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# Price dynamics in electricity spot markets

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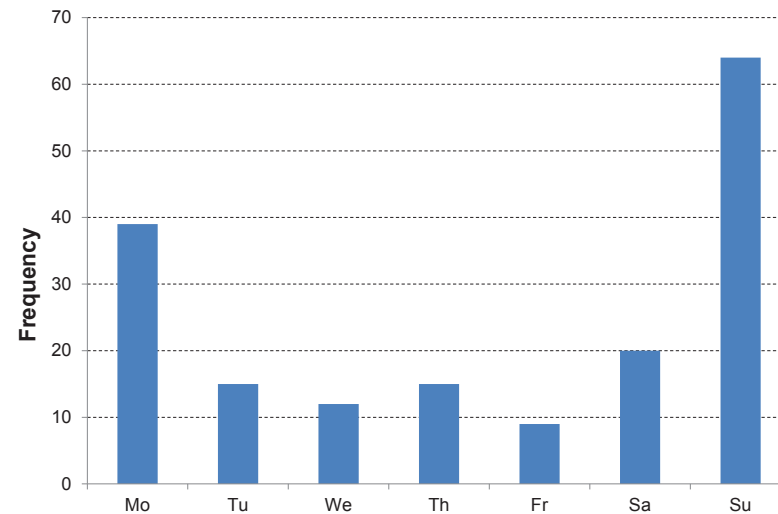
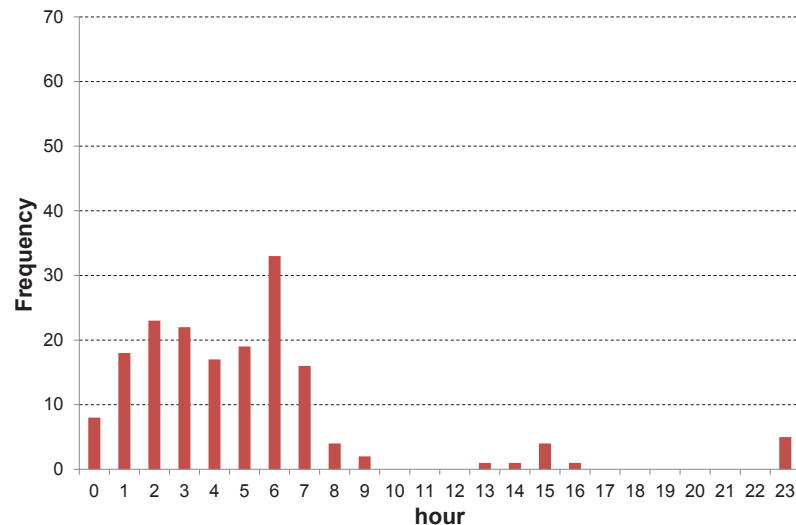
# Particularities of electricity prices (1)

## Extremely large price movements (spikes):

- Caused by small changes in demand and supply
- Electricity is not storable, load and generation are balanced on a knife's edge

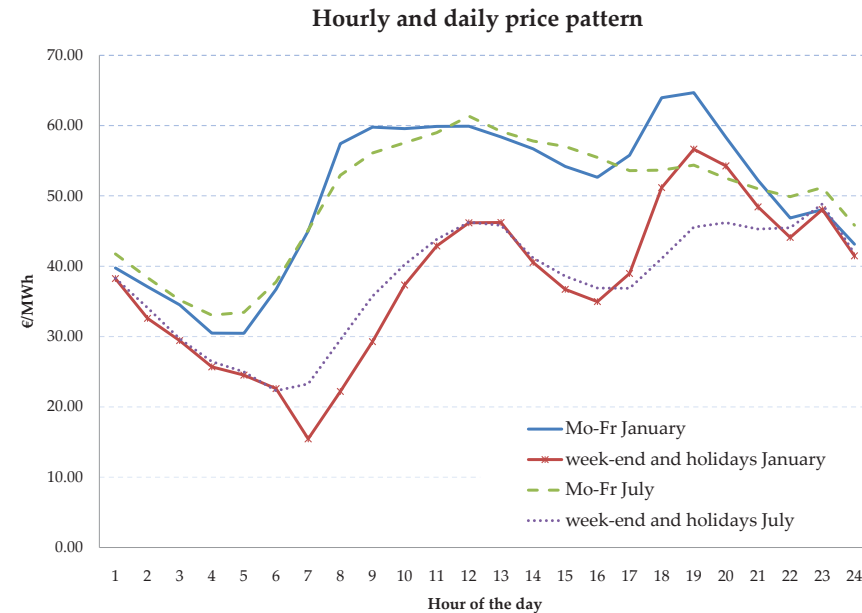
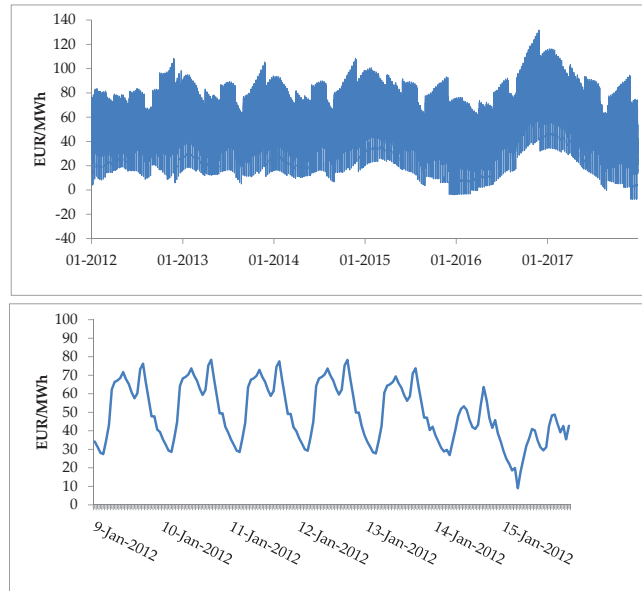
## Negative prices:

- Occur because of wind infeed, on the other hand limited load change flexibility
- More frequently observed in night hours and on weekends



## Particularities of electricity prices (2)

Electricity prices show characteristic seasonal patterns:

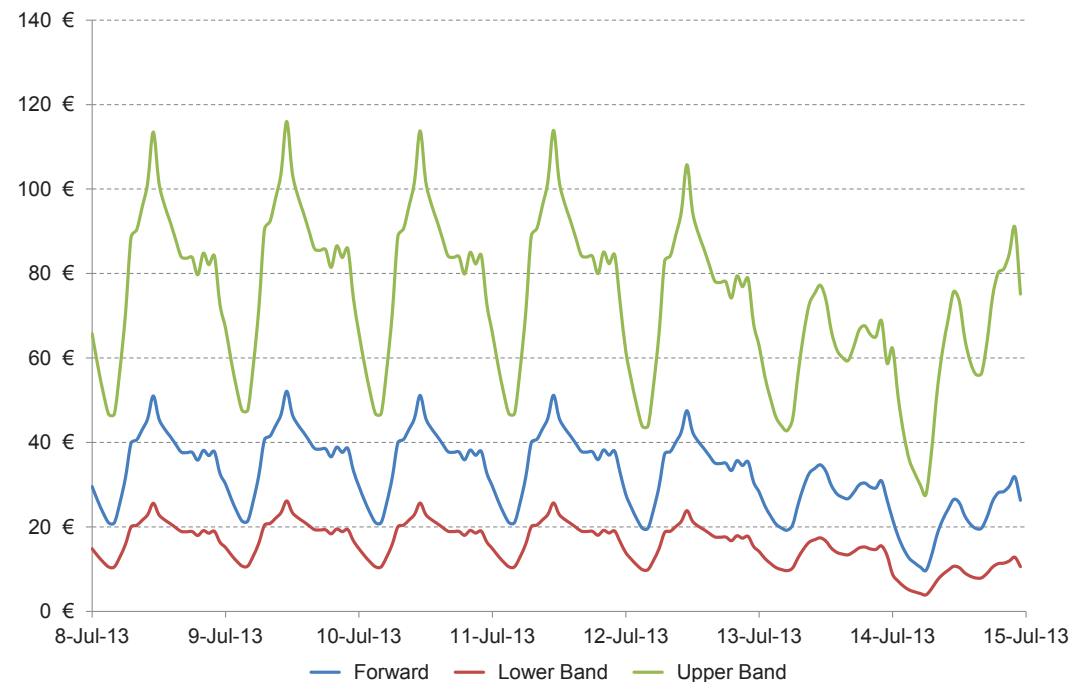


The price variation can be decomposed in a deterministic and a stochastic part:

- The deterministic part is derived as a “seasonality shape” (Blöchlinger, 2008). We determine the weights of the daily prices relative to the year as well as the hourly prices relative to the day.
- The stochastic component is modeled as variation around the hourly price-forward curve (HPFC). This takes information from the markets on future prices into account.

## Model for electricity prices: Idea

- We define a band around the hourly price-forward curve
- Within the band: Deviations of prices from the HPFC are lognormally distributed  
→ normal (“Gaussian”) regime
- Below lower or above upper limit: Prices are exponentially distributed  
→ spike regimes
- The HPFC is derived from market prices of traded futures with the procedure by Benth et al. (2007): The deviation from the seasonal component is modeled by a spline with maximum smoothness



## Model for electricity prices: Definitions

- $S_t$  market spot price for hour  $t$
- $f_t$  forward price for hour  $t$  derived from HPFC
- $f_t^L$  lower limit of normal regime
- $f_t^U$  upper limit of normal regime

The limits of the normal regime are derived as

$$f_t^L := f_t \cdot e^{-\alpha_\delta^L} \qquad f_t^U := f_t \cdot e^{\alpha_\delta^U}$$

where the parameters  $\alpha_\delta^L, \alpha_\delta^U$ ,  $\delta \in \{1, 2\}$ , must be estimated.

We distinguish between Monday to Saturday ( $\delta = 1$ ) and Sunday ( $\delta = 2$ ).

## Model for electricity prices: Time blocks

The price volatility or the probability and magnitude of spikes may be different

- in summer and winter,
- at weekdays (Mo-Fr), Saturday and Sunday,
- at different times of the day.

Times of the day are distinguished by the blocks:

| season | night | morning | high noon | afternoon | rush hour | evening |
|--------|-------|---------|-----------|-----------|-----------|---------|
| summer | 1–6   | 7–10    | 11–14     | 15–18     | –         | 19–24   |
| winter | 1–6   | 7–10    | 11–14     | 15–16     | 17–20     | 21–24   |

→  $H = 33$  hourly blocks for which parameters must be estimated (normal regime).

