Optimal Reinsurance Programs and Hedging Strategies under Default Risk

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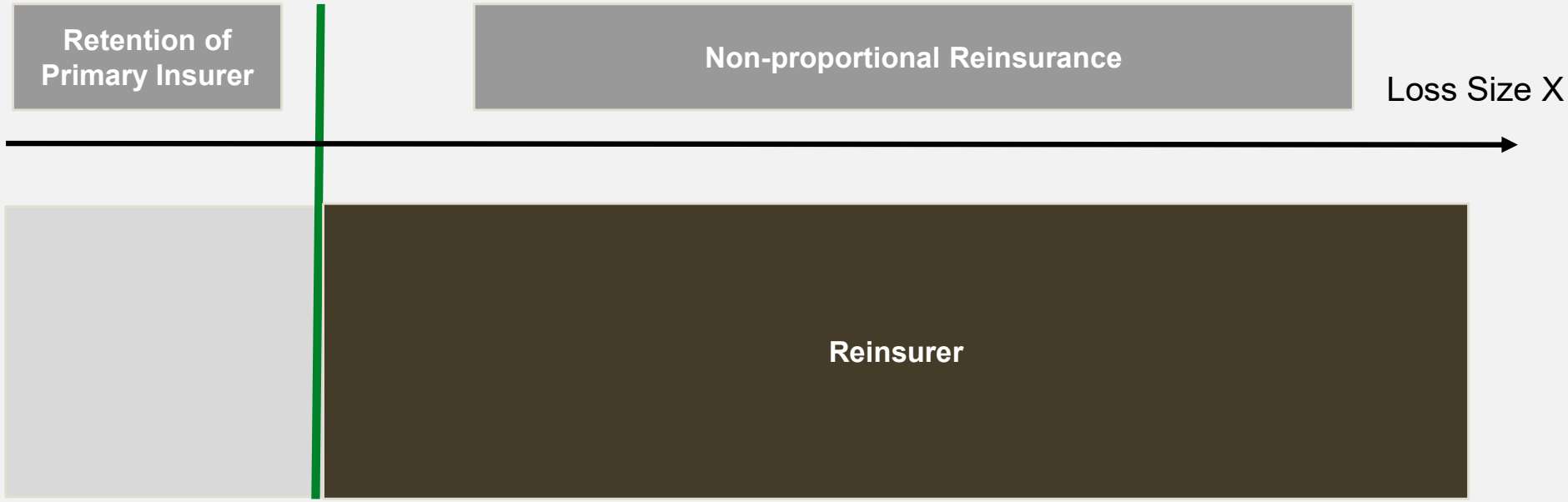
Munich, March 21-22, 2018

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Optimal Coverage Design w/o Default Risk

Arrow, K.J. (1963). *Uncertainty and the Welfare Economics of Medical Care*. AER, 55.

→ Continuous loss distribution
→ Full coverage is optimal iff premium is actuarially fair
→ With positive cost loading, full coverage above a positive deductible is optimal
→ Thus, XL-reinsurance is optimal



Optimal Coverage Design with Default Risk

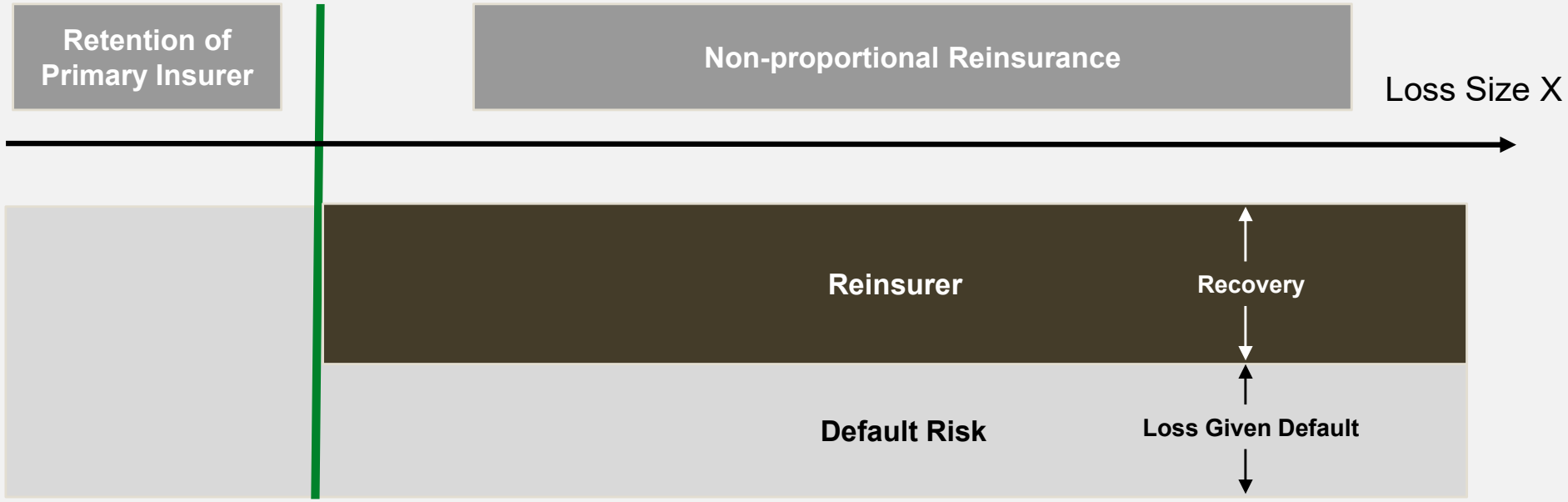
Mahul, O., Wright, B.D. (2004). *Implications of Incomplete Performance for Optimal Insurance*. Economica, 71.

→ Introduce insolvency state which occurs with probability q

→ In insolvency state: contractual payoff is reduced by LGD rate τ

→ XL-reinsurance is still optimal

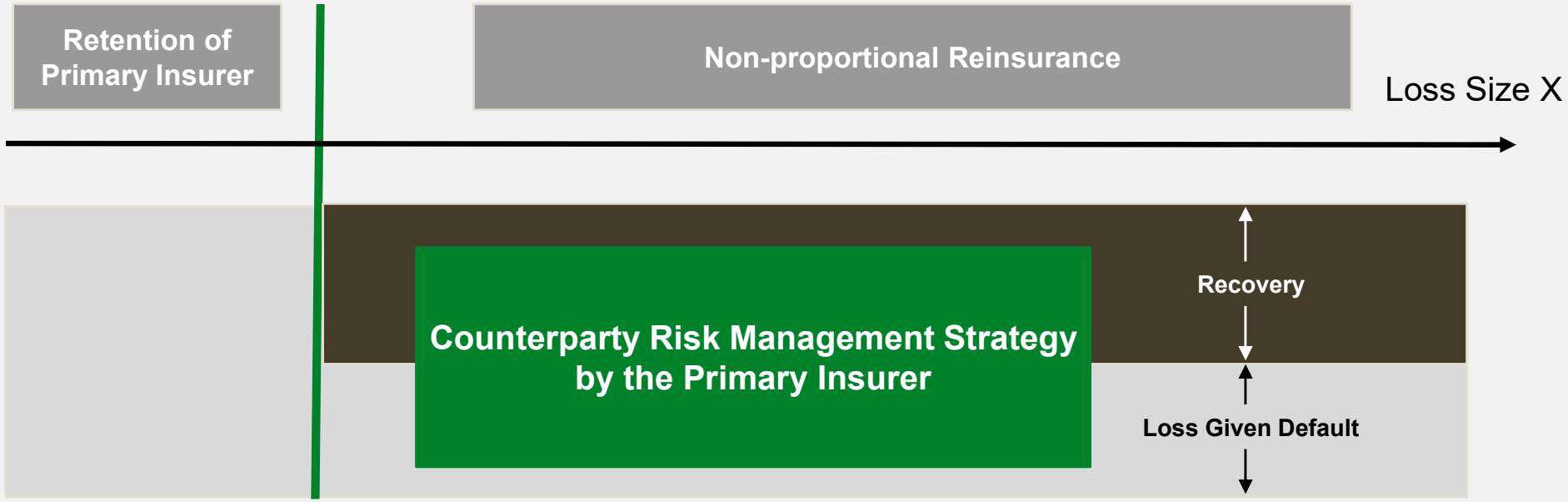
→ Deductible is always strictly positive (also for a zero cost loading)



