

Aspirational Preferences and Their Representation by Risk Measures

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We consider choice over uncertain, monetary payoffs and study a general class of preferences. These preferences favor diversification, except perhaps on a subset of sufficiently disliked acts over which concentration is instead preferred. This structure encompasses a number of known models (e.g., expected utility and several variants under a concave utility function). We show that such preferences share a representation in terms of a family of measures of risk and targets. Specifically, the choice function is equivalent to selection of a maximum index level such that the risk of beating the target at that level is acceptable. This representation may help to uncover new models of choice. One that we explore in detail is the special case when the targets are bounded. This case corresponds to a type of satisficing and has descriptive relevance. Moreover, the model is amenable to large-scale optimization.

Key words: representation of choice; risk measures; aspiration levels; decision theory paradoxes

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1. Introduction

The notion of an aspiration level rests at the core of Simon's (1955) concept of bounded rationality. Namely, because of limited cognitive resources and incomplete information, real-world decision makers may plausibly follow heuristics in the face of uncertainty. Satisficing behavior, in which the decision maker accepts the first encountered alternative that meets a sufficiently high aspiration level, may prevail.

There is ample empirical evidence that aspiration levels, or "targets," may play a key role in the decision making of many individuals. (Mao 1970, p. 353), for instance, concludes after interviewing many executives that "risk is primarily considered to be the prospect of not meeting some target rate of return." Other studies (e.g., Roy 1952; Lanzilotti 1958; Fishburn 1977; Payne et al. 1980, 1981; March and Shapira 1987) reach similar conclusions regarding the importance of targets in managerial decisions. Diecidue and van de Ven (2008) propose a model combining expected utility with loss and gain probabilities, which leads to a number of predictions consistent with empirical data. Recently, Payne (2005) showed that many decision makers would be willing to accept a decrease in a gamble's expected value

solely in exchange for a decrease in the probability of a loss.

The goal of this paper is to provide some formalism for the role of aspiration levels in choice under uncertainty and to introduce a new model in this domain. We consider the case of a decision maker choosing from a set of monetary acts (state-contingent payoffs) and define a class of choice functions over these acts. The structure of the underlying preferences is broad: the main property is the way the decision maker values mixtures of positions. In particular, we assume the decision maker prefers to diversify among acts, except possibly on a subset of sufficiently unfavorable choices for which concentration is preferred. We call these preferences *aspirational preferences* (APs). In the case when the concentration favoring set is empty, diversification is always preferred and vice versa. Our definition of diversification favoring is the standard definition of convex preferences: a mixture of two acts can never be worse than both individual acts. In this setting of monetary acts, a number of choice models, including subjective expected utility (SEU) (under a concave or convex utility function) and several variations, correspond to aspirational preferences.

The conventional way to express preferences is a choice function that ranks alternatives. In the case of

SEU, a utility function and a subjective prior form this representation. Our main result is a representation result, which states that we can equivalently express the choice function for aspirational preferences via a family of measures of risk and deterministic targets (the “target function”). In particular, the risk measures are ordered by an index, and choice is equivalent to choosing the maximum index such that the risk of beating the target at that index is acceptable. On the set of acts for which diversification is preferred, the risk measures are *convex risk measures* (Föllmer and Schied 2002, 2004).

This representation unifies a number of choice models and may be useful for establishing structural properties (e.g., stochastic dominance, robustness guarantees) as well as in optimization algorithms. It also may open the door to new models of choice. One example is the case when the target function is bounded. In this case, the lower and upper limits of the target function correspond to a minimal requirement and a satiation level, respectively. Acts that fail to attain the minimal requirement in any state are least preferred, whereas acts that attain at least the satiation level in all states are most preferred. When the two levels coincide, the target represents a single aspiration level. In this case of a bounded target function, we say the decision maker has *strongly aspirational preferences* and the choice function is a *strong aspiration measure* (SAM). Although the general AP model includes some normatively motivated models, choice in the special case of SAM has some descriptive merits. With properly chosen targets, SAM can address the classical violations of SEU from Allais (1953) and Ellsberg (1961). This model also addresses more recent paradoxes, such as an Ellsberg-like example from Machina (2009), and can explain choice patterns in a set of experiments related to gain-loss separability in Wu and Markle (2008).

