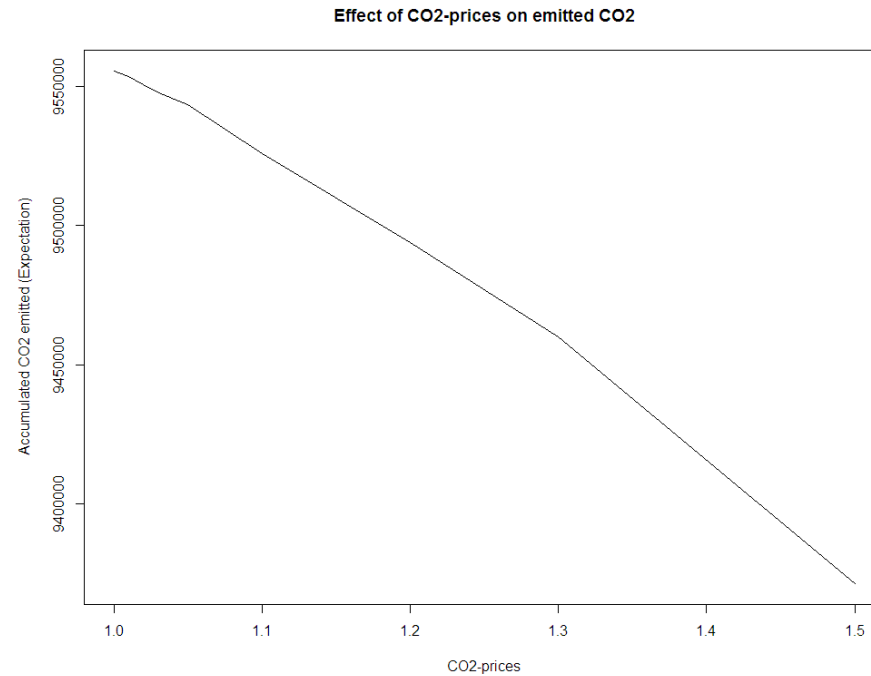
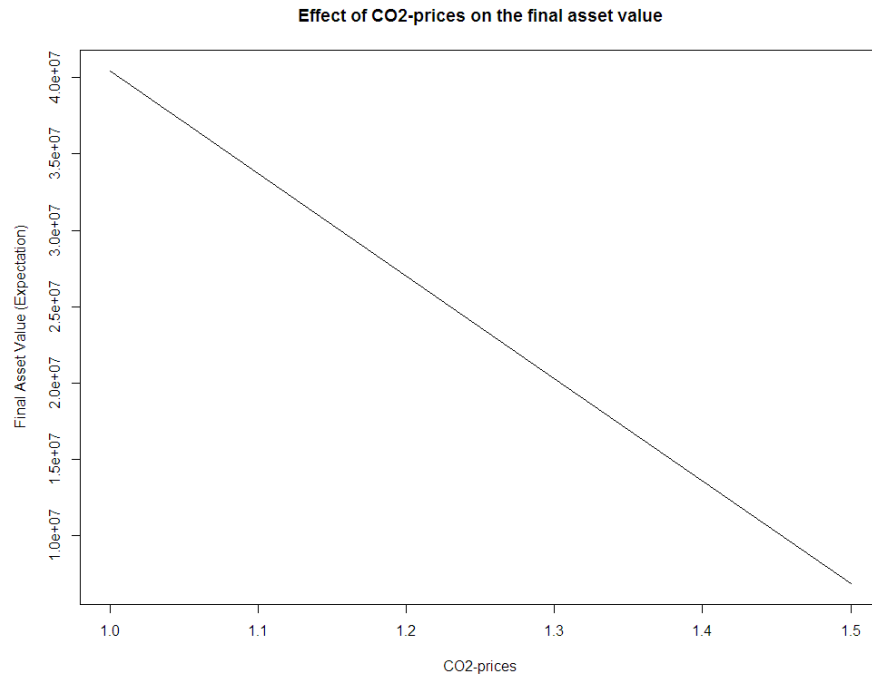
Efficient frontier. Tradeoff expected end value vs. riskiness of the end value