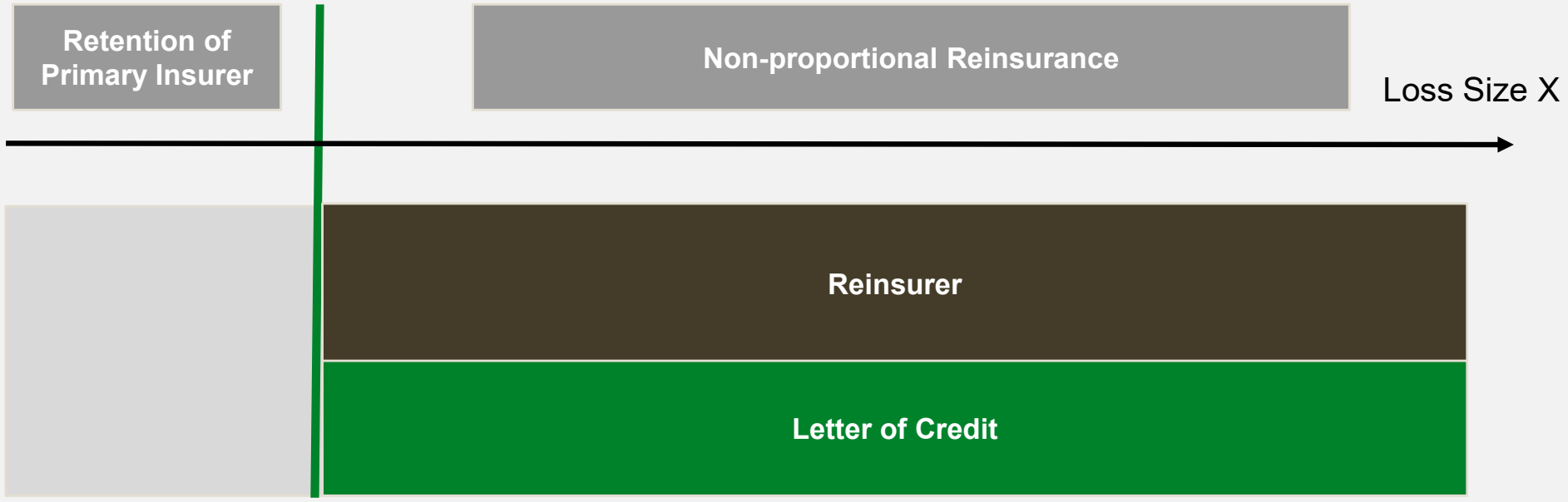
Optimal Management of the Default Risk

Our research project

- → Introduce measures to mitigate the default risk
- → Optimal design of these measures?
- → Effect on the optimal coverage?
- → Presumed strategies: Explicit hedging and diversification in the reinsurance portfolio
- → Optimal mix of measures?

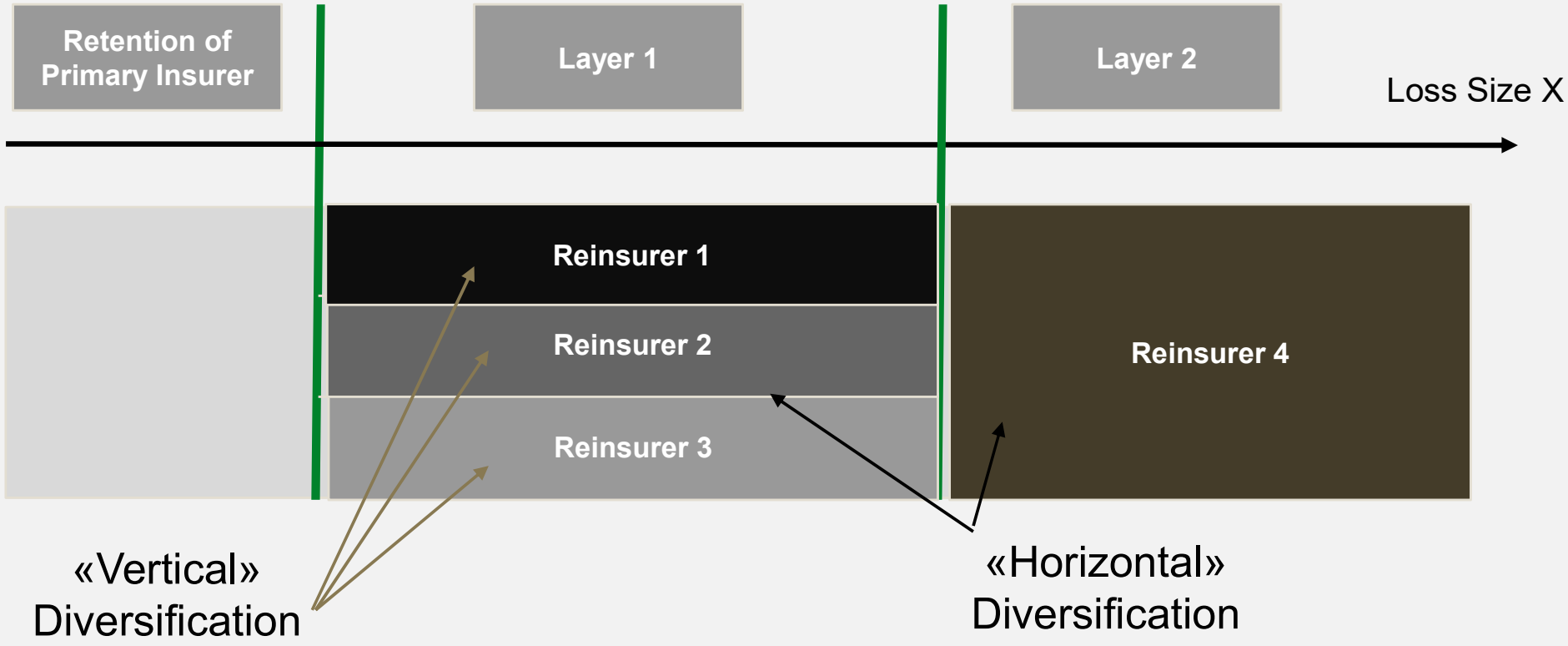


Hedging of Default Risk: Letter of Credit



Letter of Credit: A third party issues a guarantee to resume the primary insurer's claims deficit, if the reinsurer goes bankrupt.

Diversification of Default Risk



Literature on Multi-Layering (without consideration of default risk)

Boyer, M., Nyce, C.M. (2013) *An Industrial Organization Theory of Risk Sharing*. North American Actuarial Journal, 17, 283-296.

Malamud, S., Rui, H., Whinston, A. (2016) *Optimal Reinsurance with Multiple Tranches*. Journal of mathematical Economics, 65, 71-82.

Boonen, T.J., Tan, K.S., Zhuang, S.C. (2016) *The Role of a Representative Reinsurer in Optimal Reinsurance*. IME, 70, 196-204.