Two recent papers study related preference structures. Cerreia-Vioglio et al. (2011) axiomatize a model of uncertainty averse preferences, which are convex and monotone, and provide a general representation result in an Anscombe–Aumann (1963) framework. Drapeau and Kupper (2009) develop a similar representation on general topological spaces. Their representation is presented primarily in terms of “acceptance sets,” whereas ours is in the form of risk measures and targets. Another distinction is that the AP model does not require convex preferences everywhere.

The idea of representing choice in terms of targets is not new. Interestingly, it has been shown that the Savage axioms for SEU also have an equivalent representation in terms of probability of beating a stochastically independent target (see Castagnoli and LiCalzi 2006, Bordley and LiCalzi 2000), and there is

a related stream of papers on “target-oriented utility” (e.g., Abbas et al. 2008, Bordley and Kirkwood 2004). Recently, Bouyssou and Marchant (2011) axiomatized a model based on partitioning acts into “attractive” and “unattractive” sets and show a representation in terms of SEU (with a continuous utility function) and a threshold utility level. Though the AP model here includes SEU under a concave (or convex) utility function, it generally excludes utility functions that are neither convex nor concave (e.g., an S-shaped utility function). More broadly, aspiration measures need not have the form of the expected value of a utility function—SAM is one such example that is not in this form. One feature of the AP model that is useful for optimization is that preference for diversification and concentration is explicitly separated across a partition. This is crucial for tractability when the dimension of the choice set is high (e.g., portfolio choice, financial planning).

Finally, it is important to frame the contributions here relative to the *satisficing measures* in Brown and Sim (2009)—these are developed as a model of “target-oriented” choice and shown to be representable by families of risk measures. Although this paper builds upon Brown and Sim (2009), there are a number of important differences. First, Brown and Sim (2009) assume a given probability distribution and therefore cannot handle ambiguity; the AP model does not rely on a given probability distribution. Second, Brown and Sim explicitly enforce properties (which they call “attainment content” and “nonattainment apathy”) relative to an exogenous target. In contrast, the AP model does not require this and is therefore much more general (in addition to satisficing measures and SEU under convex or concave utility, the model encompasses, under concave utility, maxmin expected utility, Choquet utility, and variational preferences). Moreover, targets arise endogenously from the representation. Third, the AP model is more general in that allows for the possibility of some concentration-favoring behavior. In the SAM case, this is crucial to be able to make choices among gambles that have an expected value below the target; as we later discuss, one cannot distinguish among such gambles using satisficing measures. This can be important in applications. For instance, Brown and Sim (2009) discuss a debt management problem and mention that when the liabilities are too large, they cannot use satisficing measures to choose among investment strategies because all options have a net negative expected value and are indistinguishable in such a model. More recently, Chen and Long (2012) show that a decision bias in newsvendor problems (reported empirically in Schweitzer and Cachon 2000), which cannot be resolved by expected utility theory, can be explained by SAM. This bias involves a switch

between risk attitudes and therefore relies on a model that has both risk aversion and risk seeking. Lastly, Brown and Sim (2009) do not explore any descriptive implications, whereas here we do.

We start by describing the choice setting and defining aspirational preferences and then present the representation result. Section 3 is devoted to the SAM model and its properties. Section 4 applies this model to the decision theory paradoxes mentioned earlier, and §5 discusses optimization of the model with a portfolio choice example. The appendix contains the proof of Theorem 1 and tables for §4; all other proofs and tables are in the online appendix (available at <http://dx.doi.org/10.2139/ssrn.2005194>).

