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Optimization of hydro storage systems and indifference pricing of power contracts

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Outlook

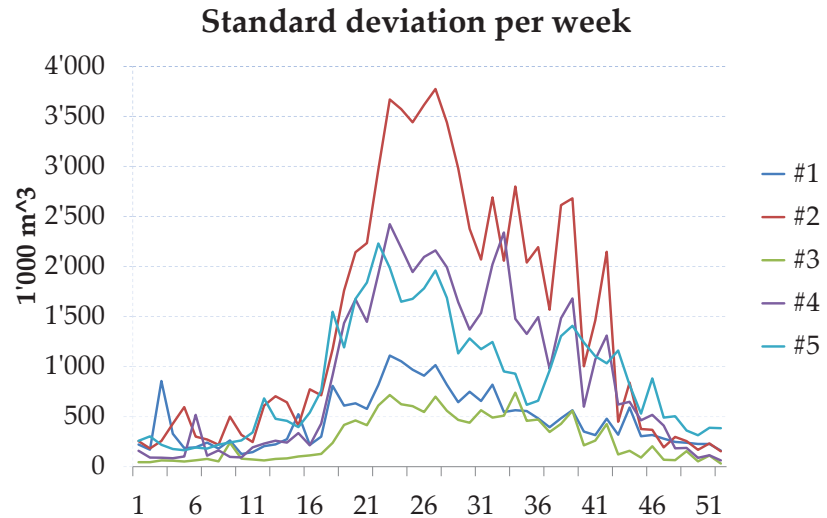
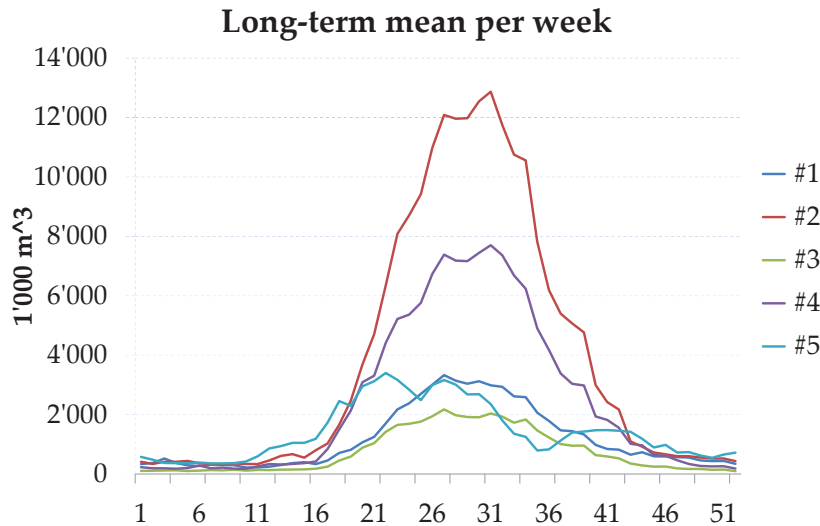
- Producer owns **pumped-storage hydropower plants** (PSHPs). These consist of large (seasonal) connected reservoirs
- We must decide on a production schedule
- We aim at a **mid-term planning model for hydropower production** based on **multistage stochastic programming**
 - Stochastic inflows; stochastic electricity prices
 - Produced electricity is sold at the spot market
- Valuation of non-standard power contracts with indifference pricing (work in progress)
 - Stochastic load profiles
 - Indifference price: entering the contracts with individual consumers or selling only at the spot market

Inflow data

- Daily inflows are given for **a system with five connected reservoirs** (sample period: 26 years)
- Data are aggregated to weekly inflows
- The inflows show a characteristic seasonal pattern:
 - Winters are dry
 - Inflows are higher in the middle of the year
- Thus, **a seasonal adjustment will be performed**