The Model: One Reinsurer, Default Risk and Hedging

Basic Setting (cf. Arrow, 1963)

a_0: non-stochastic initial wealth, e.g., the primary insurer's assets

X: loss size, modeled as a non-negative random variable with density $f(x)$ on the support $[0, \bar{x}]$

$r(X)$: reinsurance reimbursement as contracted given loss size X, constraints: $r(0) = 0$ and $r(X) \geq 0$

Default Risk of the Reinsurer (cf. Mahul/Wright, 2004)

$\mathcal{r}(X, D) = (1 - D)r(X)$: actual reimbursement with default risk D

D is an indicator variable with $\mathbb{P}(D = \tau) = q$ and $\mathbb{P}(D = 0) = 1 - q$

τ is the loss-given-default (non-stochastic, $1 - \tau$ is the recovery rate); X and D are stoch. independent

π_r is the premium: $\pi_r = (1 + \lambda_r)\mathbb{E}[\mathcal{r}(X, D)] = (1 + \lambda_r)(1 - q\tau) \int_0^{\bar{x}} r(x)dx$, with cost loading $\lambda_r \geq 0$

Extension: Hedging Instrument

Provides payoff $\hbar(X, D)$, with $\hbar(X, 0) = 0$ and $\hbar(X, \tau) = h(X) \geq 0$ (no default risk!)

π_h is the hedging fee: $\pi_h = (1 + \lambda_h)\mathbb{E}[\hbar(X, D)] = (1 + \lambda_h)q \int_0^{\bar{x}} h(x)dx$, with cost loading $\lambda_h \geq 0$

The Model: Two Reinsurers, Default Risk and Diversification

Default Risks of Two Reinsurers under Diversification

Two reinsurers: $r_i(X, D) = (1 - D_i) r_i(X),\ i = 1,2$

Total payoff from the reinsurance portfolio: $r_1(X, D) + r_2(X, D) = (1 - D_1) r_1(X) + (1 - D_2) r_2(X)$

D_i indicates the insolvency of reinsurer i, with $\mathbb{P}(D_i = \tau_i) = q_i$ and $\mathbb{P}(D_i = 0) = 1 - q_i$

q_i is the marginal default probability of reinsurer i; the joint probabilities are defined as:

$$Q_i := \mathbb{P}(D_i = \tau_i, D_j = 0),\ i \neq j; \quad Q_\emptyset := \mathbb{P}(D_1 = 0, D_2 = 0); \quad Q_{12} := \mathbb{P}(D_1 = \tau_1, D_2 = \tau_2)$$

π_i is the premium or reinsurer i: $\pi_i = (1 + \lambda_i)(1 - q_i \tau_i) \int_0^{\bar{x}} r_i(X) dx$, with cost loading $\lambda_i \geq 0$

Assumption on Heterogeneity

$$q_1 \geq q_2$$
$$\tau_1 \geq \tau_2$$

Reinsurer 1 is assumed to have an equal or **higher default risk**

$$\lambda_1 \leq \lambda_2$$

Reinsurer 1 is assumed to have an equal or **lower cost loading**

Expected-Utility-Optimization

- The primary insurer's equity after reinsurance and hedging (diversification) is:

$$E(X, D; r, h) = a_0 - \pi_r - \pi_h - X + r(X, D) + h(X, D)$$

$$E(X, D; r_1, r_2) = a_0 - \pi_r - \pi_h - X + r_1(X, D) + r_2(X, D)$$

- It is assumed that the primary insurer (or its risk-managers) behaves like a risk-averse decision-maker with concave utility function u (cf. Froot/Stein, 1998).
- It is aimed at finding the reinsurance-hedging combination $(r^*(x), h^*(x))$, and the reinsurance portfolio strategy $(r_1{}^*(x), r_2{}^*(x))$, respectively, that maximize the expected utility:

$$\max_{r,h,\pi_r,\pi_h} \mathbb{E}\big[u\big(E(X, D; r, h)\big)\big] \text{ subject to}$$

$$r(x) \geq 0, \text{ for } x \in [0, \bar{x}],$$

$$h(x) \geq 0, \text{ for } x \in [0, \bar{x}],$$

$$\pi_r = (1 + \lambda_r)(1 - q\tau) \int_0^{\bar{x}} r(x)dx,$$

$$\pi_h = (1 + \lambda_h) q \int_0^{\bar{x}} h(x)dx.$$

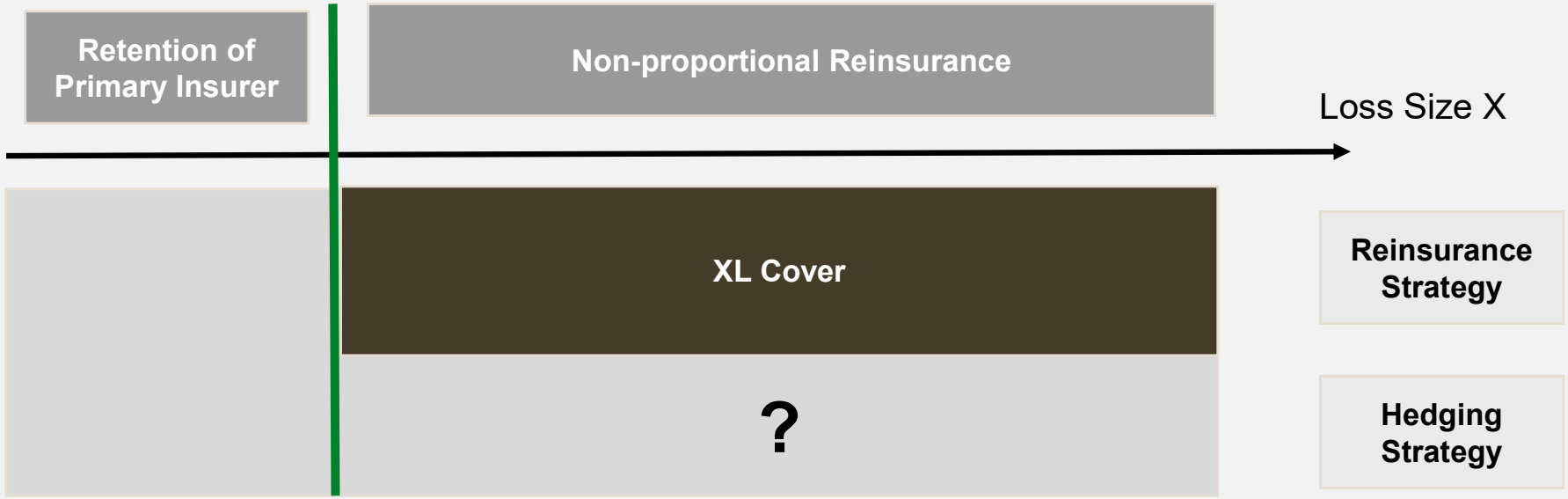
$$\max_{r_1,r_2,\pi} \mathbb{E}\big[u\big(E(X, D; r_1, r_2)\big)\big] \text{ subject to}$$

$$r_i(x) \geq 0, \text{ for } x \in [0, \bar{x}], i = 1,2$$

$$\pi = \sum_{i=1}^{2} (1 + \lambda_i)(1 - q_i\tau_i) \int_0^{\bar{x}} r_i(X)dx.$$

