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Energy Business and Finance Policy

Parallels in Methodology and Duties

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Energy Business and Finance Policy — Parallels in Methodology and Duties

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Abstract

We report on sophisticated methods to price electricity derivatives. These methods originate from financial applications and are suitably generalized to cope with the special challenges of electricity markets. We proceed in four steps. First, we identify the major obstacles to pricing electricity options. In a second step, we present a simplified Black-Scholes valuation framework; thereby we ignore the problematic peculiarities of electricity markets. Subsequently, we develop a more advanced pricing scheme that overcomes some of the new challenges. This scheme is well-suited for the valuation of simple European-style electricity derivatives. Finally, we propose a stochastic programming framework to price path-dependent and American-style electricity options.

1 Introduction

The ongoing deregulation of electricity markets worldwide has a major impact on the power industry. New price risks require new risk management tools and new methods for the valuation of generation and transmission assets as well as existing (physical) electricity contracts. As far as risk management is concerned, many derivative instruments have been designed to hedge against spot price risk or different types of liability risk exposure. For instance, one observes the emergence of markets for simple financial and physical derivative contracts such as futures, forwards, call and put options, etc. In addition, there is an immense variety of derivative contracts with American-style and path-dependent payoff structure; these contracts are usually traded over-the-counter.

Without significant modifications, the methods of classical finance are not applicable for the valuation of electricity derivatives and generation assets.[1] In this article we will address several pricing problems in the energy business, identify the major methodological difficulties, and sketch sophisticated concepts for their solution. Both analytical and numerical approaches are considered.

[1] Roughly speaking, a power plant can be considered as a special type of call option whose strike price is determined by the variable generation costs.

2 Major Difficulties

When dealing with the valuation of electricity derivatives, one basically faces four major difficulties:

- *Liquidity:* At present, markets for electricity derivatives generally suffer from insufficient liquidity. Concretely speaking, a market participant cannot quickly effect the decision to trade and find a counterparty, the market will restore equilibrium slowly after a temporary shift in supply or demand, and major purchases or sales can substantially affect prices. Moreover, there is a large difference between bid and offer prices.
- *Spot-price behavior:* Electricity spot prices are mean-reverting and exhibit jumps and spikes. In addition, the parameters of the spot price process are subject to regime-switching behavior. These peculiarities result from the non-storability and the grid bound nature of power.
- *Non-storability:* Electricity cannot be stored efficiently, and production has to cover demand on an instantaneous basis. Therefore, the traditional, storage based no-arbitrage methods of valuing commodity derivatives are unavailable.
- *Contract structures:* Many of the most wide-spread derivative contracts in the electricity industry are non-standard instruments. Their payoff structure is American-style and path-dependent, which severely complicates the valuation procedure.

Subsequently we will present several approaches to handle these problems.

3 Black-Scholes Approach

At the outset, one should ignore the difficulties inherent to electricity markets and start from the assumptions of classical finance theory. Thus, let us assume that the markets for electricity derivatives are perfectly liquid and frictionless. Moreover, approximate the dynamics of the electricity spot price S_t as a geometric Brownian motion

$$\frac{dS_t}{S_t} = \mu\, dt + \sigma\, dw_t,$$

where w_t denotes a standard Wiener process. After all, assume electric energy to be a storable and tradable commodity. Under these highly unrealistic assumptions there is a unique martingale measure Q, under which the discounted spot

price is driftless (for simplicity, assume the short rate r to be constant). Then, arguing as in finance theory, European-style contingent claims are priced by discounting the known terminal value $V(S_T, T)$ at maturity T and taking conditional expectation with respect to the risk-neutral measure Q.

$$V(S_t, t) = E^Q\Big[e^{-r(T-t)}V(S_T, T)\Big|\mathcal{F}_t\Big]$$

All methods for valuing electricity derivatives presented below are most conveniently formulated in terms of the logarithmic spot price $X_t := \ln S_t$. For the sake of transparent notation, we consider X_t as the fundamental state process throughout this work. Under the risk-adjusted measure its dynamics is given by

$$dX_t = \Big(r - \frac{1}{2}\sigma^2\Big)dt + \sigma\, dw_t^Q.$$

The forward price of electricity at time t is defined as the price f_t paid at delivery that sets the price of a derivative with the payoff $e^{X_T} - f_t$ to zero. With the above conventions we find $f_t = e^{r(T-t)+X_t}$, which implies that forward and spot price coincide at expiry. Furthermore, the value of a European call option on one unit of energy delivered at time T is given by the classical Black-Scholes formula [2]; because of the identity $f_T = S_T$ we may consider the forward contract as the underlying security:

$$C(f_t, t; k) = e^{-r(T-t)}\Big[f_t N(d_1) - kN(d_2)\Big]$$

where

$$d_1 := \frac{\ln\frac{f_t}{k} + \frac{1}{2}\sigma^2(T-t)}{\sqrt{\sigma^2(T-t)}}, \quad \text{and} \quad d_2 := d_1 - \sqrt{\sigma^2(T-t)}.$$