For the spike regimes we distinguish only between days and seasons

→  $D = 6$  parameter sets

We define two functions

$$h(t) : t \rightarrow \{1, \dots, H\} \qquad d(t) : t \rightarrow \{1, \dots, D\}$$

that assign to the hour  $t$  the index  $h$  or  $d$ , respectively.

## Model for electricity prices

$$S_t := \begin{cases} f_t^L - spike_t^- & \text{if system in in lower spike regime} \\ f_t \cdot \exp(r_t) & \text{if system is in normal regime} \\ f_t^U + spike_t^+ & \text{if system is in upper spike regime} \end{cases}$$

with

$$spike_t^- \sim \text{Exp}(\lambda_d^-), \quad spike_t^+ \sim \text{Exp}(\lambda_d^+), \quad d = 1, \dots, D$$

$$r_t = a_h + \sum_{i=1}^6 b_{i,h} \cdot \hat{r}_{t-i} + b_{24,h} \cdot \hat{r}_{t-24} + \varepsilon_t, \quad \varepsilon_t \sim \text{N}(0, \sigma_h^2), \quad h = 1, \dots, H$$

$$\hat{r}_{t-i} = \begin{cases} \ln S_{t-i} - \ln f_{t-i} & \text{if system is in normal regime at time } t-i, \\ \text{E}(r_{t-i}) & \text{otherwise.} \end{cases}$$

Transition matrix:

$$\Pi_h = \begin{pmatrix} 1 - \pi_h^{LG} & \pi_h^{LG} & 0 \\ \pi_h^{GL} & 1 - \pi_h^{GL} - \pi_h^{GU} & \pi_h^{GU} \\ 0 & \pi_h^{UG} & 1 - \pi_h^{UG} \end{pmatrix}.$$

## Estimation: Assigning observations to regimes

The allocation of observations to regimes depends on the choice of  $\alpha_\delta^L$  and  $\alpha_\delta^U$ ,  $\delta \in \{1, 2\}$ .

Estimation of all model parameters in two steps:

1. A log-likelihood function  $\ln \mathcal{L}(\alpha_1^L, \alpha_2^L, \alpha_1^U, \alpha_2^U)$  is maximized with respect to the four parameters
2. Estimation of parameters for the three regimes based on current values of  $\alpha_1^L, \alpha_2^L, \alpha_1^U$  and  $\alpha_2^U$

Observations given at hourly time points  $t = 1, \dots, T$  are assigned to sets

$$\mathcal{D}^G(h') := \{t = 1, \dots, T \mid h(t) = h' \wedge f_t^L \leq S_t \leq f_t^U\} \quad h' = 1, \dots, H,$$

$$\mathcal{D}^L(d') := \{t = 1, \dots, T \mid d(t) = d' \wedge S_t < f_t^L\} \quad d' = 1, \dots, D$$

$$\mathcal{D}^U(d') := \{t = 1, \dots, T \mid d(t) = d' \wedge S_t > f_t^U\} \quad d' = 1, \dots, D$$

Parameters of the exponential distributions of the spike magnitudes:

$$\hat{\lambda}_{d'}^- = \frac{\#\text{elements in } \mathcal{D}^L(d')}{\sum_{t \in \mathcal{D}^L(d')} (f_t^L - S_t)}, \quad \hat{\lambda}_{d'}^+ = \frac{\#\text{elements in } \mathcal{D}^U(d')}{\sum_{t \in \mathcal{D}^U(d')} (S_t - f_t^U)}.$$

## Estimation: Parameters for the normal regime

Residuals of the autoregressive process for the blocks in the normal regime:

$$e_t = \begin{cases} r_t - a_{h(t)} - \sum_{i=1}^6 b_{i,h(t)} \cdot \hat{r}_{t-i} - b_{24,h(t)} \cdot \hat{r}_{t-24}, & t \in \mathcal{D}^G(h(t)) \\ 0, & \text{otherwise} \end{cases}$$

Missing values (because a spike occurred at this time) are replaced by their expectations:

$$\hat{r}_\tau = \begin{cases} \ln S_\tau - \ln f_\tau, & \tau \in \mathcal{D}^G(h(\tau)) \\ a_{h(\tau)} + \sum_{j=1}^6 b_{j,h(\tau)} \cdot \hat{r}_{\tau-j} + b_{24,h(\tau)} \cdot \hat{r}_{\tau-24}, & \text{otherwise} \end{cases}$$

Parameters are estimated by minimizing the sum of squared errors:

$$\min \sum_{t=1}^T e_t^2$$

Unbiased estimator for the volatilities of block  $h' = 1, \dots, H$ :

$$\hat{\sigma}_{h'}^2 = \frac{\sum_{t \in \mathcal{D}^G(h')} e_t^2}{\#\text{elements in } \mathcal{D}^G(h') - 8}$$

## Estimation: Log-likelihood function

$$\begin{aligned} \ln \mathcal{L}(\alpha_1^L, \alpha_2^L, \alpha_1^U, \alpha_2^U) &= \sum_{h'=1}^H \sum_{t \in \mathcal{D}^G(h')} \ln \phi(e_t | 0, \hat{\sigma}_{h'}) \\ &+ \sum_{d'=1}^D \sum_{t \in \mathcal{D}^L(d')} \ln \varphi(f_t^L - S_t | \hat{\lambda}_{d'}^-) + \sum_{d'=1}^D \sum_{t \in \mathcal{D}^U(d')} \ln \varphi(S_t - f_t^U | \hat{\lambda}_{d'}^+) \end{aligned}$$