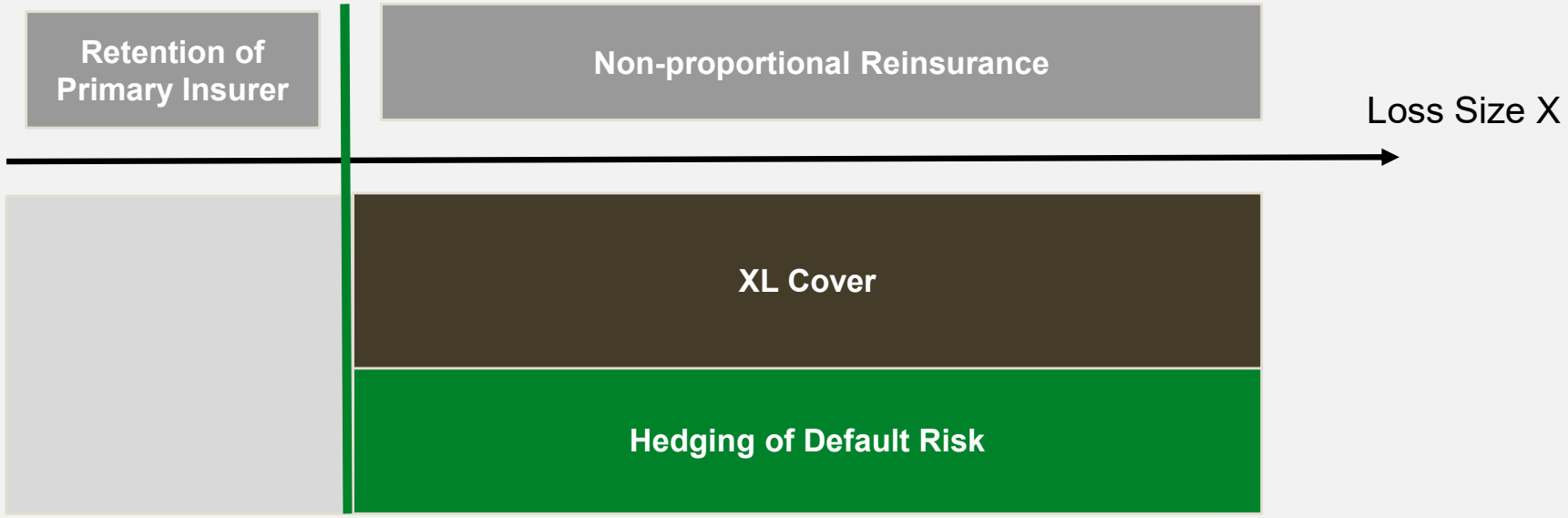
Optimal Reinsurance-Hedging Combination

Spread $\Delta_\lambda := \lambda_h - \lambda_r = ?$



Optimal Reinsurance-Hedging Combination (1/3)

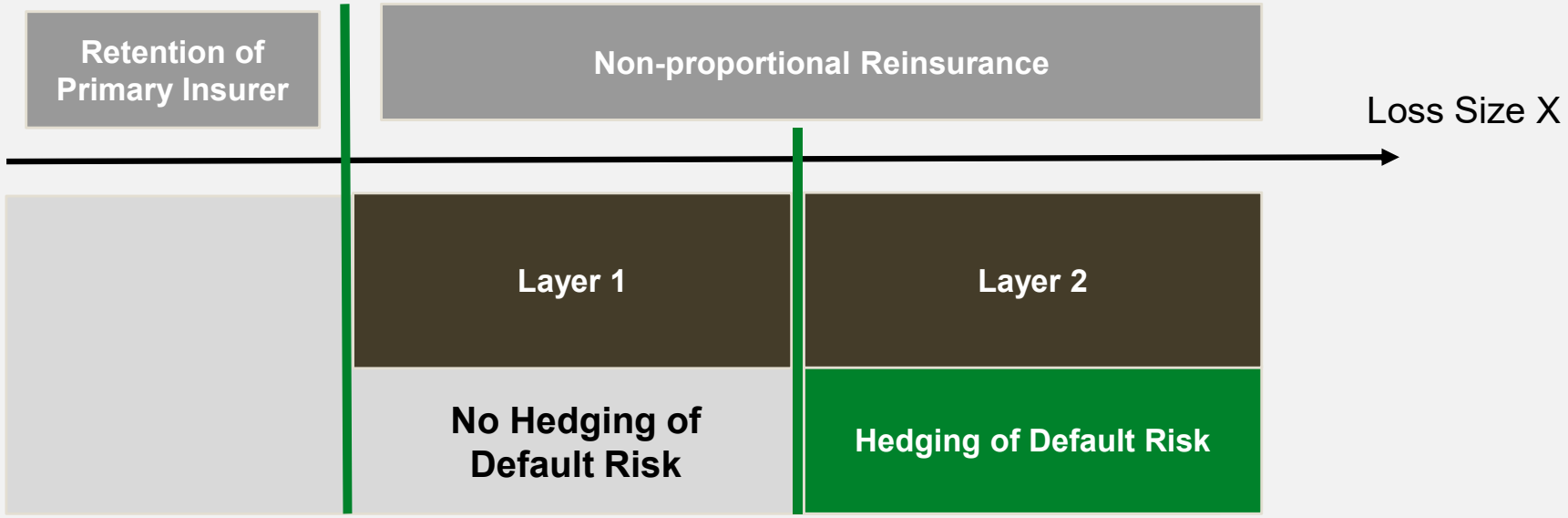
Spread $\Delta_\lambda := \lambda_h - \lambda_r = 0$ → Full hedging is optimal



- XL cover is the optimal reinsurance policy
- Full hedging of default risk from XL cover is optimal
- Deductible is the same as for the default-free demand model

Optimal Reinsurance-Hedging Combination (2/3)

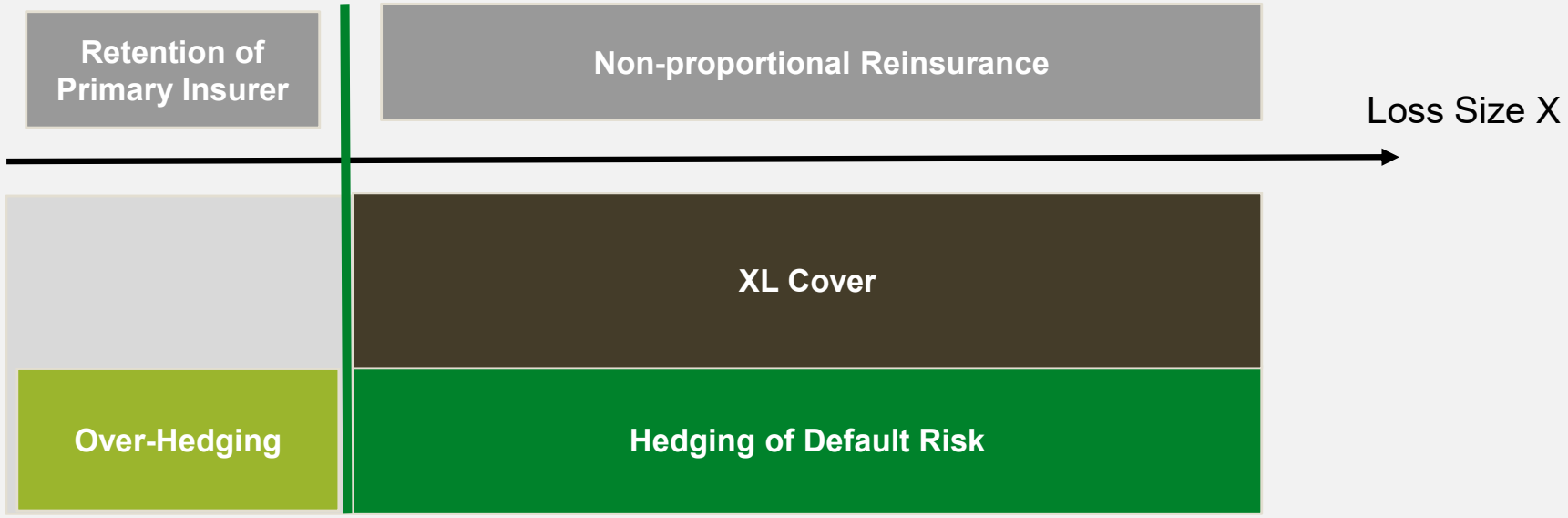
Spread $\Delta_\lambda := \lambda_h - \lambda_r > 0$ → Under-hedging is optimal



- Coverage is split in two layers: first layer remains unhedged; second layer is fully hedged
- Deductible is slightly higher than in the default-free model
- More-than-full coverage for layer 1 is optimal, if $0 < \tau < 1$; for $\tau = 1$: full coverage

Optimal Reinsurance-Hedging Combination (3/3)