4 Advanced Valuation Scheme

We still adhere to the efficient market assumption. This can be justified since liquidity of markets for electricity derivatives is likely to rise in the nearer future. However, we relax the unrealistic assumption that electric energy is storable and traded. Instead, we consider the logarithmic spot price X_t as an exogenous state variable driving the electricity market. A market is arbitrage-free if and only if there exists a martingale measure Q, under which the discounted price processes of all investment goods are driftless [5]. The goods must be traded; i.e. their prices must be determined by matching demand and supply in a liquid market. In particular, it is always assumed implicitly that the investment goods are storable. This requirement is trivially satisfied for securities but not for energy. As there is no problem with storing a financial electricity contract and

since such contracts are traded there exists a martingale measure Q under which all contingent claims on electric energy are martingales. In contrast, the spot price of electricity — as a non-tradable state variable — is not necessarily a martingale under Q. Alternatively we can postulate that electricity sold at different times must be viewed as different commodities. The corresponding forward contracts are traded and storable; and they constitute a set of basis securities, whose price processes must be given exogenously. If these contracts define a complete market we can replicate any contingent claim by dynamically trading in the forwards and a riskless instrument.[2]

Within a sophisticated valuation scheme the spot price of electricity cannot be modeled as a geometric Brownian motion. In reality, electricity prices do not grow exponentially but fluctuate around a time-varying mean value. Thus, it is more appropriate to use a mean reverting process for the spot price. Furthermore, non-storability of electricity causes steep changes in prices with changing demand and supply conditions. Hence, the simple Black-Scholes model should be extended to include jump behavior in the price process. As our interest is in the pricing of contingent claims on the underlying state process we work directly under an equivalent martingale measure.[3] By assumption, the natural logarithm of the spot price is governed by a stochastic process of the form

$$dX_t = \kappa(\theta - X_t)dt + \sigma\, dw_t^Q + \sum_{i=1}^{2} k_i\, dq_{it}, \tag{1}$$

where the parameter $\kappa > 0$ stands for the magnitude of the speed of adjustment to the long run mean log price θ, and q_{it} represents a Poisson processes with jump frequency λ_i $(i = 1, 2)$. The jump amplitudes are exponentially distributed, $k_i \sim \mathcal{EXP}(1/\mu_i)$. We use two Poisson processes with positive and negative jump values and different intensities to model unexpected increase and decrease in prices, respectively. Moreover, we assume that dw_t^Q, dq_{it}, and k_i are mutually independent. Below, we will express the values of forward and option contracts in terms of the transform

$$\Psi(\nu, t, T, X_t) := E^Q\left[e^{-r(T-t)} \exp\{\nu X_T\}\middle|\, \mathcal{F}_t\right],$$

where ν is an arbitrary complex number such that Ψ is finite, cf. also [3, 6]. Except for the discount factor Ψ is the moment generating function of the distribution of X_T conditioned on X_t. Under some standard regularity conditions it can be shown that Ψ has an exponential affine form [4]

$$\Psi(\nu, t, T, X_t) = \exp\left[\alpha(T - t, \nu) + \beta(T - t, \nu) \cdot X_t\right], \tag{2}$$

[2]It should be emphasized that any contingent claim on the spot price can be interpreted as an equivalent claim on the corresponding forward contract since $f_T = S_T$.