Seasonality

- Let $f_i(\tau)$ be the inflow in reservoir $i = 1, \dots, 5$ at time $\tau = 1, \dots, T$
- Let $\bar{f}_i(w')$ long-term mean of the inflows, $w' = 1, \dots, 52$ and $\hat{\sigma}_i(w')$ the standard deviations of the inflows after seasonal adjustment for all reservoirs over one year
- Allocate each time point to one week w' : $\mathcal{T}(w') := \{\tau = 1, \dots, T \mid w(\tau) = w'\}$
 - Long-term seasonal adjustment: $f_i^{adj}(\tau) = f_i(\tau) - \bar{f}_i(w(\tau))$
 - Correct for time-varying volatility: $f_i^{std}(\tau) = \frac{f_i^{adj}(\tau)}{\hat{\sigma}_i(w(\tau))}$



Principal component analysis

- A principal component analysis is performed on the stochastic component of the inflows
- According to the eigenvalue criterion, only the first factor should be considered
- Then 75% of the variation can be explained; with 2(3) factors we explain about 86(93) percent of the variation

Table 1: Principal component analysis

Component	Eigenvalue	% of Variance	Cumulative %
1	3.744	74.9	74.9
2	0.573	11.5	86.3
3	0.371	7.4	93.8
4	0.279	5.6	99.3
5	0.033	0.7	100.0

Spot model for electricity prices

The formulation of the regime-switching model reads:

$$X_t = \begin{cases} \ell_t^L - \xi_t^-, & \text{if the system is in the lower spike regime,} \\ s_t \cdot \exp(r_t), & \text{if the system is in the base regime or} \\ \ell_t^U + \xi_t^+, & \text{if the system is in the upper spike regime.} \end{cases} \quad (1)$$

$$dr_t = -\kappa_\alpha r_t dt + \sigma_\alpha dZ_t \quad (2)$$

$$\ell_t^L = s_t / \exp(a_\gamma^L), \quad \ell_t^U = s_t \cdot \exp(a_\gamma^U). \quad (3)$$

$$\xi_t^- \sim \text{Wei}(\lambda_\beta^-, k_\beta^-), \quad \xi_t^+ \sim \text{Wei}(\lambda_\beta^+, k_\beta^+)$$

X_t spot price in hour t

s_t seasonality shape (deterministic component)

r_t stochastic component of the spot price in the base regime

ξ_t^+ upward spike

ξ_t^- downward spike

ℓ_t^U upper limit of the base regime in hour t

ℓ_t^L lower limit of the base regime in hour t

Transition probabilities and results

- The probabilities of remaining in the current regime or switching to another one from one hour to the next are modeled by a (time-dependent) transition matrix:

$$\Pi_{\delta} = \begin{pmatrix} 1 - \pi_{\delta}^{LB} & \pi_{\delta}^{LB} & 0 \\ \pi_{\delta}^{BL} & 1 - \pi_{\delta}^{BL} - \pi_{\delta}^{BU} & \pi_{\delta}^{BU} \\ 0 & \pi_{\delta}^{UB} & 1 - \pi_{\delta}^{UB} \end{pmatrix}. \quad (4)$$

where $\delta := \delta(t)$ is a function that maps t to a set of indices (seasonality)

- Estimation procedure: **two-step maximum likelihood**

Estimation results

Estimation sample: Hourly EEX spot prices from 1 Jan 2009 to 31 Dec 2013

Param.	a_{γ}^L	a_{γ}^U	κ_{α}	σ_{α}	λ_{β}^{-}	k_{β}^{-}	λ_{β}^{+}	k_{β}^{+}	$E(\xi_t^{-})$	$E(\xi_t^{+})$
Overall	0.697	0.875	2.848 (0.05)	0.566 (0.001)	0.127 (0.005)	0.737 (0.01)	0.09 (0.018)	0.798 (0.101)	9.493	12.681
Summer	0.649	0.894	3.045 (0.075)	0.555 (0.002)	0.184 (0.008)	0.936 (0.015)	0.105 (0.042)	1.326 (0.899)	5.591	8.759
Winter	0.89	1.054	2.538 (0.057)	0.606 (0.002)	0.101 (0.006)	0.646 (0.015)	0.065 (0.027)	1.134 (0.542)	13.657	14.771