Spread $\Delta_\lambda := \lambda_h - \lambda_r < 0$ → Over-hedging is optimal

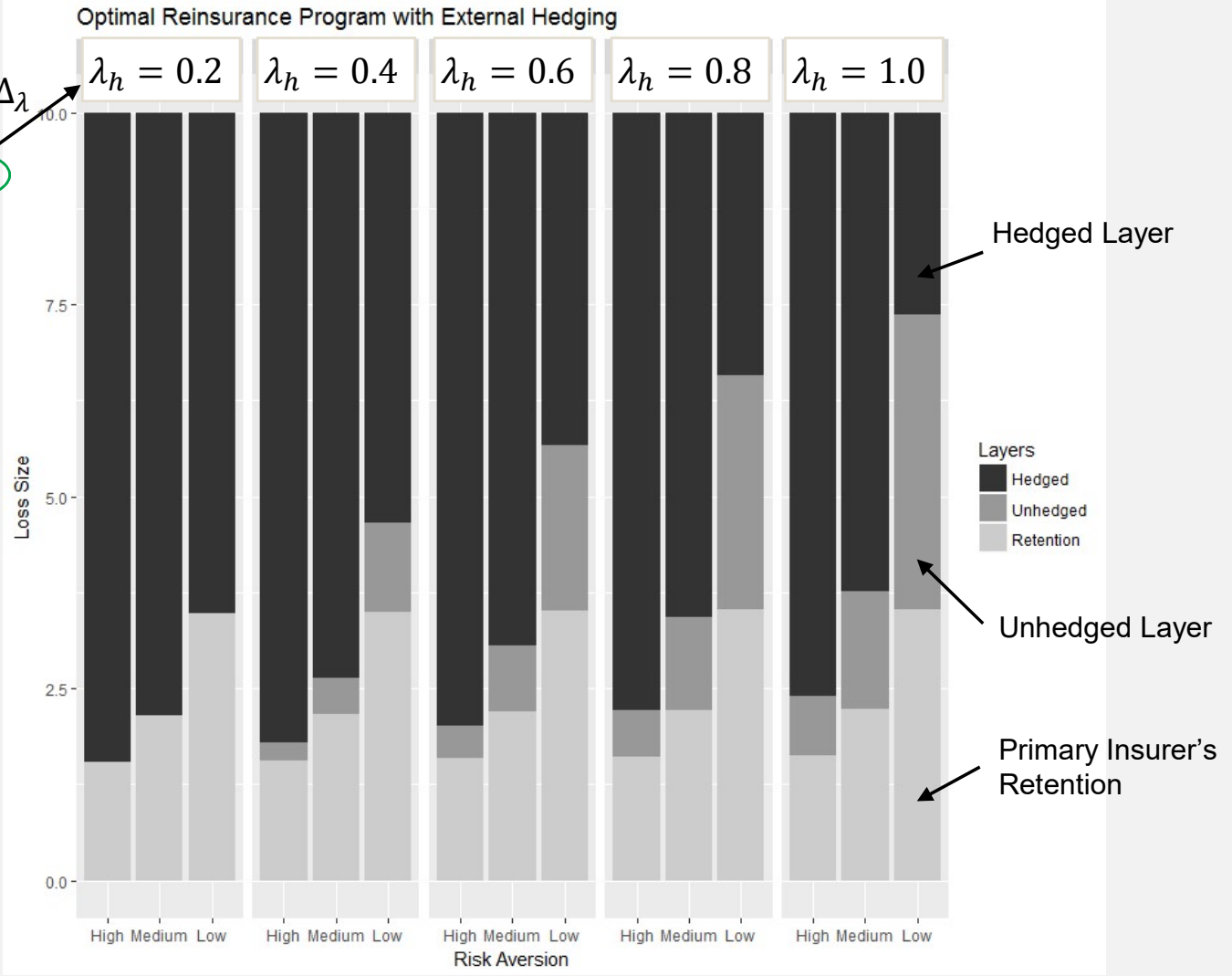


- XL cover is optimal reinsurance policy
- More-than full hedging is optimal (speculating on reinsurer's default)
- Deductible is slightly smaller than in the default-free case

Numeric Example

Parameter Setting

- Initial assets $a_0 = 15$
- Reinsurance cost loading: $\lambda_r = 0.2$
- Default risk of reinsurer: $q = 0.05$, $\tau = 1.0$ (total default)
- Loss size X has a truncated exponential distribution with $\bar{x} = 10$
- Primary insurer has exponential utility function with risk-aversion-parameter β
- High Risk Aversion: $\beta = 10/a_0$
- Medium Risk Aversion: $\beta = 5/a_0$
- Low Risk Aversion: $\beta = 2/a_0$



$Risk\ Aversion: High - Medium - Low$

Optimal Diversification Strategies

Cost loading spread:

$$\Delta_r = \lambda_2 - \lambda_1 \geq 0$$

Exchange rate to substitute one unit of coverage from reinsurer j by one unit of coverage from reinsurer i:

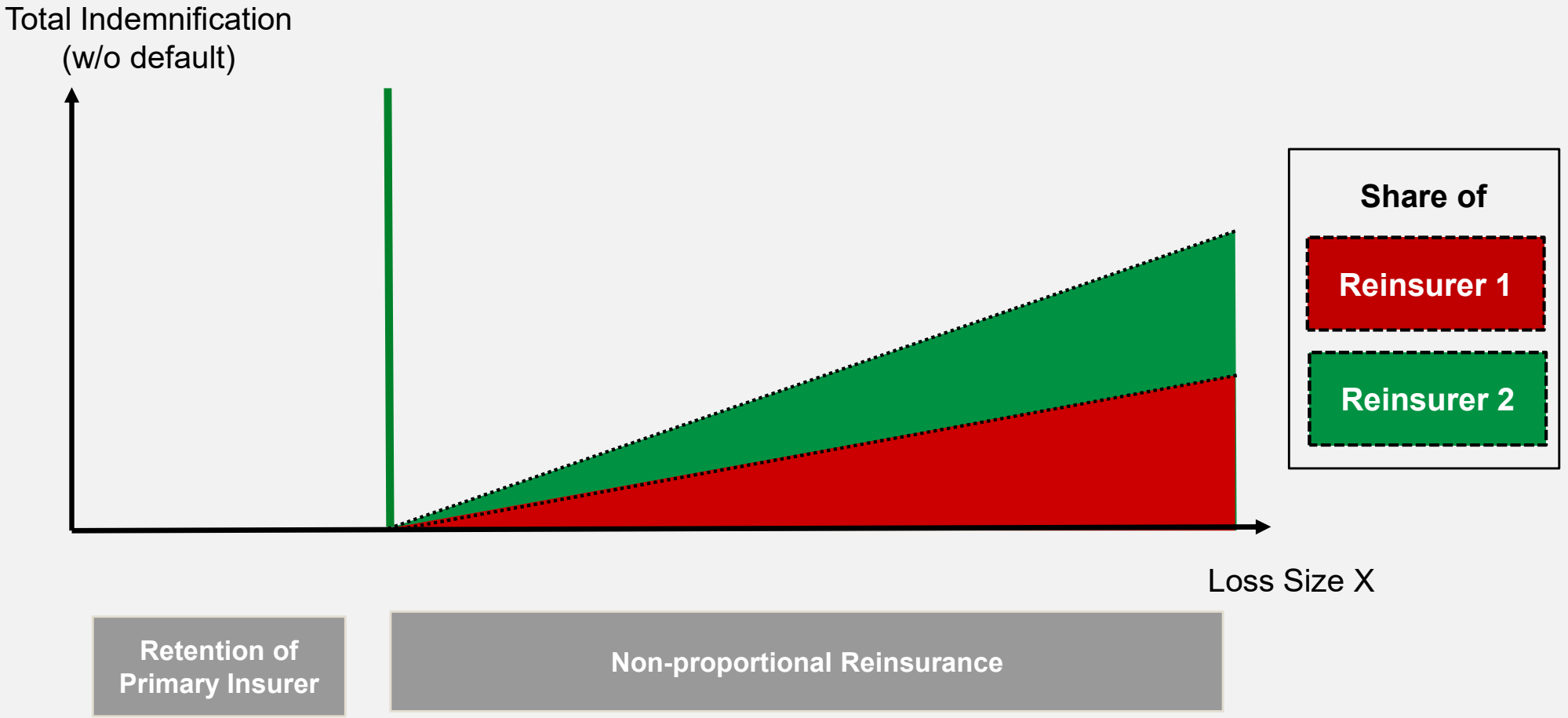
$$\gamma_{ij} = \frac{(1+\lambda_i)(1-q_i\tau_i)}{(1+\lambda_j)(1-q_j\tau_j)}$$