[3]The incorporation of jumps has the effect that the market becomes incomplete. Thus, arbitrary claims cannot generally be replicated by trading forwards.

where the functions α and β solve the system of ordinary differential equations

$$\begin{aligned}\frac{d}{ds}\beta &= -\kappa\beta \\ \frac{d}{ds}\alpha &= \kappa\theta\beta + \frac{\sigma^2}{2}\beta^2 + \sum_{i=1}^{2}\lambda_i\frac{\mu_i\beta}{1-\mu_i\beta} - r\end{aligned}$$

for $s := T - t$ and with boundary conditions $\beta(0,\nu) = \nu$ and $\alpha(0,\alpha) = 0$. Integration yields explicit solutions

$$\beta(s,\nu) = \nu e^{-\kappa s}, \tag{3}$$

$$\begin{aligned}\alpha(s,\nu) &= \theta\nu(1-e^{-\kappa s}) + \frac{\sigma^2\nu^2}{4\kappa}(1-e^{-2\kappa s}) - rs \\ &\quad - \sum_{i=1}^{2}\frac{\lambda_i}{\kappa}\ln\left[\frac{\mu_i\nu - 1}{\mu_i\nu e^{-\kappa s} - 1}\right].\end{aligned} \tag{4}$$

Next, the transform Ψ is used to determine contingent claim prices. For instance, one can verify that the forward price reduces to $f_t = \exp(r(T-t))\Psi(1,t,T,X_t)$ under the given assumptions. Notice that f_t inherits the jump behavior of the spot price X_t; however, for large values of $s = T - t$ the jumps are increasingly damped, and the forward price shows almost smooth sample paths. In order to price European call options we have to look at a family of functionals G closely related to the transform Ψ.

$$G_{a,b}(y) := \frac{\Psi(a,t,T,X_t)}{2} - \frac{1}{\pi}\int_0^\infty \frac{\mathrm{Im}[\Psi(a+i\nu b,t,T,X_t)e^{-i\nu y}]}{\nu}d\nu \tag{5}$$

As before, the parameters a and b are arbitrary complex numbers such that G is finite. By using the residue theorem of complex analysis one can prove that $G_{a,b}(y)$ is the value of a claim which pays off $\exp\{aX_T\}$ at time T in the event $b \cdot X_T \leq y$ and zero otherwise, i.e.

$$G_{a,b}(y) = E^Q\left(e^{-r(T-t)}\exp\{aX_T\}\mathbf{1}_{\{bX_T - y \leq 0\}}\big|\,\mathcal{F}_t\right).$$

Thus, the price of a European call option with strike price k amounts to

$$C(f_t,t;k) = G_{1,-1}(-\ln k) - kG_{0,-1}(-\ln k).$$

In order to obtain the call price explicitly we express the G-functionals in terms of the well-known transform Ψ

$$\begin{aligned}C(f_t,t;k) &= e^{-r(T-t)}f_t\left(\frac{1}{2} - \int_0^\infty \frac{\mathrm{Im}[\Psi(1-i\nu,t,T,X_t)e^{r(T-t)+i\ln k}]}{S\nu}d\nu\right) \\ &\quad - e^{-r(T-t)}k\left(\frac{1}{2} - \int_0^\infty \frac{\mathrm{Im}[\Psi(-i\nu,t,T,X_t)e^{r(T-t)+i\nu\ln k}]}{S\nu}d\nu\right)\end{aligned}$$

Thus, we have found a closed form solution for the prices of European options. However, the involved integrals usually must be evaluated numerically.

5 American-style Path-dependent Options

American-style path-dependent options are much more widespread in the energy business then in finance. Probably the most important examples are swing options which have been traded in electricity over-the-counter markets for a long time. Swing options are sometimes referred to as virtual power plants. In fact, the problem of finding an optimal exercise strategy for a swing option is basically equivalent to the problem of finding an optimal generation schedule for a real power plant.

A swing option is an agreement to purchase electric energy at a predetermined strike price K during a fixed period [1, 7]. Without loss of generality, we may assume that the delivery period is given by the time interval $[0, T]$. With this convention, the trading period simply reduces to the interval $(-\infty, 0)$. Typically, a swing option appears along with limitations on power and cumulative energy. Concretely speaking, the option holder has the right to buy a maximum amount of energy $\overline{e}$ over the delivery period at maximum power $\overline{p}$. On the other hand, she has the obligation to purchase a minimal quantity of energy $\underline{e}$ over the delivery period, and her load pattern is restricted from below by $\underline{p}$. The value of the swing option will be positive if the associated rights outweigh the obligations; otherwise it will be negative.