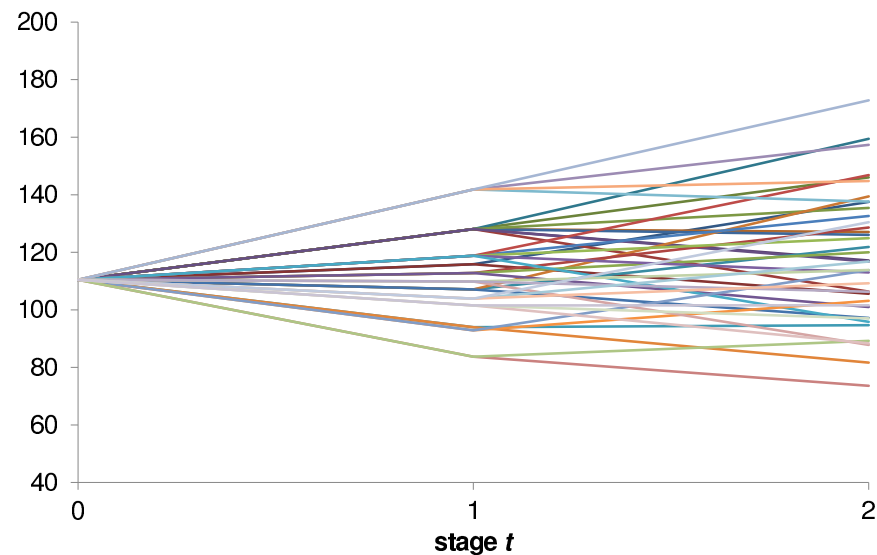
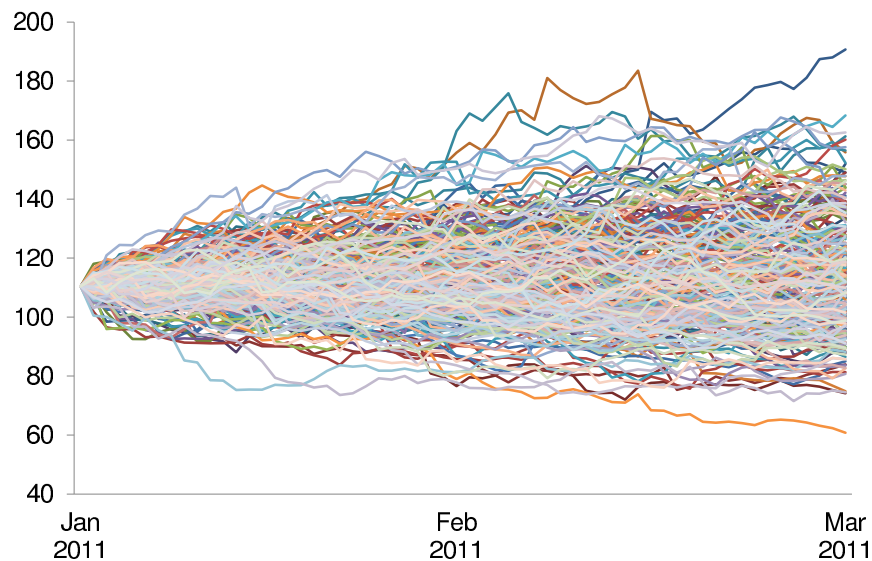
Table 2: Estimation results#1 (standard errors in parenthesis). $E(\xi_t^{+/-})$ shows the expected value of upwards/downwards spike magnitudes

Param.	$\pi_{\delta}^{BL/LB}$	$\pi_{\delta}^{BU/UB}$	#base	#lower	#upper	loglike
Overall	1.053% 24.061%	0.078% 73.333%	41937	1837	45	27156
Summer Base	1.631% 28.800%	0.015% 66.666%	21130	826	4	27338
Summer Peak	0.193% 22.368%	0.013% 100%				
Winter Base	1.309% 24.386%	0.007% 100%	21064	788	7	
Winter Peak	0.078% 12.631%	0.064% 80%				

Table 3: Estimation results#2 (standard errors in parenthesis)