Payoff ratio of one unit of coverage from reinsurer i against one unit of coverage from reinsurer j **under a default of reinsurer** i:

$$\phi_{ij}^{i} = \frac{(1-\tau_i)Q_i + (1-\tau_i)Q_{12}}{Q_i + (1-\tau_j)Q_{12}}$$

Identical Cost Loadings: $q_1 \geq q_2\ \tau_1 \geq \tau_2\ , \Delta_r = 0$ (1/2)

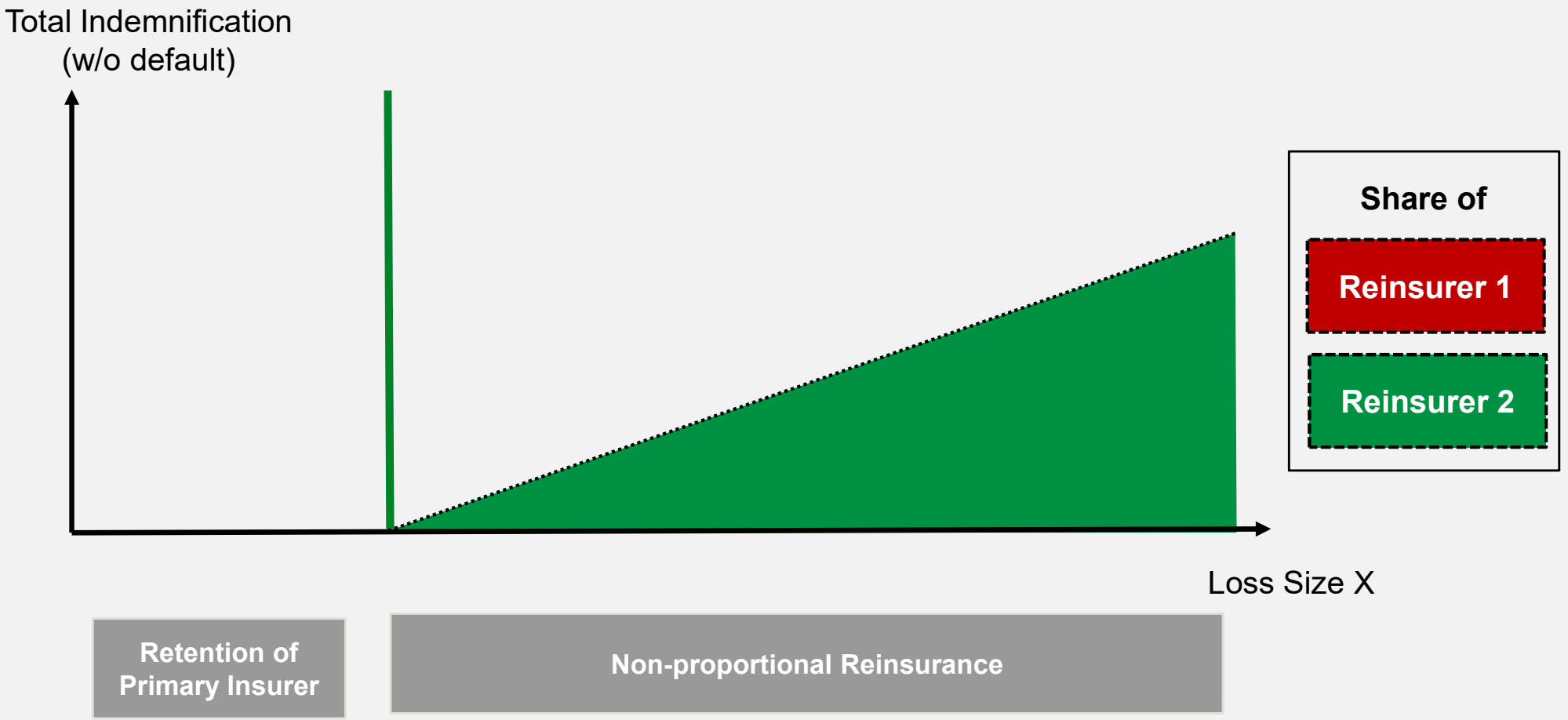
$\gamma_{21} \geq \phi_{21}^2$ → **Vertically diversified XL cover is optimal**



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Identical Cost Loadings: $q_1 \geq q_2\ \tau_1 \geq \tau_2, \Delta_r = 0$ (2/2)

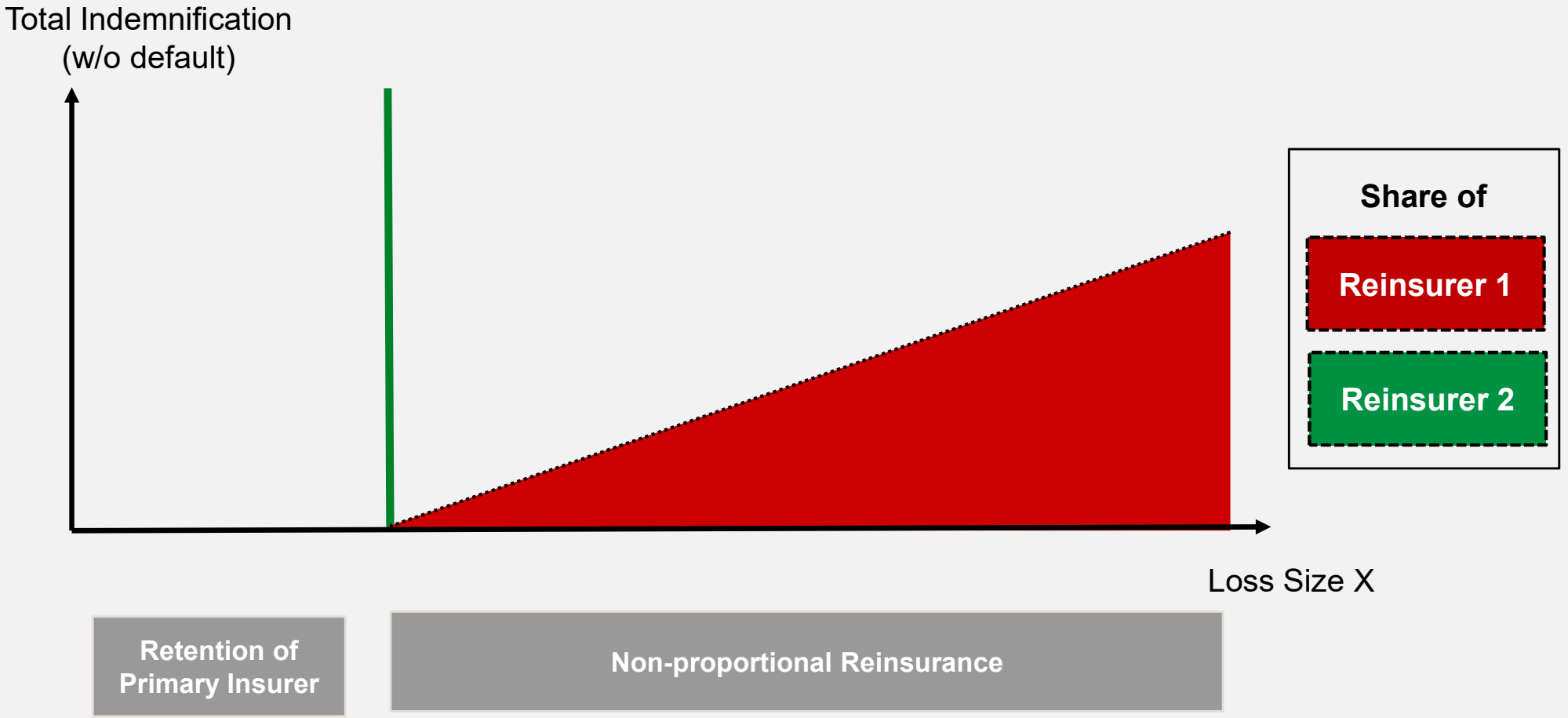
$\gamma_{21} < \phi_{21}^2$ → **Non-diversification is optimal; saver reinsurer only**