2. Aspirational Preferences and Representation

2.1. Model Preliminaries and Aspirational Preferences

Our general setup considers the problem of individual choice under uncertainty. A set of “monetary acts” describes the choice set. Specifically, we consider a set S of states of the world, endowed with a sigma-algebra Σ . An element $s \in S$ is an individual state of the world, whereas an element $A \in \Sigma$ is an event. An act, typically denoted f, g , or h , is a measurable function from S to $\mathbb{R}$, with the value of the act in a given state, $f(s)$, interpreted as an amount of money. An act f is *constant* if $f(s) = x$ for some payoff value $x \in \mathbb{R}$ for all $s \in S$. For any two acts f and g , we say that f *dominates* g if $f(s) \geq g(s)$ for all $s \in S$ and use the shorthand $f \geq g$ to denote this. For any two acts f, g and $\lambda \in [0, 1]$, the convex combination $h = \lambda f + (1 - \lambda)g$ is defined state by state; i.e., $h(s) = \lambda f(s) + (1 - \lambda)g(s)$. This setup follows Savage (1954) in that no “objective” probability distribution is specified in advance. The decision maker may, of course, make choices according to his or her own subjective, probabilistic beliefs; this will often be the case in the examples that follow.

We consider a decision maker who wants to choose an act from a closed and convex set F of bounded acts. Preferences are such that there exists an upper semicontinuous (with respect to the sup-norm topology) choice function $\rho: F \rightarrow \bar{\mathbb{R}}$, with f (weakly) preferred to g if and only if $\rho(f) \geq \rho(g)$. Lack of continuity allows us to accommodate preferences in the spirit of satisficing, e.g., $\rho(f) > 0$ on $\{f \geq 0\}$ (satisfactory payoffs that attain at least the target) and $\rho(f) < 0$ on $\{f < 0\}$ (unsatisfactory payoffs).

We then require the following.

PROPERTY 1 (MONOTONICITY). For all $f, g \in F$, if $f \geq g$, then $\rho(f) \geq \rho(g)$.

PROPERTY 2 (MIXING). There exists a partition of F into three disjoint subsets F_{++} , F_{--} , and F_0 , some possibly

empty, termed the *diversification favoring*, *concentration favoring*, and *neutral sets of acts*, respectively, such that for all $f \in F_{++}$, $g_1, g_2 \in F_0$, $h \in F_{--}$, we have $\rho(f) > \rho(g_1) = \rho(g_2) > \rho(h)$, in addition to the following:

(i) *Diversification favoring set:* For all $f, g \in F_{++}$, $\lambda \in [0, 1]$, $\rho(\lambda f + (1 - \lambda)g) \geq \min\{\rho(f), \rho(g)\}$.

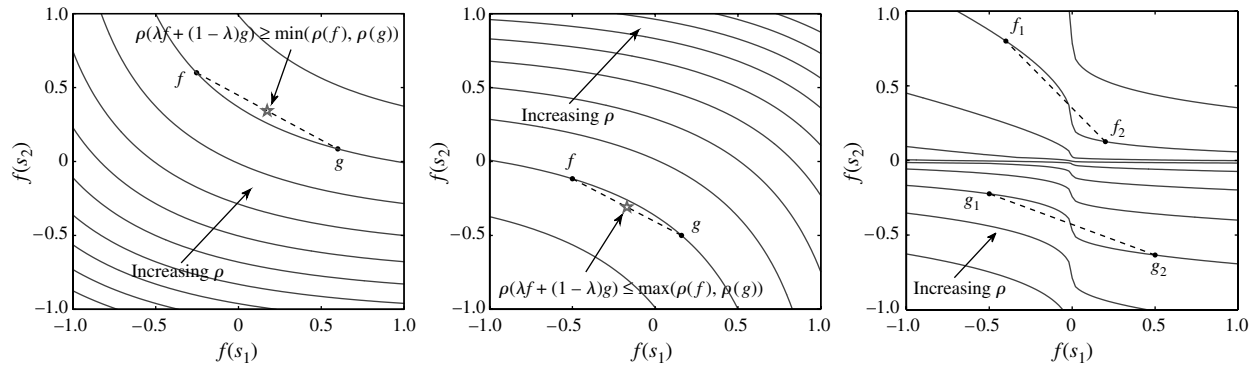
(ii) *Concentration favoring set:* For all $f, g \in F_{--}$, $\lambda \in [0, 1]$, $\max\{\rho(f), \rho(g)\} \geq \rho(\lambda f + (1 - \lambda)g)$.