Effect of increases in CO2 prices on the accumulated CO2 emissions

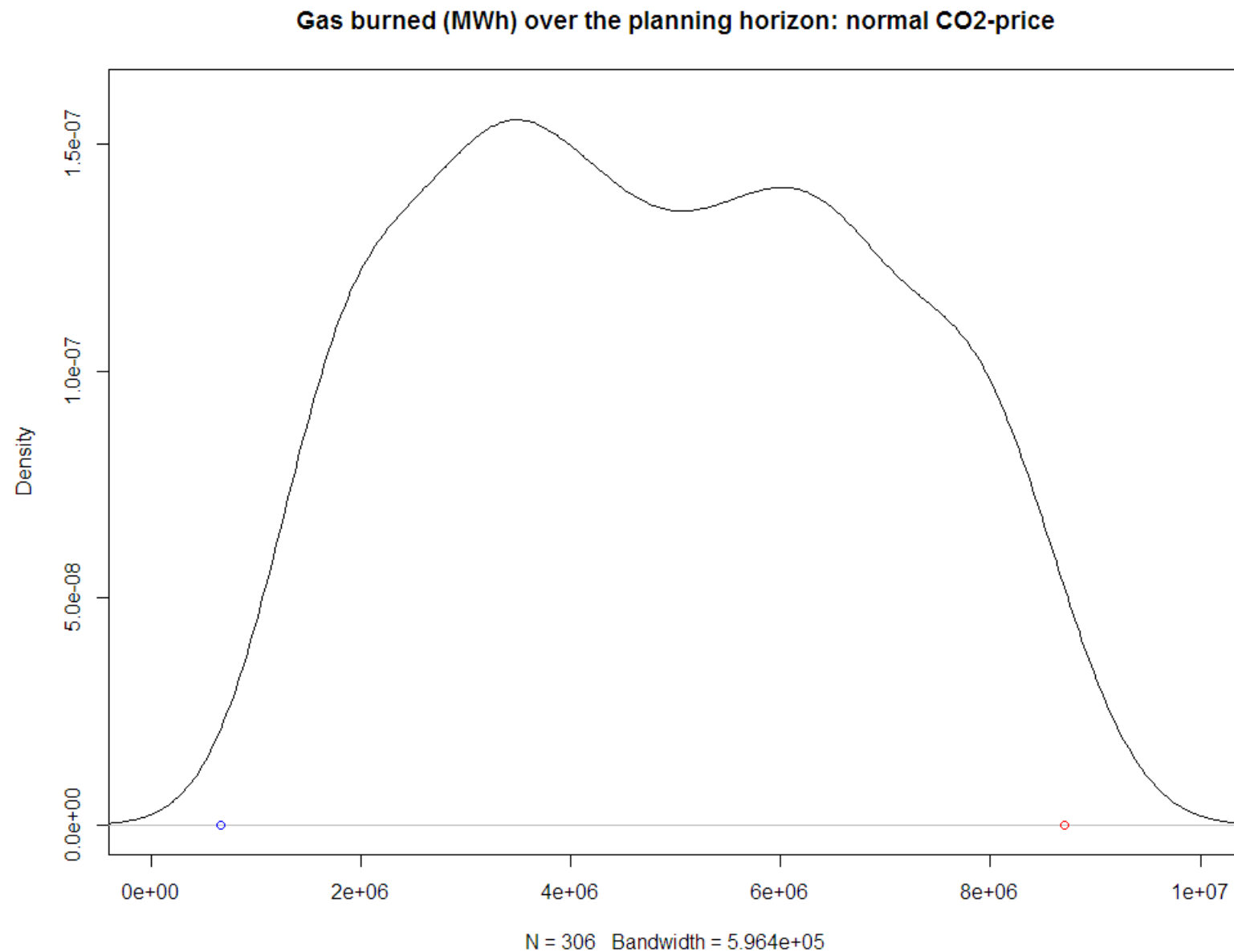


Effect of CO2 prices

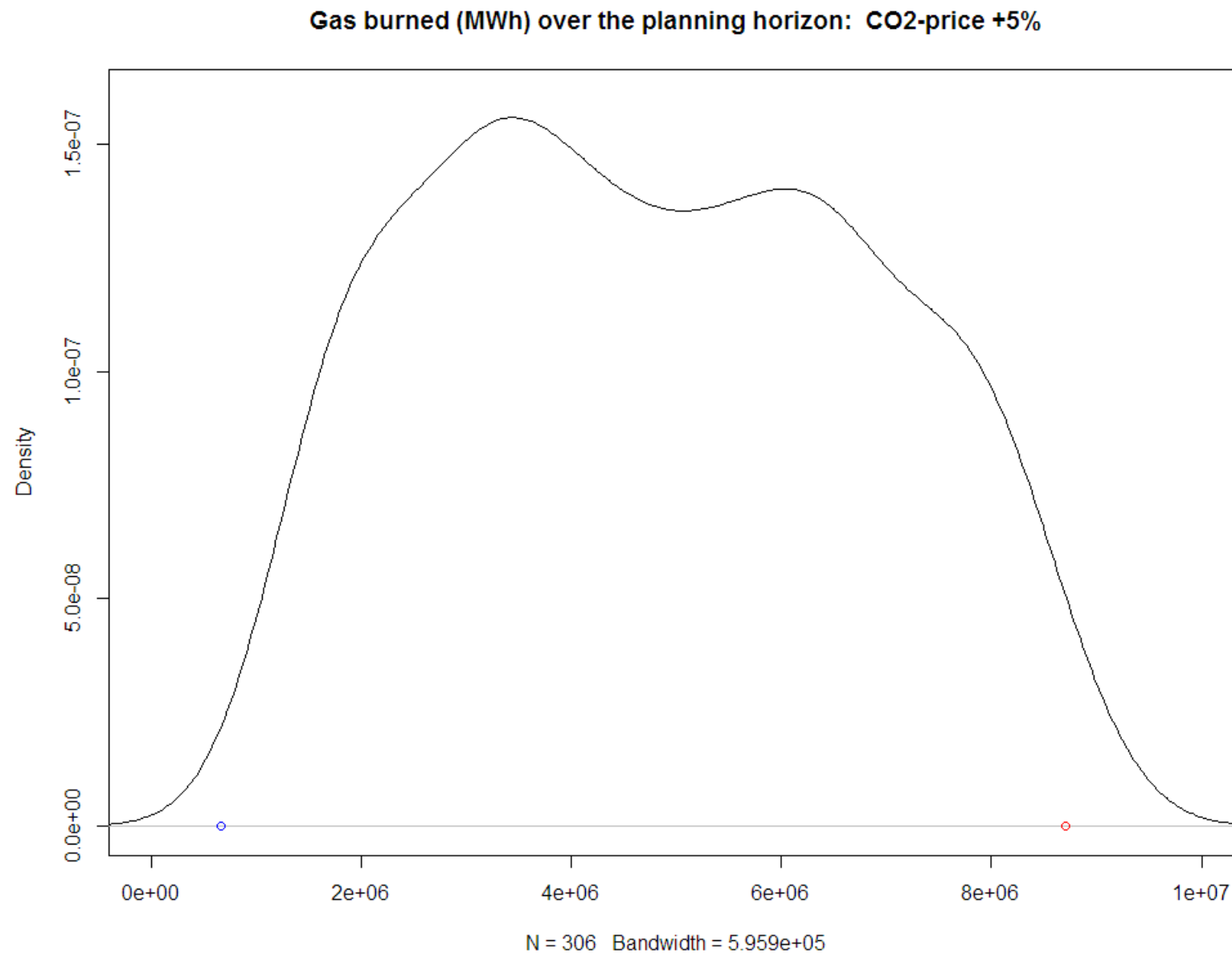


- A (general) increase of 1% in CO2 prices decreases the final asset value by $\sim 1.66\%$
- The decrease in CO2-emissions is (on average) $\sim 0.035\%$