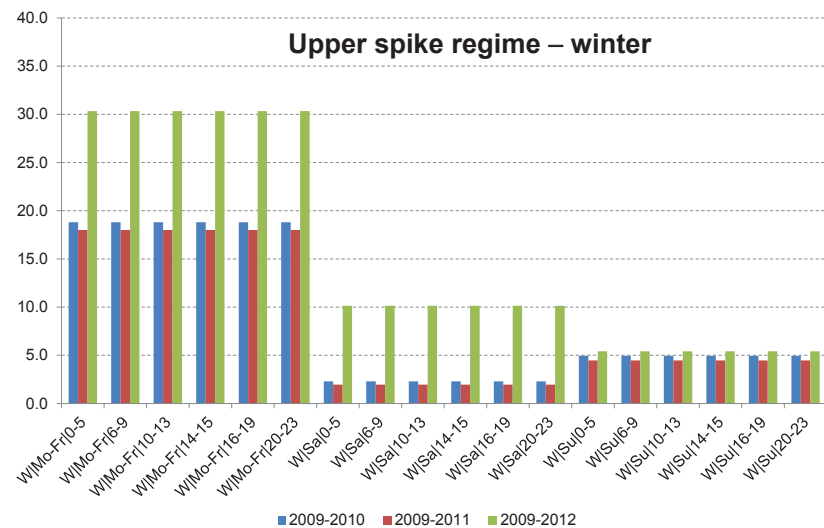
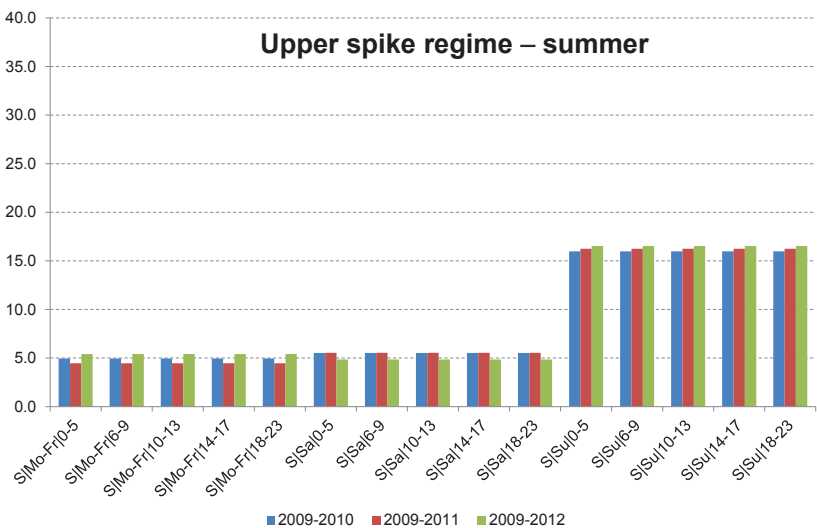
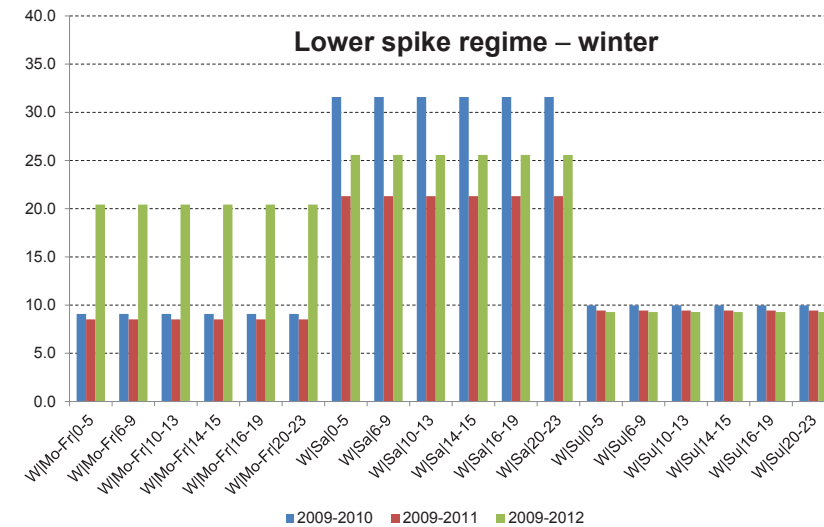
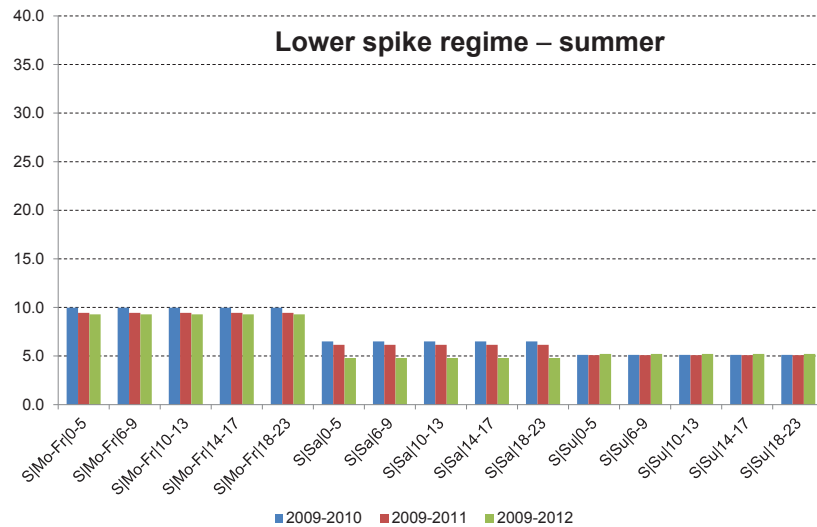
$\phi$ : density of the normal distribution with parameters  $\mu, \sigma$