Generating scenarios

- A scenario tree approximates the underlying stochastic process for the risk factors
- The Wasserstein distance is employed to identify the best scenario approximation
- We generate scenarios in weekly steps for
 - the stochastic component of the spot price process for the base regime
 - two principal components of the inflow model
- An initial tree is generated with the procedure in Hochreiter/Pflug (2007); it is improved using the stochastic approximation algorithm in Pflug/Pichler (2015) (*implementation ongoing*)



Time aggregation: price level approach

- The valuation of non-standard power contracts requires an hourly time resolution
- But: The computational effort would explode since scenario trees grow exponentially with the number of stages
- Thus, an **aggregation technique** is required!
- We apply the price level approach (a.k.a. occupation times)

Optimization model prerequisites

- Notation:

$\mathcal{I} = \{1, \dots, I\}$ set of reservoirs

$\mathcal{J} = \{1, \dots, J\}$ set of connections between reservoirs

$\mathcal{T} \subset \mathcal{J}$ set of turbines

$\mathcal{P} \subset \mathcal{J}$ set of pumps

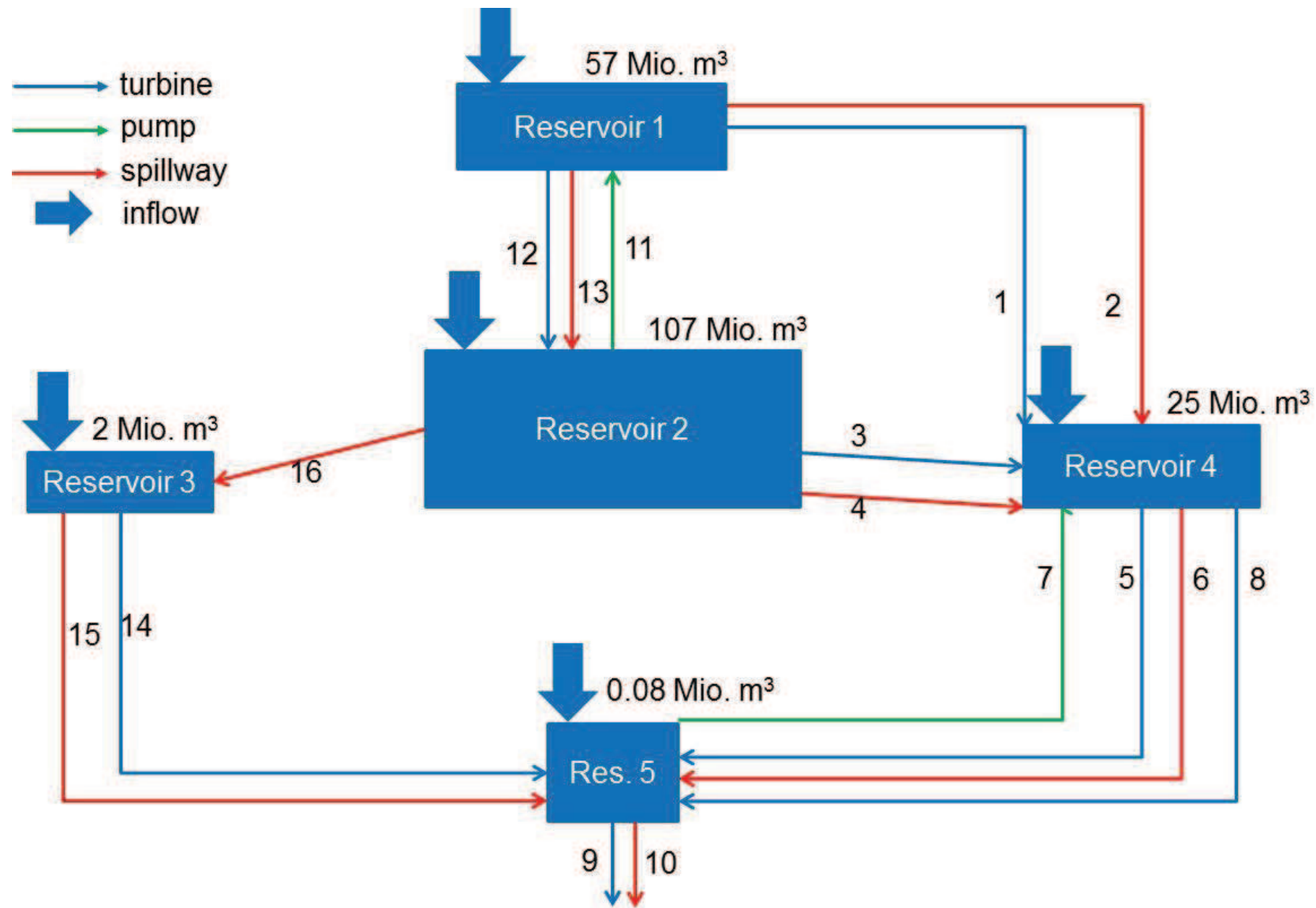
$\mathcal{W} \subset \mathcal{J}$ set of spillways