In the remainder, assume that the spot price process under the risk neutral measure Q is governed by the stochastic differential equation

$$dS_t = \mu(S_t, t)dt + \sigma(S_t, t)dw_t^Q,$$

where w_t^Q is a one-dimensional Wiener process, and the real-valued functions μ and σ comply with some standard regularity conditions (see e.g. [8]). To determine the option's value we adopt the perspective of the option holder. At time $s \in [0, T]$ an infinitesimal amount of energy $p \cdot ds$ purchased at the strike price K can immediately be sold on the spot market at the price S_s. This results in an infinitesimal cash flow of magnitude $(S_s - K) \cdot p_s \, ds$. The net present value of this cash flow at time $t < s$ is calculated by discounting and taking conditional expectation with respect to the risk-adjusted measure Q. As a rational agent, the option holder tries to maximize the gains from the contract by selecting (nonanticipatively) an optimal load pattern p^* among the patterns p that satisfy the constraints for energy and power. Thus, the pricing problem of a swing option reads

$$\begin{aligned} \Phi(e, S, t) = \sup \; & E^Q_{t,S,e} \int_{t \vee 0}^{T} e^{-r(s-t)} (S_s - K) \cdot p_s \, ds \\ \text{s.t.} \quad & \dot{e}_s = p_s && \forall s \in [t, T] \\ & \underline{p} \le p_s \le \overline{p} && \forall s \in [t, T] \\ & \underline{e} \le e_T \le \overline{e} \\ & p_s, e_s \; \mathcal{F}_s\text{-adapted.} \end{aligned}$$

Thereby, $\Phi(e,S,t)$ is the price of the swing option given that the current spot price at time t is S and the energy purchased in the interval $[0\wedge t, t]$ is e. Moreover, $\{\mathcal{F}_t \,|\, t \geq 0\}$ represents the filtration induced by the spot price process, and $E^Q_{t,S,e}$ is the expectation operator conditional on $S_t = S$ and $e_t = e$. The pricing problem can be viewed as a stochastic program in continuous time. It has relatively complete recourse if we take account of the induced constraints [9, 10]

$$\underline{e} - \overline{p} \cdot (T-t) \leq e_t \leq \overline{e} - \underline{p} \cdot (T-t) \quad \forall t \in [0,T].$$

Below, without loss of generality we set $\underline{p} = 0$. This causes a shift of the value function by the price F of a portfolio of forward contracts

$$F(S,t) = \underline{p} \cdot E^Q_{t,S} \int_{t\vee 0}^{T} e^{-r(s-t)}(S_s - K)\, ds.$$

The natural domain of the value function Φ can then be defined as

$$Z := \{(e,S,t) \in \mathbb{R}\times\mathbb{R}\times[0,T] \,|\, \underline{e} - \overline{p}\cdot(T-t) \leq e \leq \overline{e}\}.$$

Notice that the value function has no physical meaning on the complement of Z, and the induced constraints are not active in the interior of Z. Thus, under suitable differentiability conditions one can show that the value function Φ satisfies a Hamilton-Jacobi-Bellman (HJB) equation in the interior of Z.

$$0 = \overline{p} \cdot [S - K + \Phi_e]^+ + \mu\, \Phi_S + \frac{1}{2}\sigma^2 \Phi_{SS} + \Phi_t - r\Phi$$

Note that the swing option is worthless at expiry, and the exercise strategy is necessarily given by 0 and $\overline{p}$ on the upper and lower induced constraint, respectively. These requirements translate to the boundary conditions (see also Fig. 1)

$$\begin{aligned}
&\Phi(e,S,T) = 0 \quad \forall (e,S) \in [\underline{e},\overline{e}]\times\mathbb{R} \\
&\Phi(\overline{e},S,t) = 0 \quad \forall (S,t) \in \mathbb{R}\times[0,T] \\
&\Phi(\underline{e} - \overline{p}\cdot(T-t), S, t) = \overline{p}\cdot E^Q_{t,S}\textstyle\int_t^T e^{-r(s-t)(S_s-K)} ds \quad \forall (S,t) \in \mathbb{R}\times[0,T].
\end{aligned}$$

In addition, the optimal exercise strategy can be shown to have a particularly simple form:

$$p^*(e,S,t) = \begin{cases} \overline{p} & \text{for } S - K + \Phi_e(e,S,t) \geq 0 \\ 0 & \text{else} \end{cases}$$

Concretely speaking, it is optimal to purchase energy at maximum rate $\overline{p}$ if the shadow price of energy is lower or equal to the difference of the current spot price and the strike price. Otherwise, one should buy nothing (see Fig. 2). However, determination of the optimal exercise strategy p^* requires knowledge of the value