$\varphi$ : density of the exponential distribution with parameter  $\lambda$

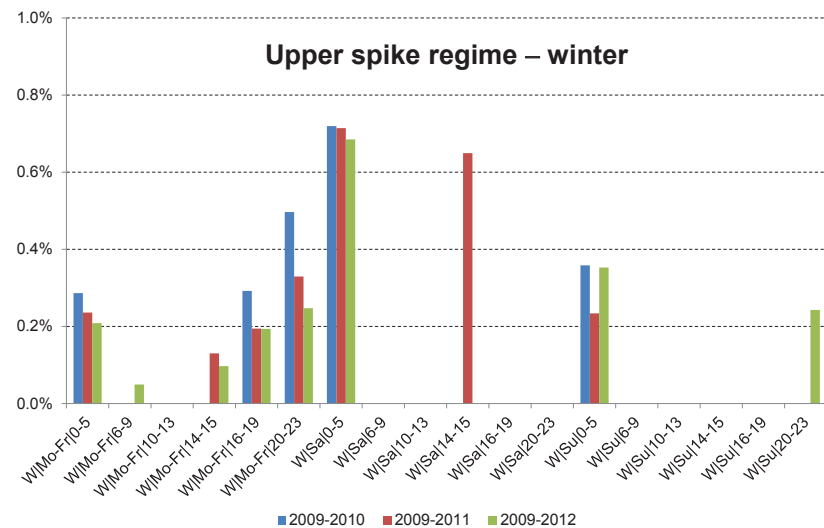
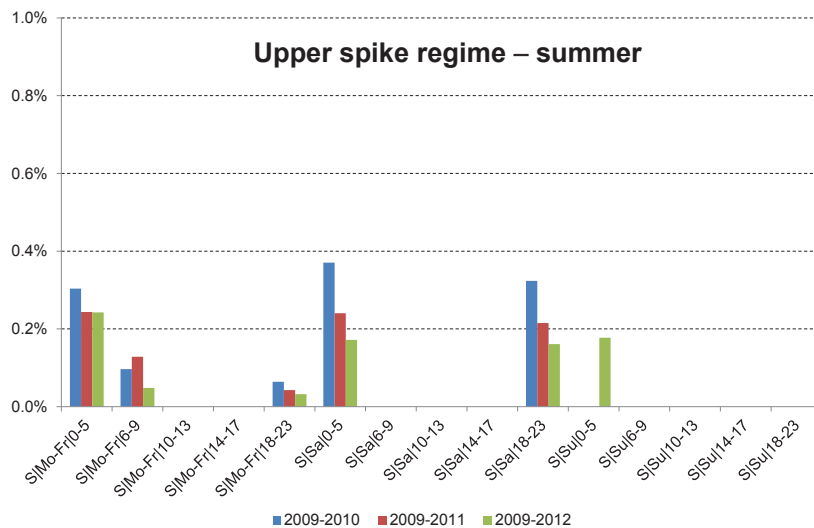
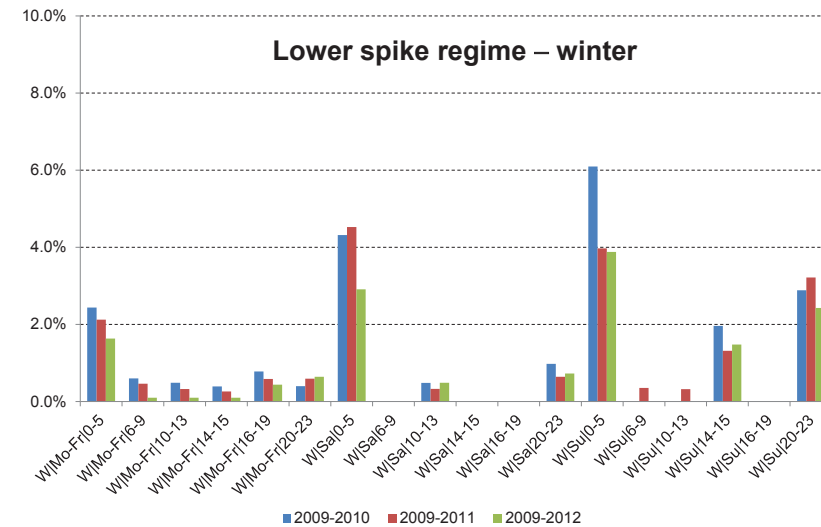
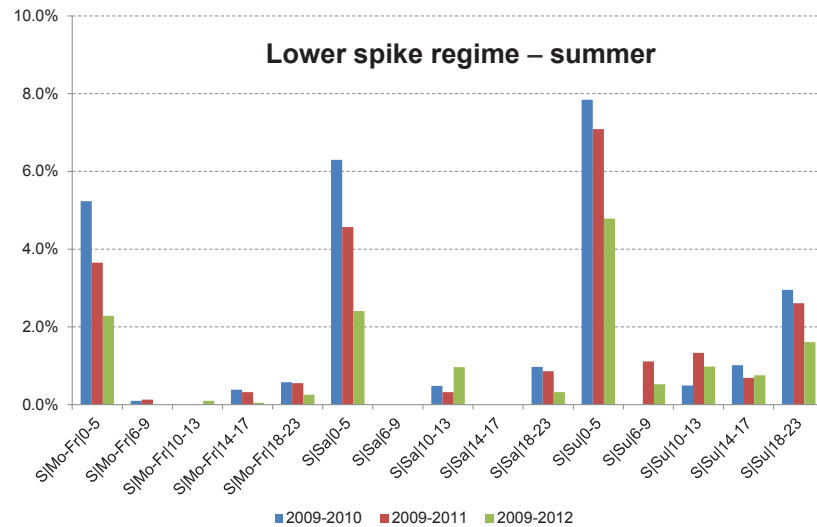
Determination of  $\alpha_1^L, \alpha_2^L, \alpha_1^U, \alpha_2^U$ :