$\mathcal{L} = \{1, \dots, L\}$ price level index set

- The system topology is described by an incidence matrix A such that

$$A_{ji} = \begin{cases} 1 & \text{if water flows into reservoir } i \text{ over connection } j, \\ -1 & \text{if water flows out of reservoir } i \text{ over connection } j, \\ 0 & \text{if connection } j \text{ is not related to reservoir } i. \end{cases}$$

Reservoirs layout



System parameters

- **System parameters (deterministic):**

$\bar{s}_i$	maximum storage volume (minimum is zero) in m^3
g_i	minimum storage level (end of planning horizon) in m^3
$\underline{q}_j, \bar{q}_j$	minimum/maximum water flow in m^3/h
η_j	production efficiency in MWh/m^3
$\underline{p}_j, \bar{p}_j$	lower/upper bounds on produced power in MWh

- **Remarks:**

- Turbines have positive efficiency (conversion from water to electricity)
- Pumps have negative efficiency, negative minimum power and maximum power of zero
- Spillways have zero efficiency and nonnegative flow capacity

- **System parameters (stochastic):**

S_t	electricity spot price at time t in EUR/MWh
F_{it}	inflow in reservoir i (precipitation, snowmelt etc.) in m^3

Tree notation

- The (joint) stochastic process of the uncertain factors is represented by a finite tree with node set $\mathcal{N} = \{0, 1, \dots, N\}$
- $\mathcal{N}_t$ is the set of nodes at level $t = 0, \dots, T$
- The tree structure represents the filtration of the process and can be defined by stating the unique predecessor n^- for each node n
- In the tree-based model with price levels notation changes:

$S_{n\ell}$ spot price at level ℓ in node n

F_{in} inflow in reservoir i in node n in m^3

- Further definitions:

Δ_n length of planning period beginning in node n

π_n probability of node n

$\pi_{n\ell}$ probability of price level ℓ in node n (index set: $\mathcal{L}$)

Optimization model

- For each price level ℓ at each node n the producer decides how much water $q_{jn\ell}$ flows over connection j (in m^3/h)
- This corresponds to the produced power $p_{jn\ell}$ (in MW) and energy $e_{jn\ell}$ (in MWh) over the period:

$$p_{jn\ell} = \eta_j q_{jn\ell}, \quad (5)$$

$$e_{jn\ell} = p_{jn\ell} \pi_{n\ell} \Delta_n \quad (6)$$

- Technical constraints for produced (consumed) power:

$$\underline{p}_j \leq p_{jn\ell} \leq \bar{p}_j \quad (7)$$

- The stored water s_{in} in each reservoir at the beginning of the period is restricted by the reservoir capacity:

$$0 \leq s_{in} \leq \bar{s}_i \quad (8)$$

- It starts from an initial value s_{i0} and develops over time as:

$$s_{in} = s_{in-} + \sum_{j \in \mathcal{J}} A_{ji} \sum_{\ell \in \mathcal{L}} \pi_{n\ell} q_{jn-\ell} \Delta_n + F_{in} \quad (9)$$