Property (i) is equivalent to convex preferences over F_{++} . This states that mixing among any two acts in F_{++} never results in a position that is worse than both individual acts. This is a form of “risk aversion” and encourages diversification over indifferent acts in this set. Property (ii) says the opposite: mixing among any two acts in F_{--} can never result in a position that is better than both individual acts. This can be viewed as a form of “risk seeking” preferences over F_{--} and encourages the decision maker to concentrate toward more extreme choices. Any concentration favoring that is permitted, however, is localized entirely on a set of less preferable acts. Intuitively, the decision maker will want to diversify, except possibly in situations when the set of available choices is sufficiently unfavorable. This is expressed in Property 2, with all acts in F_{++} strictly preferred to those in F_{--} . The neutral set F_0 is the boundary set between the set of acts where diversification is favored and the set of acts where concentration is favored.

As a simple example of a situation when concentration may be sensible, consider a decision maker who is selecting among a set of investments with the goal of attaining a target payoff level τ . This could be the case, for instance, for a firm selecting among risky income streams in order to cover existing liabilities. Consider two possible income streams, f and g , with f attaining τ only in one state of the world s_f and g attaining τ only in one state of the world $s_g \neq s_f$. For any mixture $h = \lambda f + (1 - \lambda)g$ with $\lambda \in (0, 1)$, h can never attain the desired payoff level in any state, i.e., $h < \tau$. Therefore, if the decision maker diversifies, he will always attain a mixed position that never attains the desired payoff level. In this case, if the goal is to attain the desired payoff level, it seems reasonable to prefer at least one of f or g individually, each of which have some chance of attaining this desired level, over any mixture.

Although this is a simple example purely for illustrative purposes, it makes an important point. Namely, there may be situations, particularly in models of choice that focus on aspiration levels, in which some concentration favoring may be sensible.

DEFINITION 1. A decision maker with an upper semicontinuous choice function $\rho: F \rightarrow \bar{\mathbb{R}}$ satisfying Properties 1 and 2 is said to have *aspirational preferences* on F with partition F_{++} , F_0 , F_{--} , and we call the choice function ρ an *aspiration measure* (AM).

Figure 1 Indifference Curves for Three Utility Functions: Concave (Left), Convex (Center), and S-Shaped (Right)

Note. The first two are aspiration measures (with $F_{--} = \emptyset$ and $F_{++} = \emptyset$, respectively); the S-shaped utility function is not.

Figure 1 provides an illustration, in two states, of two choice functions that are aspiration measures and one that is not. These plots represent the preferences of all acts with payoffs bounded between -1 and $+1$; each point in the two-dimensional space represents an act f with point $(f(s_1), f(s_2))$ corresponding to the payoffs of f in each state, and the indifference curves across these acts are plotted for each choice function. In all three cases in the figure, the choice function is the expected value of a utility function under some probability measure. On the left are the indifference curves for a concave utility function; naturally, diversification is always weakly preferred for concave utility, and here we have an aspiration measure with $F_{--} = \emptyset$. For a convex utility function, concentration is always weakly preferred, and we have an aspiration measure with $F_{++} = \emptyset$ instead.

The right figure illustrates indifference curves for an S-shaped utility function à la prospect theory.¹ This is not an aspiration measure because we cannot partition the set of acts $F = [-1, 1]^2$ into disjoint sets where diversification is preferred and where concentration is preferred as well as indifference to all acts between these sets. We can argue this formally as follows. Consider the acts f_1 and f_2 ; because the decision maker is indifferent (they lie on the same indifference curve) between these acts, both must be in the same part of the partition. But we can have neither $f_1, f_2 \in F_{++}$ nor $f_1, f_2 \in F_{--}$ because neither diversification nor concentration is preferred between these two acts (see the dashed line; some mixtures are strictly worse, whereas others are strictly better). This requires $f_1, f_2 \in F_0$. By a similar argument, $g_1, g_2 \in F_0$. But then we must have $f_1 \sim f_2 \sim g_1 \sim g_2$, which is a contradiction because both f_1 and f_2 are strictly preferred to g_1 and g_2 . Thus, SEU with an S-shaped utility function is generally not an aspiration measure.