- $\ln \mathcal{L}$  is maximized to find the optimal values for the regime limits
- In each step the other parameters are estimated as outlined above
- The optimal values for the limits are fixed and the log-likelihood function is maximized again with respect to the other parameters
- Standard errors are approximated from the outer products of the gradients
- Probabilities of transitions and of remaining in regimes are calculated from the number of corresponding observations.

# Estimation results: Expected spike magnitudes



# Probabilities of transitions from normal to spike regimes

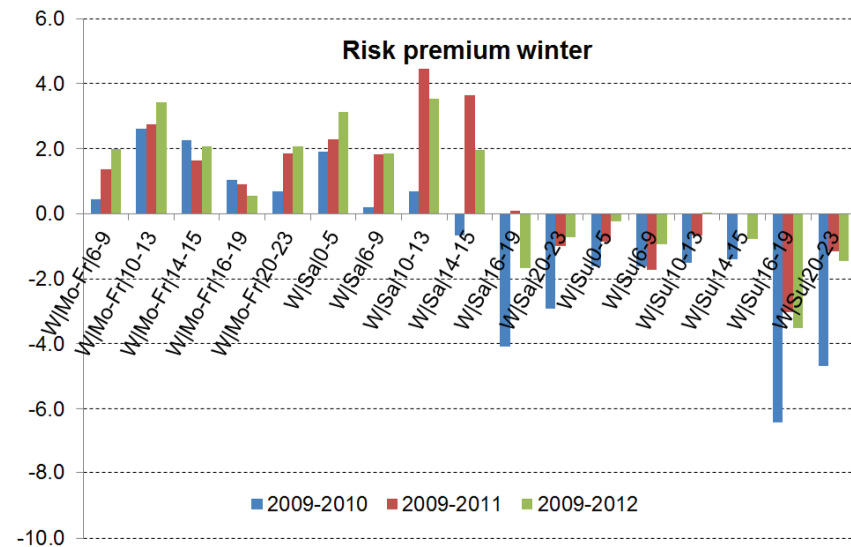
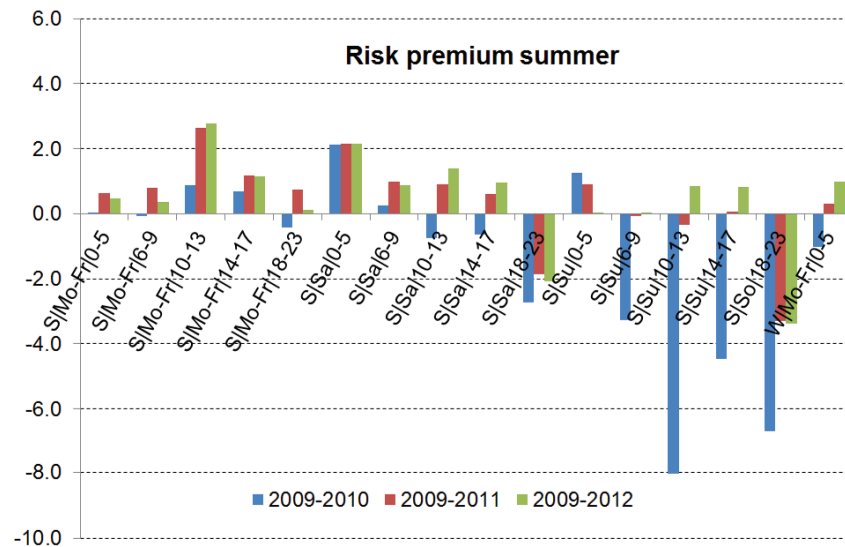