- At time T , the stored water volume has to meet the minimum level:

$$s_{in} \geq \bar{g}_i \quad (n \in \mathcal{N}_T) \quad (10)$$

Optimization model

- We start with an initial cash amount c_0 ; for later time points $n > 0$ cash evolves according to the budget equation:

$$c_n = R_n c_{n-} + \sum_{j \in \mathcal{T} \cup \mathcal{P}} \sum_{\ell \in \mathcal{L}} e_{jn-\ell} S_{n-\ell} \quad (11)$$

where R_n is the interest rate in node n over the period Δ_n (equal for lending and borrowing)

- **The objective is defined as a mix of expectation and average value at risk** in view of the available cash at the end of the planning horizon in T :

$$\max \lambda E(\text{"cash in } T") + (1 - \lambda) AVaR(\text{"cash in } T") \quad (12)$$

Optimization model overview

- The complete model in the tree-based notation is:

$$\begin{aligned} & \max \lambda \sum_{n \in \mathcal{N}_T} \pi_n c_n + (1 - \lambda) z & (13) \\ \text{s.t.} & & \\ & z \leq y_0 - \frac{1}{\alpha} \sum_{n \in \mathcal{N}_T} \pi_n y_n \\ & n \in \mathcal{N}_T : & c_n - y_0 + y_n \geq 0 \\ & n \in \mathcal{N}_T : & y_n \geq 0 \\ & j \in \mathcal{J}; \ell \in \mathcal{L}; n \in \mathcal{N} : & p_{jn\ell} = \eta q_{jn\ell} \\ & j \in \mathcal{J}; \ell \in \mathcal{L}; n \in \mathcal{N} : & e_{jn\ell} = p_{jn\ell} \pi_{n\ell} \Delta_n \\ & j \in \mathcal{J}; \ell \in \mathcal{L}; n \in \mathcal{N} : & \underline{p}_j \leq p_{jn\ell} \leq \bar{p}_j \\ & i \in \mathcal{I}; n \in \mathcal{N} \setminus \{0\} : & s_{in} = s_{in-} + \sum_{j \in \mathcal{J}} A_{ji} \sum_{\ell \in \mathcal{L}} \pi_{n\ell} q_{jn-\ell} \Delta_n + F_{in} \\ & i \in \mathcal{I}; n \in \mathcal{N} \setminus \{0\} : & 0 \leq s_{in} \leq \bar{s}_i \\ & i \in \mathcal{I}; n \in \mathcal{N}_T : & s_{in} \geq \bar{g}_i \\ & n \in \mathcal{N} \setminus \{0\} : & c_n = R_{n-} c_{n-} + \sum_{j \in \mathcal{T} \cup \mathcal{P}} \sum_{\ell \in \mathcal{L}} e_{jn-\ell} S_{n-\ell} \end{aligned}$$

Indifference pricing

- Consider M delivery contracts $m \in \mathcal{M} = \{1, \dots, M\}$ of individual consumers
 - Different demand levels D_{mnl} for different price levels
 - Electricity is delivered at fixed prices K_m (EUR/MWh)
- **Indifference pricing:** Determine the smallest possible price such that the producer meets the obligations from the contracts; new mix of expectation and AVaR γ should not be worse than without the contract