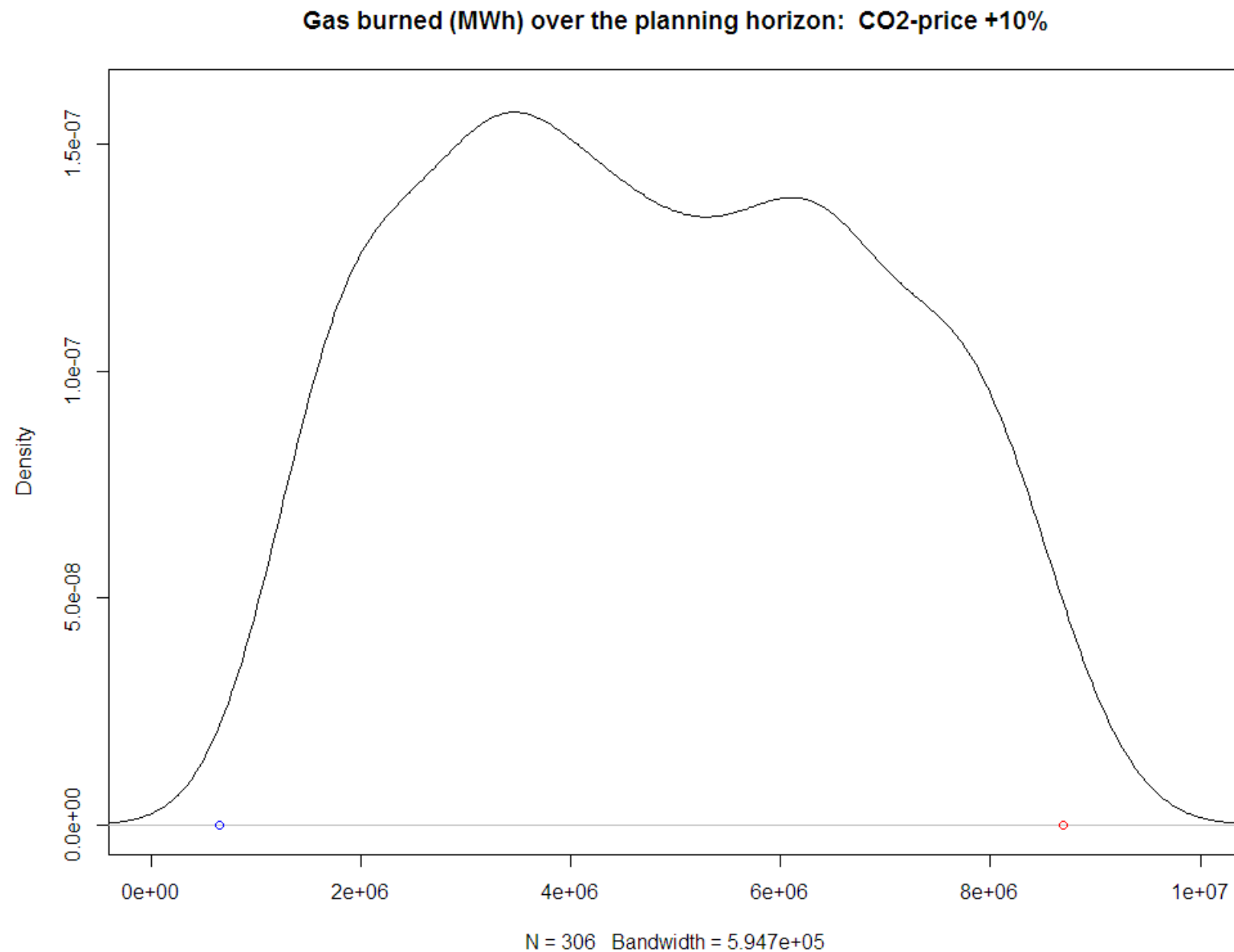
Effect of CO2 prices



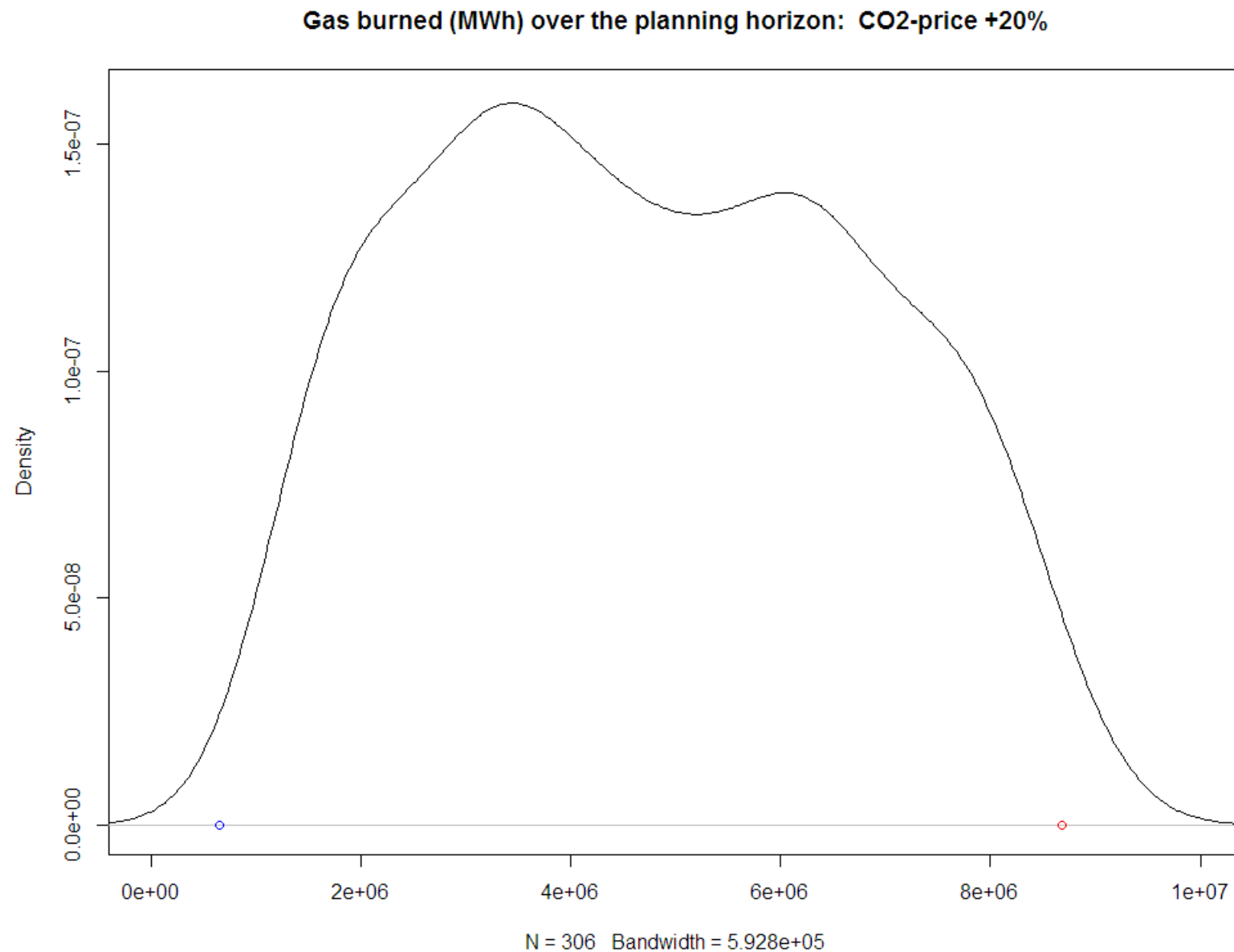
Effect of CO2 prices



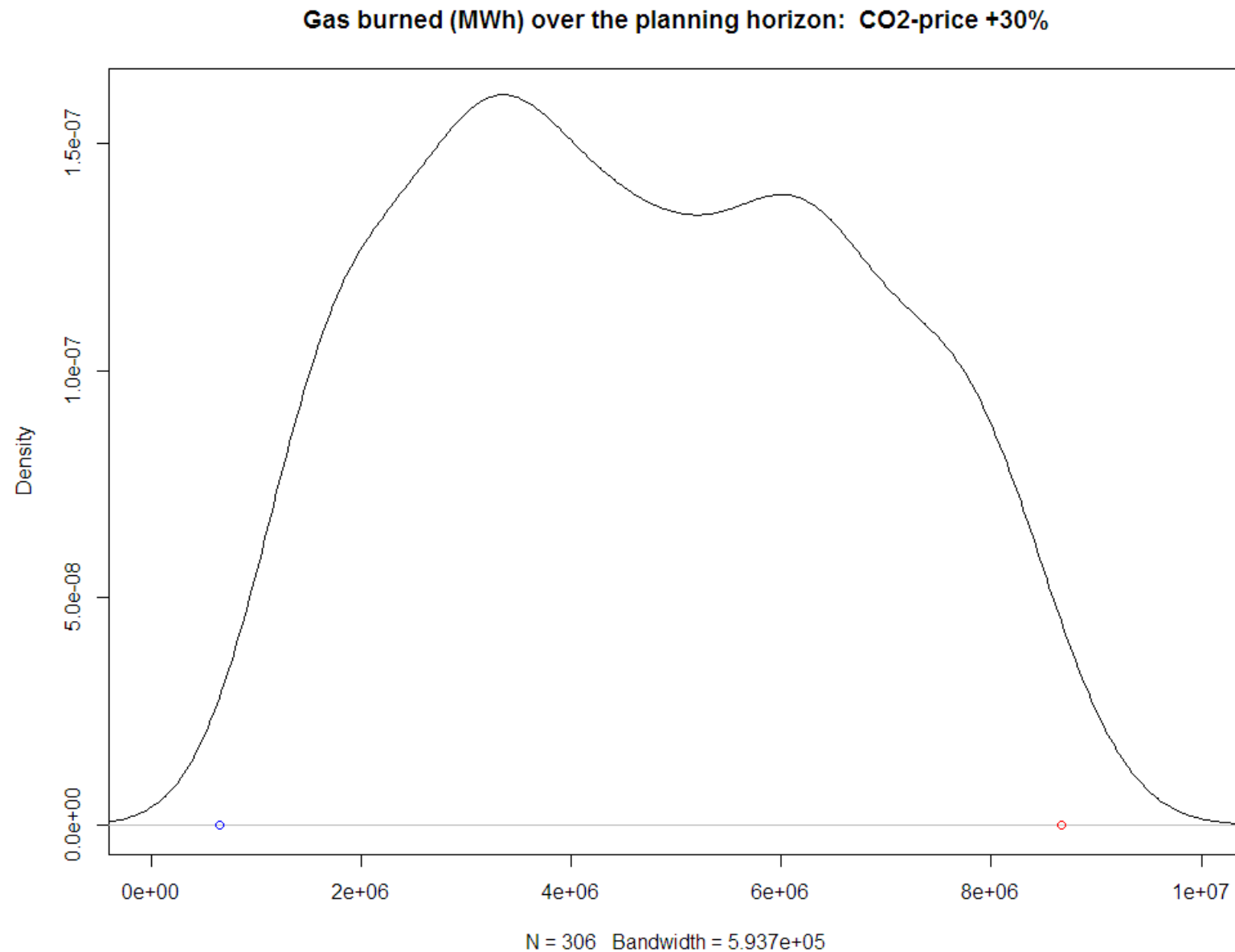
Effect of CO2 prices