Intuitively, what rules out an S-shaped utility function from being an AM is the strong requirement of indifference across all acts neither in the diversification favoring set nor the concentration favoring set. More broadly, a necessary condition for a choice function to be an AM is that for all acts f , either the set of acts (weakly) preferred to f be convex, or the set of acts to which f is preferred be convex.²

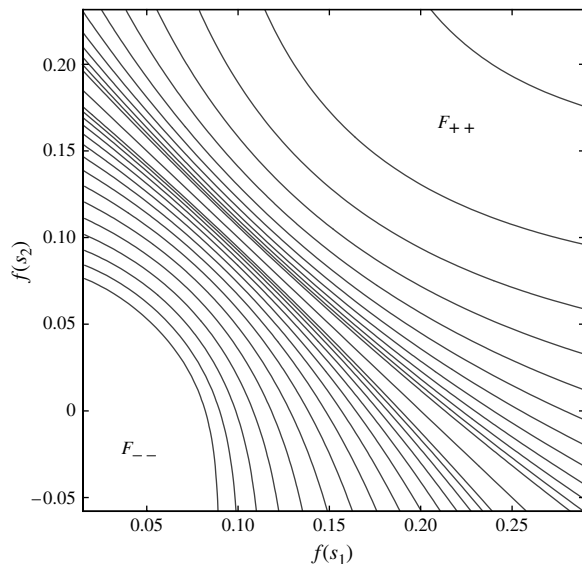
Figure 2 provides a two-dimensional illustration of the more general case of an AM with both F_{++} and F_{--} nonempty. Here, we can clearly see the regions in which diversification and concentration are favored, respectively; the neutral set F_0 divides the two sets. We see that F_{++} and F_{--} must be convex sets. Indeed, consider any $f, g \in F_{++}$ and assume without loss of generality that $\rho(f) \geq \rho(g)$. Then for any $\lambda \in [0, 1]$, we have by Property 2, $\rho(\lambda f + (1 - \lambda)g) \geq \rho(g)$, which implies that $\lambda f + (1 - \lambda)g \in F_{++}$ because all elements in the diversification favoring set are favored to all elements in the neutral and concentration favoring sets. A similar argument applies for F_{--} .

A decision maker with aspirational preferences must specify the partition because it is part of the preference structure. This specification may be cognitively challenging in general. In some cases, however, the structure of the partition simplifies greatly. A trivial (but important) case is when diversification is everywhere preferred, as is the case for an SEU maximizer with concave utility. Here, $F_{--} = \emptyset$. The partition is equally simple when $F_{++} = \emptyset$, as is the case with convex utility. A more interesting situation is the case of strongly aspirational preferences, which we later discuss. In this case, when the decision maker

¹ The specific utility function here is a convex-concave power utility with loss aversion: $u(y) = \max(0, y)^\alpha - \lambda \max(0, -y)^\alpha$. The parameters in this example are $\mathbb{P}(s_1) = 0.2$, $\alpha = 0.4$, and $\lambda = 2$.

² Indeed, consider any $f \in F_{++}$. For any $g_1 \geq f$, $g_2 \geq f$, $\lambda \in [0, 1]$. Then, we must have $g_1, g_2 \in F_{++}$ as well, and thus, by Property 2, $\rho(\lambda g_1 + (1 - \lambda)g_2) \geq \min(\rho(g_1), \rho(g_2)) \geq \rho(f)$, which shows that the set of acts weakly preferred to f is convex. An analogous argument shows that for $f \in F_{--}$, the set of acts to which f is preferred is convex. Finally, the set of acts less preferred to any $f \in F_0$ is just F_{--} , which by similar arguments is convex.

Figure 2 Indifference Curves for an Aspiration Measure with F_{++} and F_{--} Both Nonempty



Note. In this case, the neutral set F_0 is the separating hyperplane dividing the two sets.

has a subjective prior distribution, we will find that the partition is formed simply by calculating expected values with respect to the given prior.

2.2. Risk Measures and Representation of Aspirational Preferences

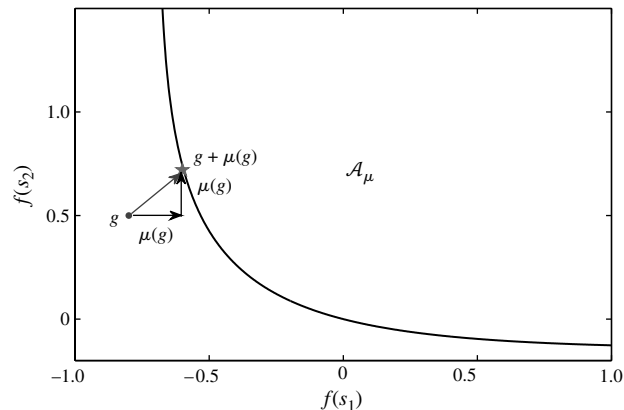
We now show how to represent aspirational preferences in terms of a standard definition of a “risk measure.” Risk measures are motivated from the perspective of minimal capital requirements to make positions acceptable.