$$\min_{K, q, p, e, c, s, h} \sum_{m \in \mathcal{M}} K_m$$

$$\text{s.t. (4) – (10)}$$

$$\text{AVaR-constraints}$$

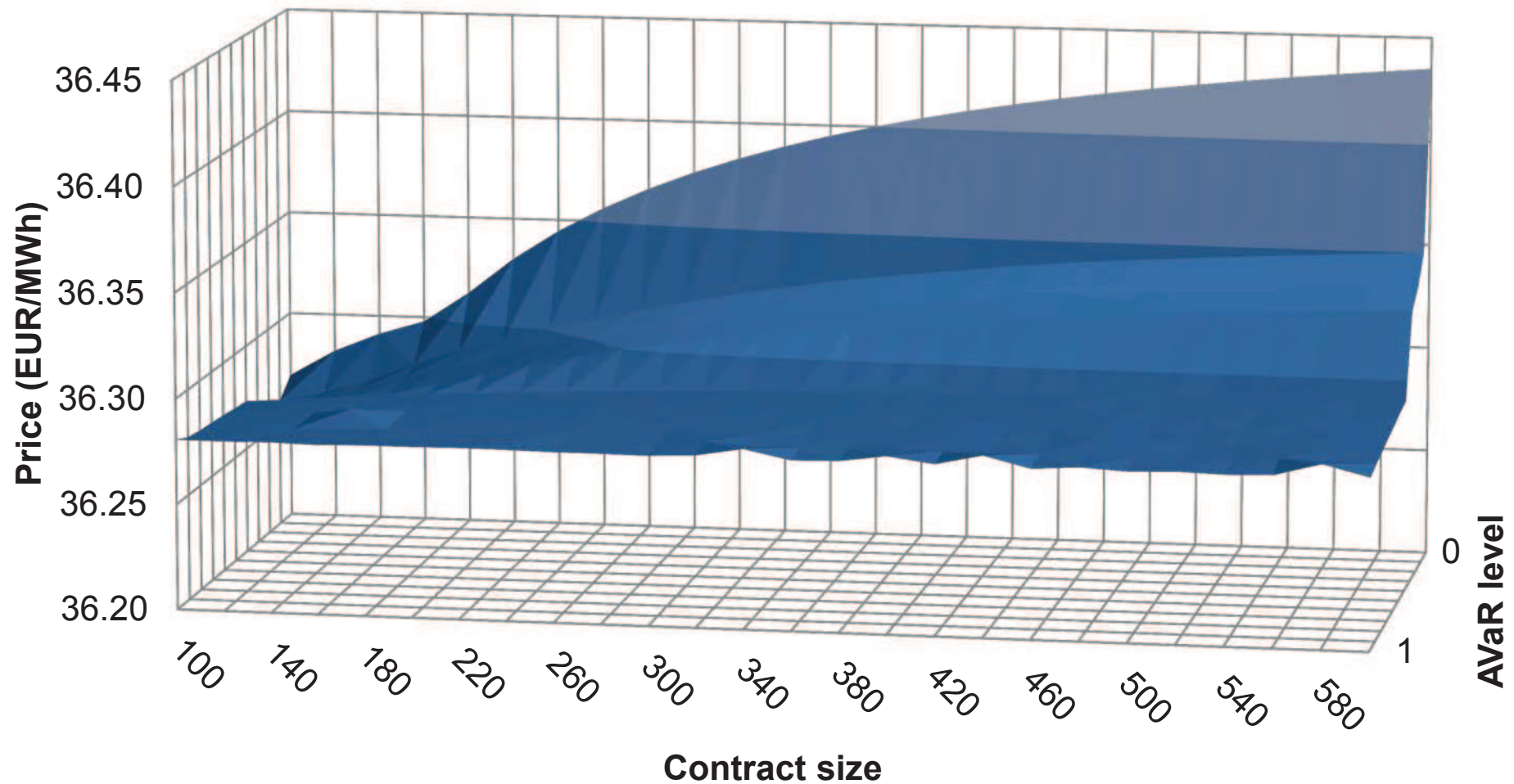
$$\gamma \geq \gamma^*$$

$$\sum_{j \in \mathcal{T} \cup \mathcal{P}} e_{jnl} = h_{nl} + \Delta_n \pi_{nl} \sum_{m \in \mathcal{M}} D_{mnl}$$

$$c_n = R_{n-} - c_{n-} + \sum_{\ell \in \mathcal{L}} S_{n\ell} h_{n\ell} + \Delta_n \sum_{\ell \in \mathcal{L}} \pi_{n\ell} \sum_{m \in \mathcal{M}} K_m D_{mnl}$$

$$s_{in} \geq s_{in}^* \quad (n \in \mathcal{N}_T)$$

Indifference pricing for a constant load, one contract



Summary

- Optimization of PSHP operations by multistage stochastic programming with aggregation based on price levels
- Model aims at large reservoirs (seasonal storages)
 - Regime-switching model for electricity spot prices
 - Factor model for inflows, taking into account seasonality
- Valuation of power contracts with indifference pricing
- Open:
 - Modelling stochastic load profiles
 - Extension to several contracts (portfolio effects)
 - Comparison with other approaches