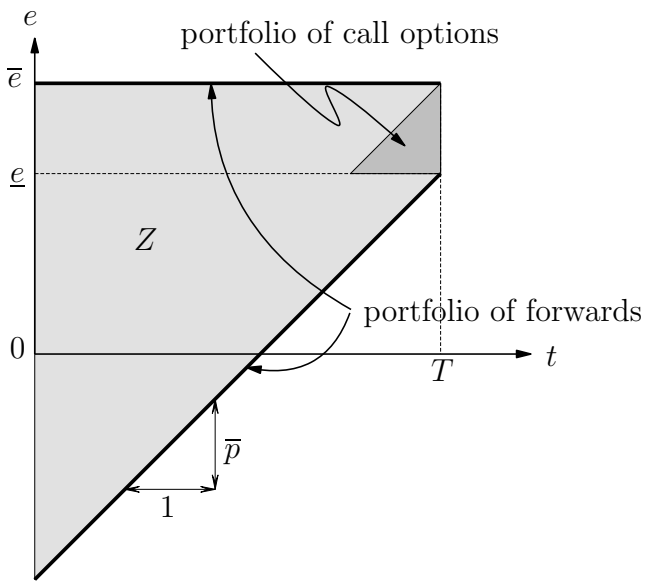


Figure 1: (t, e)-diagram

function Φ, which can conveniently be calculated by means of stochastic dynamic programming. Fig. 3 shows the optimal exercise strategy of a typical swing option with a one year delivery period, while Fig. 4 visualizes the sensitivity of the value Φ with respect to the energy limits. The numerical calculations are based on a software package developed at the institute for operations research at the university of St. Gallen.

6 Conclusions

The well-known methods of finance (i.e. stochastic processes, option theory, stochastic dynamic programming, etc.) can essentially be applied to forecasting, scheduling, and pricing problems in the energy business, as well. However, the special peculiarities of electric power lead to complications. In fact, electric energy is not storable. Thus, contingent claims must be hedged by trading in forward contracts and a risk-free asset, whereas the spot price of electric energy has to be considered as a non-tradable state variable driving the market. Moreover, electricity prices are mean-reverting and exhibit jumps and spikes, which significantly complicates the valuation of European-style derivatives. Finally, the most wide-spread derivative contracts (as well as the physical generation assets) in the energy business have an American-style and path-dependent payoff structure. The valuation of such contracts requires solution of involved stochastic programming problems.

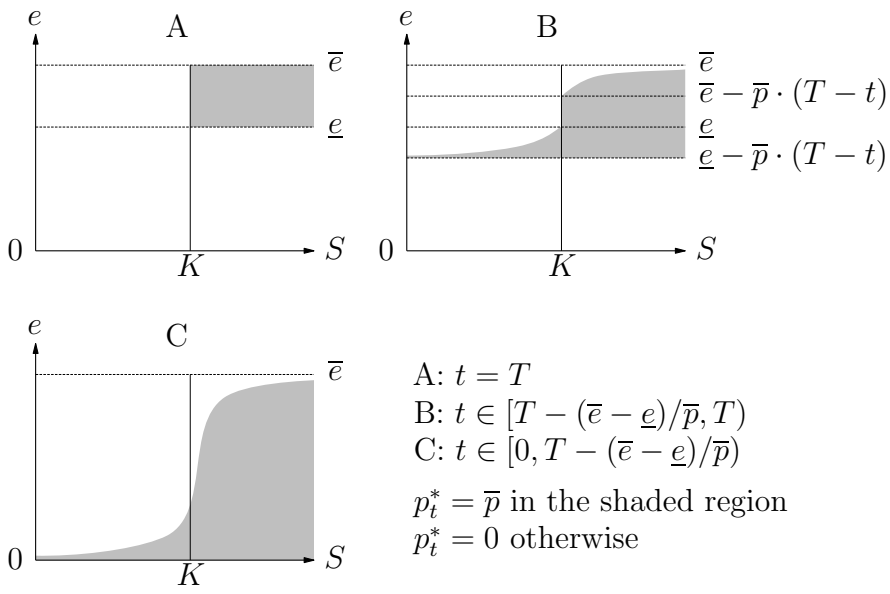


Figure 2: Exercise strategy in different regimes

References

[1] Barbieri, A., and Garman, M. Understanding the valuation of swing contracts. In *Energy and Power Risk Management* (1996).

[2] Black, F., and Scholes, M. The pricing of options and corporate liabilities. *Journal of Political Economy 81* (1973), 637–654.