DEFINITION 2. A function $\mu: F \rightarrow \mathbb{R}$ is a *risk measure* over F if it satisfies the following for all $f, g \in F$:

1. Monotonicity: If $f \geq g$, then $\mu(f) \leq \mu(g)$.
2. Translation invariance: If $x \in F$ is a constant act, then $\mu(f + x) = \mu(f) - x$.
3. Normalization: $\mu(0) = 0$.

The value $\mu(f)$ may be interpreted as the constant amount to be added to f to make f acceptable by some standard. Namely, $\mu(f + \mu(f)) = \mu(f) - \mu(f) = 0$; i.e., by adding the amount $\mu(f)$ to f , one obtains a new act $g = f + \mu(f)$ with “zero risk” (i.e., $\mu(g) = 0$). In this sense, acts with nonpositive risk can be considered as acceptable under a risk measure μ . Put another way, an act is acceptable to the decision maker if it does not require any additional, guaranteed money. The normalization property is then natural. One simple example of a risk measure is the negative of the expected value under any fixed probability measure. The following formalizes the concept of an acceptable act.

Figure 3 Illustration of a Risk Measure for a Simple Two-State Act g



Notes. Here, $\mathcal{A}_\mu$ denotes the acceptance set of the risk measure μ , and $\mu(g)$ represents the smallest amount of certain money that needs to be added to g to make the augmented position acceptable. Adding this sure amount to g in each state results in a position translated along a “45° line” that touches the acceptance set $\mathcal{A}_\mu$. In this particular example, $g(s_1) = -0.75$, $g(s_2) = 0.5$, and $\mu(g) = 0.15$, so that $g + \mu(g) = (-0.6, 0.65) \in \mathcal{A}_\mu$.

DEFINITION 3. Let $\mu: F \rightarrow \mathbb{R}$ be a risk measure. The subset $\mathcal{A}_\mu$ of F defined by $\mathcal{A}_\mu = \{f \in F: \mu(f) \leq 0\}$ is called the *acceptance set* associated with the risk measure μ and $f \in \mathcal{A}_\mu$ is an *acceptable act*.

The two properties of risk measures have clear implications for the acceptance set: if one act dominates an acceptable act, then it must be acceptable as well. In addition, if we add a constant amount to another act, then the additional money required to make this new act acceptable is reduced accordingly. Figure 3 provides an illustration. We refer the reader to Föllmer and Schied (2004) for more on risk measures and acceptance sets.

The class of convex risk measures has garnered much attention. A risk measure is *convex* if, for any $f, g \in F$, $\lambda \in [0, 1]$, $\mu(\lambda f + (1 - \lambda)g) \leq \max\{\mu(f), \mu(g)\}$ and *concave* if $\mu(\lambda f + (1 - \lambda)g) \geq \min\{\mu(f), \mu(g)\}$.³ Convex risk measures induce diversification favoring, whereas concave risk measures induce concentration favoring.

In addition to risk measures, the notion of a *target* will also be central in the representation result. We can understand the role of a target by thinking of a desired (deterministic) payoff level $\tau \in \mathbb{R}$ that the decision maker wishes to obtain. For a risk measure μ and an act f , the value $\mu(f - \tau)$ represents the risk associated with the *target premium* $f - \tau$. In such a model, the decision maker is concerned with the

³ Convex risk measures are usually defined as satisfying $\mu(\lambda f + (1 - \lambda)g) \leq \lambda\mu(f) + (1 - \lambda)\mu(g)$, which is the usual notion of convexity. However, it is not hard to show that for a risk measure, this property is equivalent to the one stated above (which is typically termed *quasi-convexity*). An analogous statement applies to concave risk measures.

payoff in excess of τ , rather than the absolute payoff. Note that $\tau_1 \geq \tau_2$ implies $f - \tau_1 \leq f - \tau_2$, and thus, by monotonicity of μ , $\mu(f - \tau_2) \leq \mu(f - \tau_1)$. Intuitively, the risk measured against a more ambitious target is never smaller relative to the risk measured against a less ambitious target.