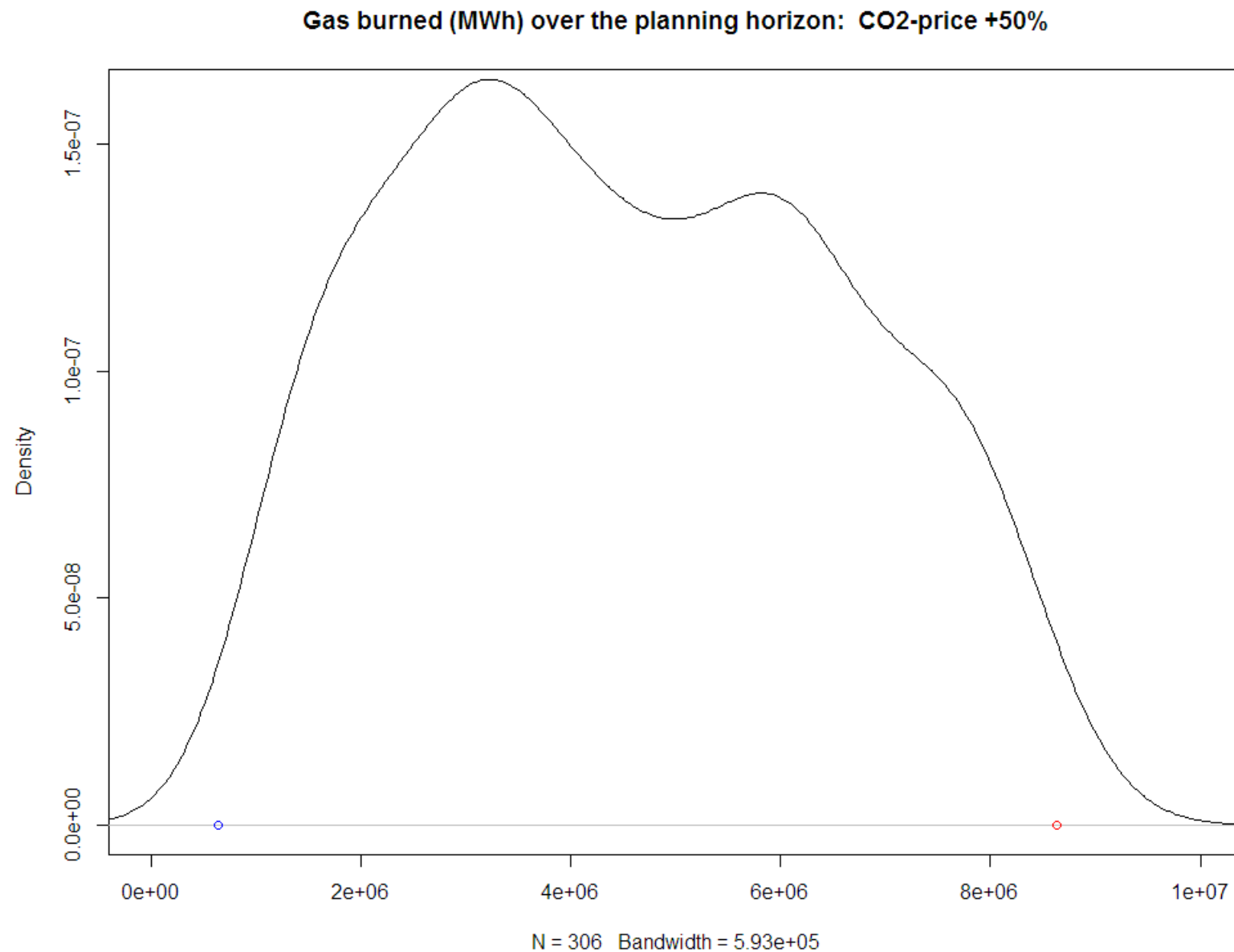
Effect of CO2 prices



Effect of CO2 prices



Effect of CO2 prices



Indifference pricing

- Given the thermal system as described above, consider in addition an electricity **delivery contract**: A fixed amount $\mathbf{E}$ of electricity has to be delivered (produced) during all weeks (52) of the planning horizon at a fixed price $\mathbf{K}$.
- Which price is the minimum price such that the producer is interested to sign the contract?
- Solve with indifference pricing:

$$\min_{K, (\dots)} K \quad (9)$$

$$s.t. \lambda \cdot \mathbb{E}[v_T] + (1 - \lambda) \cdot AV@R_\alpha(v_T) \geq v^* \quad (10)$$