## Probabilities of transitions from spike to normal regimes & regime limits

| Sample period |       | 01/01/2009<br>31/12/2010              | 01/01/2009<br>31/12/2011              | 01/01/2009<br>14/03/2013              |
|---------------|-------|---------------------------------------|---------------------------------------|---------------------------------------|
| seas.         | day   | $\hat{\pi}_h^{LG}$ $\hat{\pi}_h^{UG}$ | $\hat{\pi}_h^{LG}$ $\hat{\pi}_h^{UG}$ | $\hat{\pi}_h^{LG}$ $\hat{\pi}_h^{UG}$ |
| S             | Mo-Fr | 65.40 53.85                           | 66.77 46.67                           | 65.74 55.00                           |
| S             | Sa    | 60.78 87.50                           | 64.52 87.50                           | 64.81 87.50                           |
| S             | Su    | 70.24 95.83                           | 69.92 95.83                           | 68.33 92.00                           |
| W             | Mo-Fr | 65.22 80.36                           | 65.29 77.78                           | 69.86 76.32                           |
| W             | Sa    | 73.08 33.33                           | 75.00 33.33                           | 71.43 85.71                           |
| W             | Su    | 65.00 0.00                            | 68.92 0.00                            | 71.00 77.78                           |

| Sample period           |  | 01/01/2009<br>31/12/2010  | 01/01/2009<br>31/12/2011  | 01/01/2009<br>14/03/2013  |
|-------------------------|--|---------------------------|---------------------------|---------------------------|
| Day index               |  | $\delta = 1$ $\delta = 2$ | $\delta = 1$ $\delta = 2$ | $\delta = 1$ $\delta = 2$ |
| $\hat{\alpha}_\delta^L$ |  | 0.50 0.99                 | 0.50 0.85                 | 0.69 0.91                 |
| $\hat{\alpha}_\delta^U$ |  | 0.78 1.15                 | 0.78 1.11                 | 0.80 1.05                 |

# Risk premiums



- Risk premium = difference forward minus spot price
- Derived from simulation of 1000 scenarios with regime-switching model
- Higher magnitudes in winter than in summer
- Values are positive during the week and decrease or become negative for the weekend
- Premiums vary over time

## Comparison with ARMA and GARCH

Time series models are frequently applied for electricity price simulations:

**ARMA**(n, m)

$$X_t = c + \sum_{i=1}^n \alpha_i X_{t-i} + \sum_{j=1}^m \beta_j \varepsilon_{t-j} + \varepsilon_t$$

**GARCH**(p, q)

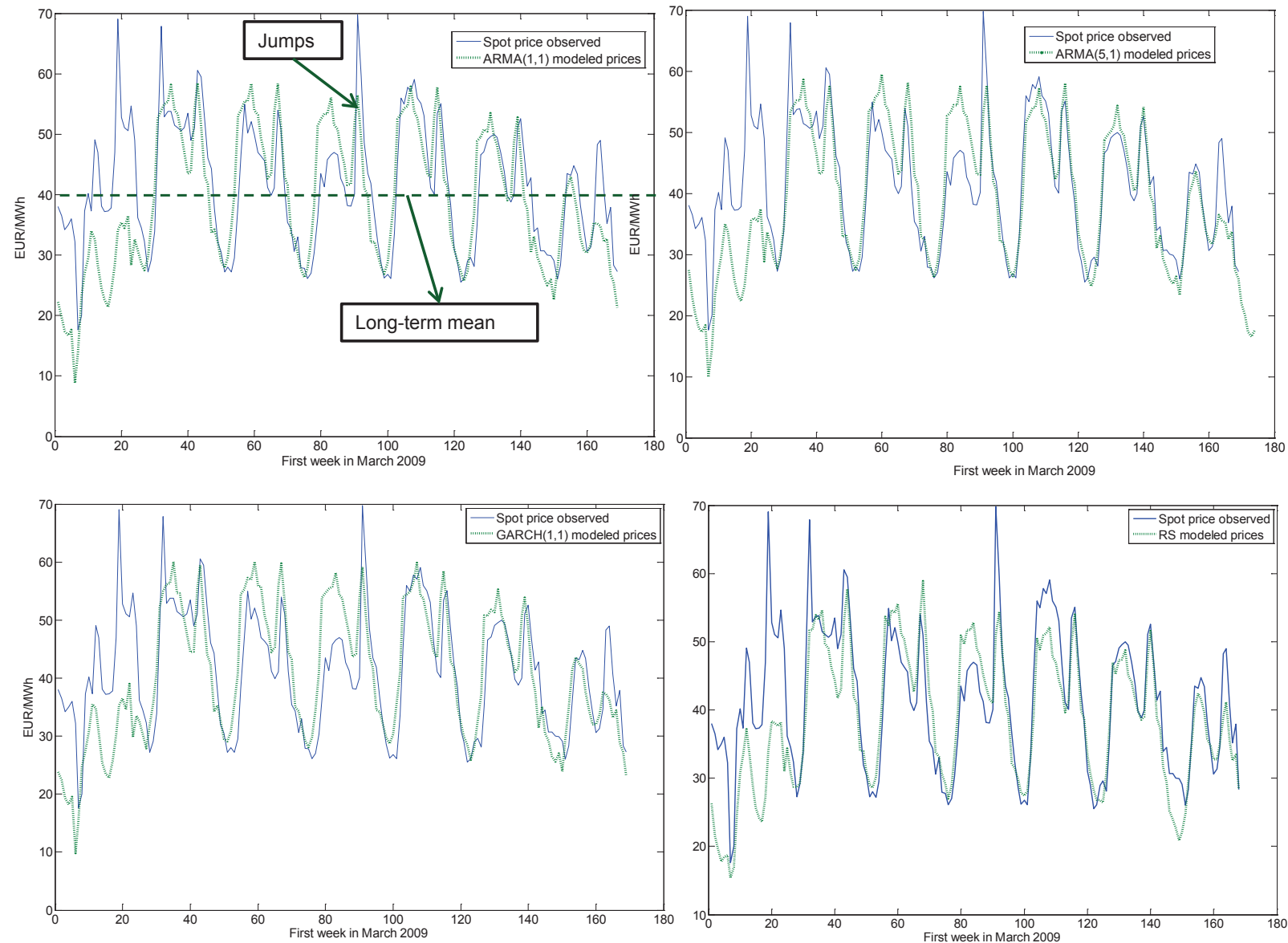
$$\sigma_t^2 = \phi_0 + \sum_{z=1}^p \phi_{1z} \sigma_{t-z}^2 + \sum_{y=1}^q \phi_{2y} \varepsilon_{t-y}^2$$