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Different Cost Loadings: $q_1 \geq q_2\ \tau_1 \geq \tau_2, \Delta_r > 0$ (1/3)

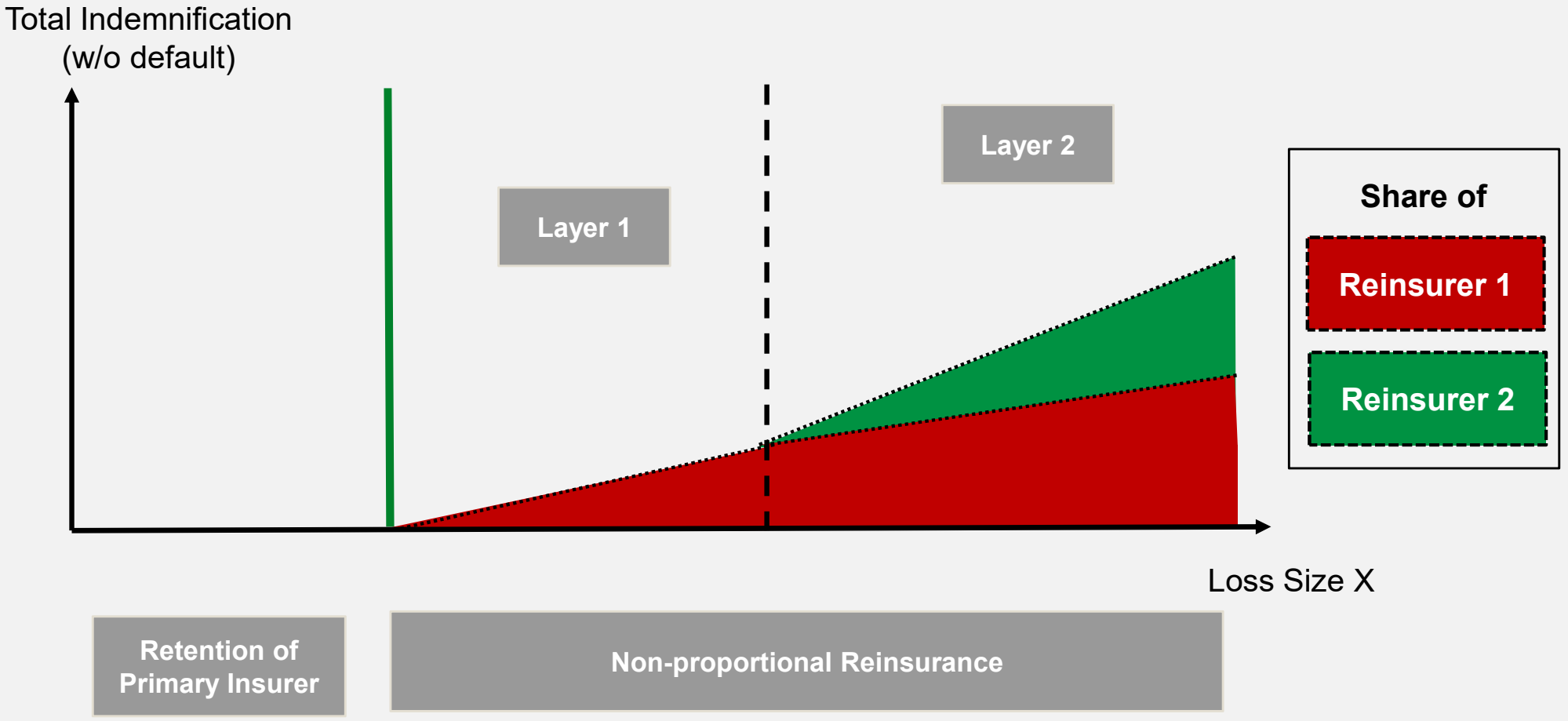
$\gamma_{12} \leq \phi_{12}^1$ → **Non-diversification is optimal; riskier reinsurer only**



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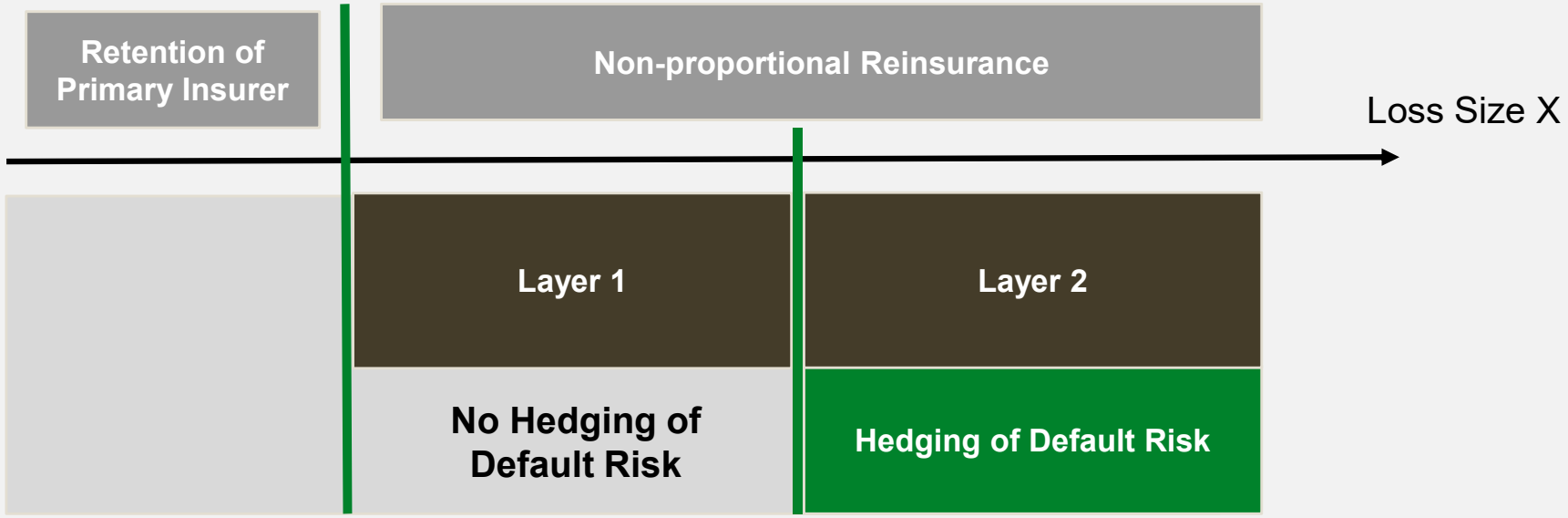
Different Cost Loadings: $q_1 \geq q_2\ \tau_1 \geq \tau_2, \Delta_r > 0$ (2/3)

$\gamma_{12} > \phi_{12}^1$ and $\gamma_{21} \geq \phi_{21}^2$ → **Horizontal and vertical diversification is optimal**



Optimal Reinsurance-Hedging Combination (2/3)