We can express all AMs in terms of risk measures and targets. Specifically, we will use families of risk measures $\{\mu_k\}$ and a family of targets (or target function) $\tau(k)$, parameterized by the index k . We use the convention $\sup \emptyset = -\infty$ in what follows.

THEOREM 1. *A choice function $\rho: F \rightarrow \bar{\mathbb{R}}$ is an aspiration measure (with partition F_{++}, F_0, F_{--}) if and only if there exists a $\hat{k} \in \bar{\mathbb{R}}$ such that for all $g \in F_0$, $\rho(g) = \hat{k}$, and*

$$\rho(f) = \sup\{k \in \mathbb{R}: \mu_k(f - \tau(k)) \leq 0\} \quad \forall f \in F, \quad (1)$$

where $\{\mu_k\}$ is a family of risk measures, convex if $k > \hat{k}$, concave if $k < \hat{k}$, with closed acceptance sets $\mathcal{A}_{\mu_k}$, and $\tau(k)$, the target function, is a constant act for all $k \in \mathbb{R}$ and nondecreasing in k . Moreover, $\mu_k(f - \tau(k))$ is nondecreasing in k for all $f \in F$.

Conversely, given an aspiration measure ρ , the underlying target function and risk measure family are given by

$$\tau(k) = \inf\{a \in \mathbb{R}: \rho(a) \geq k\}, \quad (2)$$

$$\mu_k(f) = \inf\{a \in \mathbb{R}: \rho(f + a) \geq k\} - \tau(k). \quad (3)$$

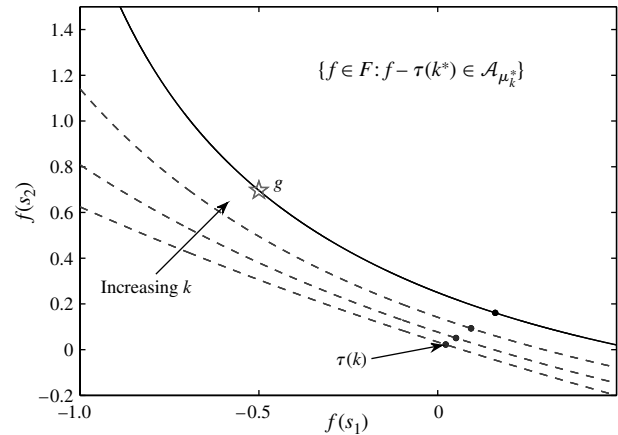
This representation provides an interpretation of choice under aspirational preferences. Such choice can be interpreted as searching over index levels k , such that at k , the risk μ_k associated with the payoff in excess of the target, $f - \tau(k)$, is acceptable. The AM $\rho(f)$ then represents the maximal level k^* at which the risk of f beating the target at that level is acceptable. Figure 4 provides an illustration.

An AM, therefore, assigns higher value to acts whose risk associated with falling short of the target remains acceptable when measured against higher targets. In some cases, μ_k may be a single risk measure that does not vary with k , in which case only the target increases with k (we will see this is true for constant absolute risk aversion (CARA) utility maximizers). On the flip side (which will be the case in our applications of aspirational preferences in §§4 and 5), the target function may be a constant τ ; in this case, only the risk according to μ_k increases in k , whereas the target remains fixed in k .