- First electricity prices are deseasonalized
- ARMA(1, 1), ARMA(5, 1) and GARCH(1, 1) models are estimated for their stochastic components
- Model orders are defined by Akaike's information criterion
- Engle's ARCH test shows significant evidence for ARCH and GARCH effects
- Model parameters estimated with ML are not sample dependent, except ARMA(5, 1)

## Estimation results for ARMA and GARCH

| Sample                      |             | ARMA(1,1) | ARMA(5,1)                                | GARCH(1,1) |
|-----------------------------|-------------|-----------|------------------------------------------|------------|
| 01/01/2009–<br>31/12/2010   | $c$         | 0.035     | 0.035                                    |            |
|                             | $\alpha_i$  | 0.825*    | 0.763*, 0.051, -0.018, 0.009*, 0.020*    |            |
|                             | $\beta_j$   | 0.168*    | 0.228                                    |            |
|                             | $\phi_0$    |           |                                          | -2.167*    |
|                             | $\phi_{1z}$ |           |                                          | 0.046*     |
|                             | $\phi_{2y}$ |           |                                          | 0.900*     |
| 01/01/2009–<br>31/12/2011   | $c$         | -0.08     | -0.005                                   |            |
|                             | $\alpha_i$  | 0.827*    | 1.765*, -0.903*, 0.161*, -0.062*, 0.028* |            |
|                             | $\beta_j$   | 0.079*    | -0.877*                                  |            |
|                             | $\phi_0$    |           |                                          | -2.135*    |
|                             | $\phi_{1z}$ |           |                                          | 0.015*     |
|                             | $\phi_{2y}$ |           |                                          | 0.948*     |
| 01/01/2009–<br>14/03/2013   | $c$         | 0.058*    | 0.086*                                   |            |
|                             | $\alpha_i$  | 0.879*    | 0.409*, 0.394*, 0.026*, 0.021*, -0.031*  |            |
|                             | $\beta_j$   | 0.0003    | 0.470*                                   |            |
|                             | $\phi_0$    |           |                                          | 1.657*     |
|                             | $\phi_{1z}$ |           |                                          | 0.016*     |
|                             | $\phi_{2y}$ |           |                                          | 0.892*     |
| * = significant at 1% level |             |           |                                          |            |

# Simulation of 1000 scenarios



## Simulation results

- ARMA/GARCH: Stochastic component is simulated, then seasonality shape is added
- Regime-switching (RS) model: Prices are simulated directly (seasonality incorporated in HPFC)
- Goodness-of-fit is assessed by  $R^2$  and the mean average percentage error ( $MAPE$ ):

$$E(MAPE) = \frac{1}{N} \sum_{k=1}^N \frac{1}{T} \sum_{t=1}^T \frac{|S_{k,t}^{sim} - S_t|}{S_t}$$

where

- $S_{k,t}^{sim}$  is simulated price in path  $k = 1, \dots, N$  and
- $S_t$  is the real price at time  $t = 1, \dots, T$

| Sample      |        | ARMA(1,1) | ARMA(5,1) | GARCH(1,1) | RS model     |
|-------------|--------|-----------|-----------|------------|--------------|
| 01/01/2009– | $MAPE$ | 0.135     | 0.136     | 0.095      | <b>0.079</b> |
| 31/12/2010  | $R^2$  | 0.490     | 0.493     | 0.490      | <b>0.607</b> |
| 01/01/2009– | $MAPE$ | 0.144     | 0.143     | 0.096      | <b>0.084</b> |
| 31/12/2011  | $R^2$  | 0.442     | 0.442     | 0.439      | <b>0.652</b> |
| 01/01/2009– | $MAPE$ | 0.150     | 0.15      | 0.086      | <b>0.083</b> |
| 14/03/2013  | $R^2$  | 0.414     | 0.414     | 0.409      | <b>0.617</b> |

## Summary

- Classical time series models are based only on historical data, no information on future is used
- RS model is built on market view reflected by price forward curves, spot prices vary around HPFC
- A base and two spike regimes are distinguished:
  - Autoregressive process takes dependencies between consecutive hours into account (normal regime)
  - Negative prices and spike (clusters) can be reflected
  - Probabilities modeled by transition matrix
- Electricity prices show different volatilities and spike behavior depending on time of day, weekday or season
- Reflected by different parameter sets, estimated probabilities confirm this assumption
- Small estimated risk premiums, vary over time; consistent with literature
- Higher  $R^2$  and smaller errors than time series models
- Potential applications: Simulation of spot prices for medium and long term planning of electricity production, risk management, valuation of power contracts etc.