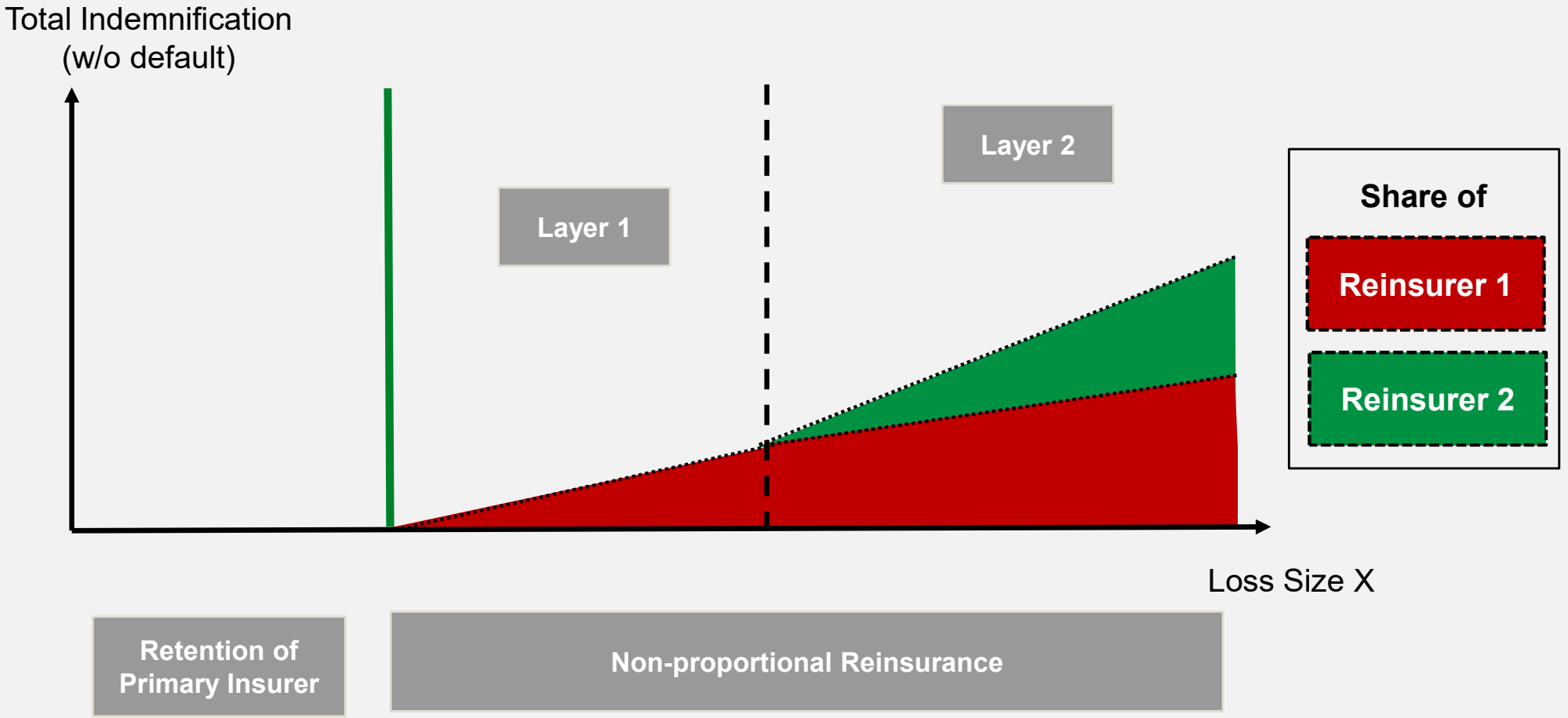
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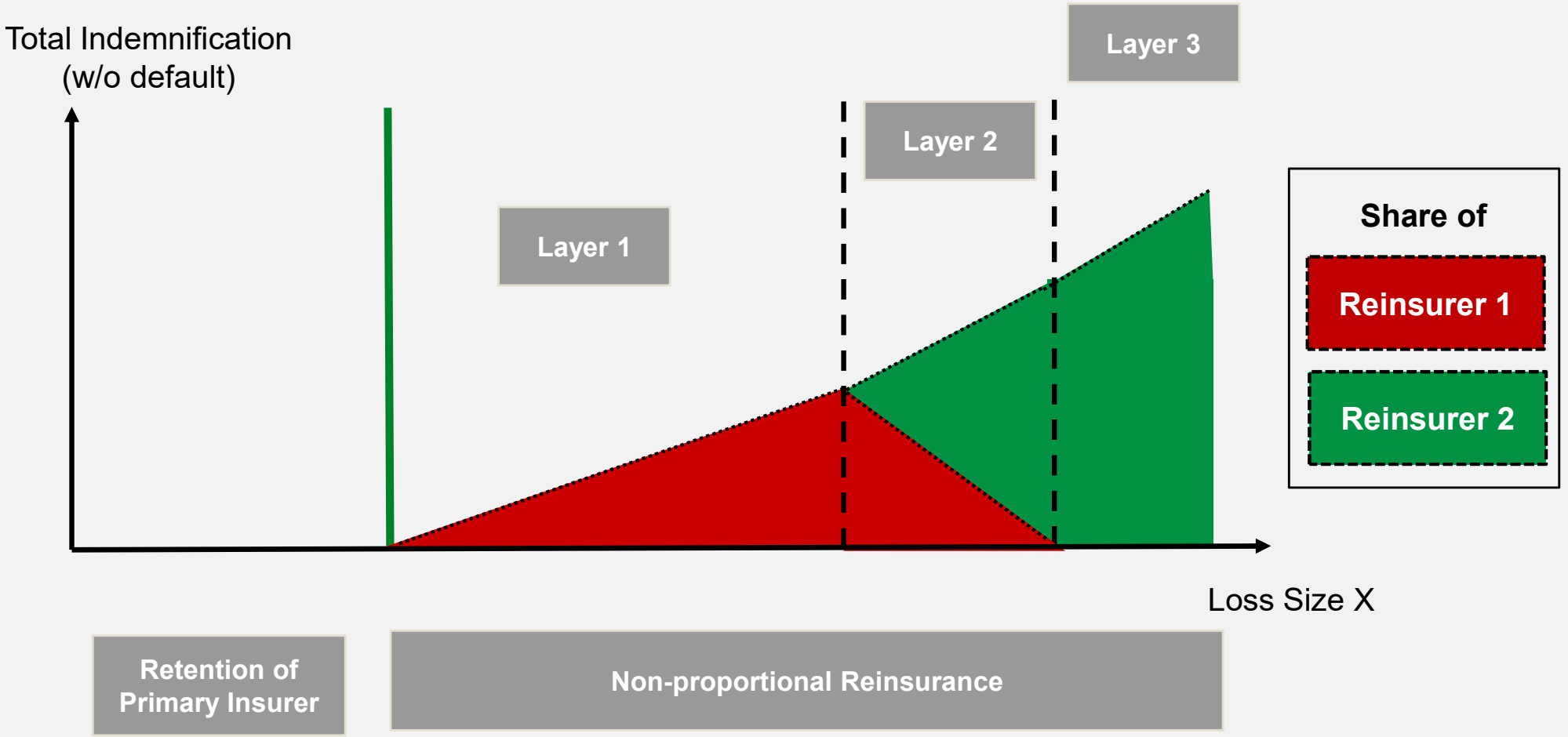
Different Cost Loadings: $q_1 \geq q_2\ \tau_1 \geq \tau_2, \Delta_r > 0$ (2/3)

$\gamma_{12} > \phi_{12}^1$ and $\gamma_{21} \geq \phi_{21}^2$ → **Horizontal and vertical diversification is optimal**



Different Cost Loadings: $q_1 \geq q_2\ \tau_1 \geq \tau_2, \Delta_r > 0$ (3/3)

$\gamma_{12} > \phi_{12}^1$ and $\gamma_{21} < \phi_{21}^2$ → **Horizontal and vertical diversification is optimal**



Conclusion

So far

- Costly management of default risk should be concentrated on higher layers
- Basically, preference for reinsurers with low default risk, but only if corresponding costs do not outweigh the utility from a lower default risk
- Combination of horizontal and vertical diversification may be optimal for placing saver, but expensive reinsurer in higher layers.

Still to be researched

- Optimal shares of riskier and saver reinsurer, respectively, under horizontal diversification
- Better understanding of cost and payoff rates γ_{ij} and ϕ_{ij}^{i}
- Length of the different layers / position of their attachment points
- Combination of hedging and diversification: substitution and optimal risk-management mix.

Thank You

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