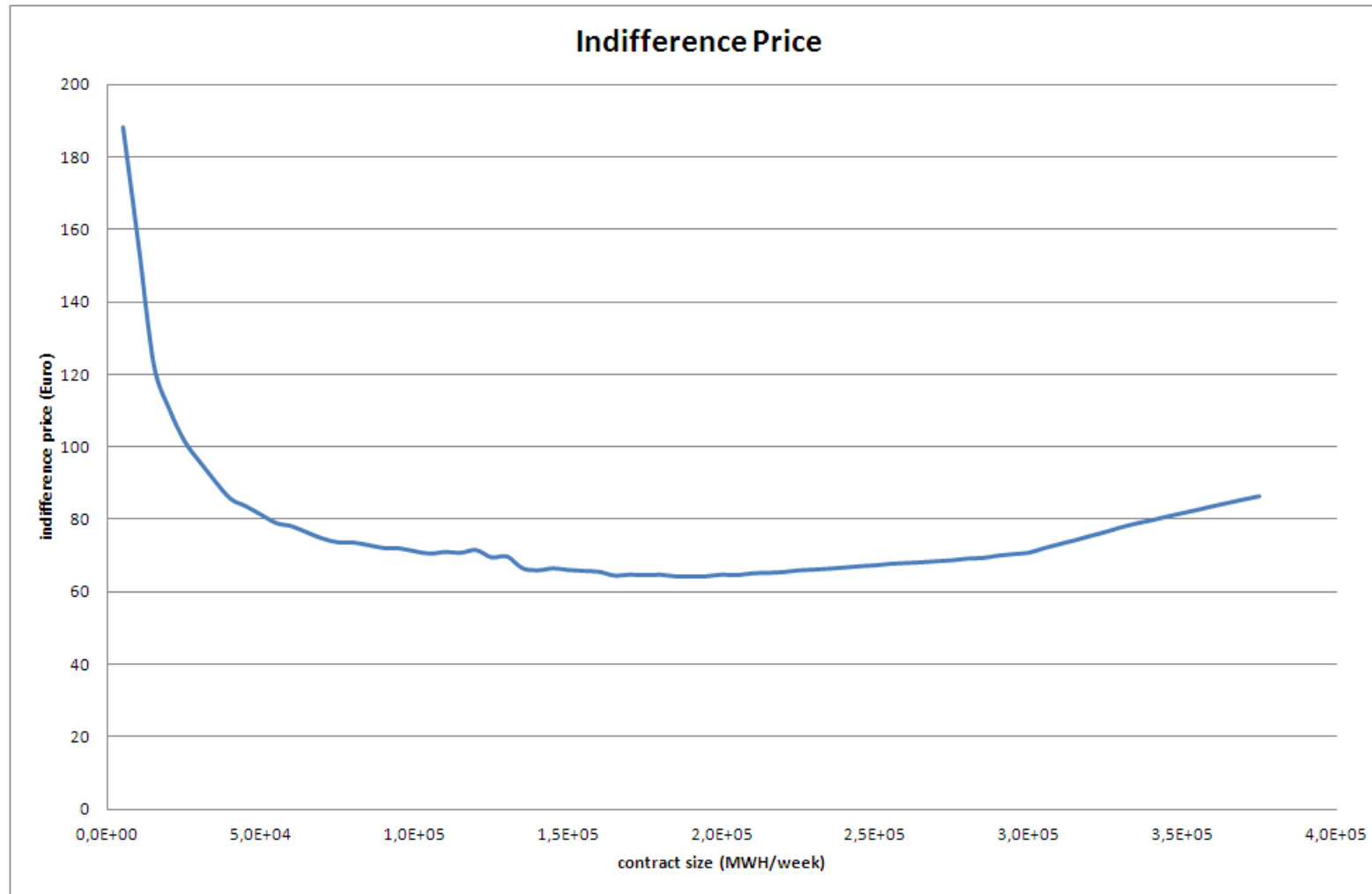
- All constraints of the original problem, except

* It is possible to buy electricity y_t at the spot market,

$$* \sum_{i \in I, j \in J} x_{t,i,j} + y_t \geq E$$

* The cash calculation has to be corrected: $P_t^x \cdot (\sum_{i=1}^I \sum_{j=1}^J x_{t-1,i,j} - E) + K \cdot E$.

Indifference pricing



Conclusion

- We specified a flexible model for mid-term planning, such that iterative analysis – repeatedly using the optimization model can be done in reasonable time
- We simulated the risk factors: oil, gas, coal and CO₂ emissions by a GBMJ process and electricity prices by a spot-forward model
- Simulated hourly/daily commodity prices were aggregated to weekly average price scenarios and reduced to stochastic trees suitable for multistage optimization
- We show the sensitivity of the asset value and of CO₂ emissions to increases in the prices of the CO₂ allowances
- We investigated the pricing of electricity delivery contracts with fixed amount and price in the framework of indifference pricing