[3] Deng, S. Stochastic models of energy commodity prices and their applications: Mean-reversion with jumps and spikes. Tech. rep., Industrial Engineering and Operations Research, University of California at Berkeley, February 1999. Presented at Uncertainty Workshop, Palo Alto (July 1999).

[4] Duffie, D., Pan, J., and Singleton, K. Transform analysis and asset pricing for affine jump-diffusions. *Econometrica 68*, 6 (2000), 1343–1376.

[5] Harrison, M., and Kreps, D. Martingales and arbitrage in multi-period securities markets. *Journal of Economic Theory 20* (1979), 381–408.

[6] Kamat, R., and Oren, S. S. Exotic options for interruptible electricity supply contracts. Report PSerc-01-13, April 3, 2001.

[7] Pilipovic, D., and Wngler, J. Getting into the swing. *Energy and Power Risk Management 2* (1998).

[8] Protter, P. *Stochastic Integration and Differential Equations: a New Approach*. Springer, New York, 1990.

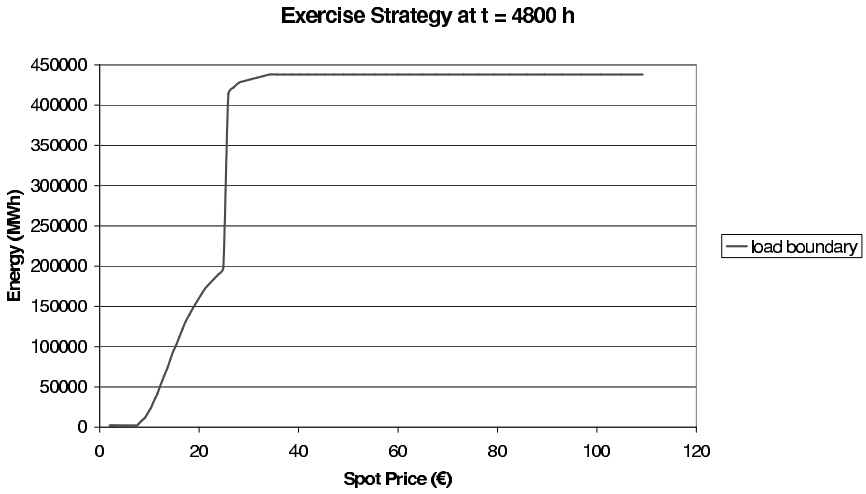


Figure 3: Optimal exercise strategy at time $t = 4800$ h. Buy electric energy at maximum rate if the current state (S, e) lies below the graph; otherwise buy nothing.

[9] Rockafellar, R., and Wets, R.-B. Stochastic convex programming: Relatively complete recourse and induced feasibility. *SIAM Journal on Control and Optimization 14* (1976), 574–589.

[10] Wets, R.-B. Induced constraints for stochastic optimization problems. In *Techniques of Optimization* (1972), A. Balakrishnan, Ed., Academic Press, pp. 433–443.

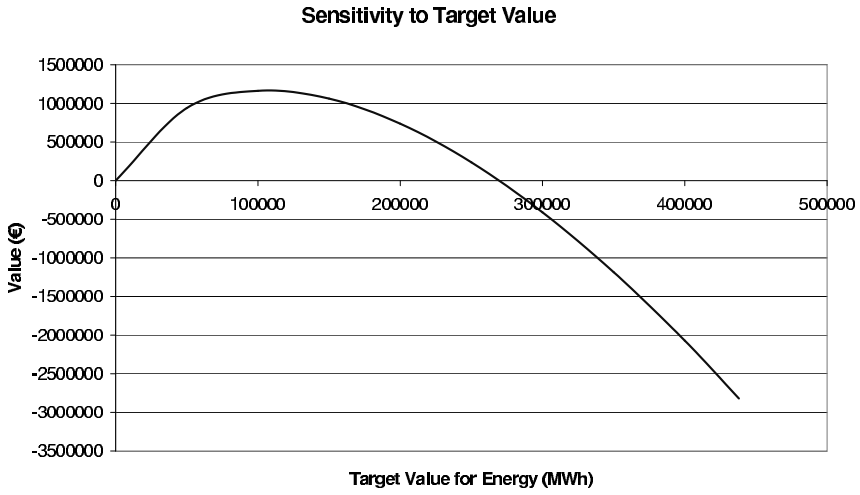


Figure 4: Sensitivity of the option value at time 0 with respect to a fixed energy target $\underline{e} = \overline{e}$.