Although Theorem 1 does not explicitly state the role of probability measures, we can invoke a well-known dual representation for convex risk measures to express aspirational preferences in terms of probability measures. Indeed, it is known (Föllmer and Schied 2004) that any convex risk measure μ on bounded acts can be represented as

$$\mu(f) = \sup_{\mathbb{Q} \in \mathcal{P}} \{-\mathbb{E}_{\mathbb{Q}}[f] - \alpha(\mathbb{Q})\}, \quad (4)$$

Figure 4 Illustration of Theorem 1 in Two States for an Act $g \in F_{++}$



Notes. The representation involves finding the maximum k such that $g - \tau(k)$ is in the acceptance set $\mathcal{A}_{\mu_k}$. As k increases, the set of acts that can meet this condition shrinks, represented with the dashed lines. The level $k^* = \rho(g)$ corresponds to the maximal level where the gamble g satisfies $g - \tau(k^*) \in \mathcal{A}_{\mu_{k^*}}$.

where $\mathcal{P}$ is the set of σ -additive measures on (S, Σ) and α is a convex function satisfying $\inf_{\mathbb{Q} \in \mathcal{P}} \alpha(\mathbb{Q}) = 0$ (this ensures $\mu(0) = 0$). Using this dual representation, for any $f \in F_{++}$, we have

$$\begin{aligned} \rho(f) &= \sup\{k > \hat{k}: \mu_k(f - \tau(k)) \leq 0\} \\ &= \sup\left\{k > \hat{k}: \sup_{\mathbb{Q} \in \mathcal{P}} \{-\mathbb{E}_{\mathbb{Q}}[f - \tau(k)] - \alpha_k(\mathbb{Q})\} \leq 0\right\} \\ &= \sup\left\{k > \hat{k}: \inf_{\mathbb{Q} \in \mathcal{P}} \{\mathbb{E}_{\mathbb{Q}}[f] + \alpha_k(\mathbb{Q})\} \geq \tau(k)\right\}, \end{aligned}$$

where α_k is a family of convex functions, nonincreasing on $k > \hat{k}$. This provides a “robustness” interpretation for evaluating $f \in F_{++}$: acts are evaluated according to the largest index k such that the worst-case expected value of f , adjusted by $\alpha_k(\mathbb{Q})$, over all probability measures, does not fall below the target. A similar representation applies on $f \in F_{--}$, using the fact that $-\mu(-f)$ is a convex risk measure if μ is a concave risk measure.

In summary, the setup does not require given probabilistic beliefs, but our representation result (1) induces such a form for the choice function in terms of probability measures.

It should also be noted that the representation in Theorem 1 holds for the more general class of monotone preferences (i.e., only Property 1, but not Property 2; this would then include models like S-shaped utility), with the important difference that the risk measures need not have any convexity or concavity properties.

We now provide some specific examples. Expectations are taken with respect to σ -additive probability measures $\mathbb{P}$ if not otherwise stated. In the first three examples, diversification is always favored

(i.e., $F_{--} = \emptyset$); the final example discusses a class of AMs that partially favor concentration.

EXAMPLE 1 (SUBJECTIVE EXPECTED UTILITY THEORY (WITH CONCAVE UTILITY)). Let $u: \mathbb{R} \rightarrow \mathbb{R}$ be a continuous, nondecreasing, concave utility function satisfying $u(0) = 0$. The choice function $\rho(f) = \mathbb{E}[u(f)]$ is an AM. Here, the concentration favoring set F_{--} and the neutral set F_0 can be assumed to be empty; i.e., we can take $F_{++} = F$ as diversification is always (weakly) preferred. The representing target function and risk family are

$$\begin{aligned}\tau(k) &= \inf\{a: u(a) \geq k\}, \\ \mu_k(f) &= \inf\{a: \mathbb{E}[u(f+a)] \geq k\} - \tau(k) \\ &= -(\sup\{a: \mathbb{E}[u(f-a)] \geq k\} + \tau(k)).\end{aligned}$$

Here, $-\mu_k(f - \tau(k))$ represents a maximum purchase price one would pay to assume the position f while still attaining a level k in expected utility. When u is invertible, we have $\tau(k) = u^{-1}(k)$; i.e., the target act is the constant act with utility k . Föllmer and Schied (2004) study the class of *shortfall risk measures* induced by convex, increasing loss functions $l: \mathbb{R} \rightarrow \mathbb{R}$:

$$\mu_v^{\text{short}}(f) = \inf\{a: \mathbb{E}[l(-f-a)] \leq v\}.$$

The risk measure μ_k here, with $l(y) = -u(-y)$ and $v = -k$, is the normalized analog of $\mu_v^{\text{short}}(f)$.

For CARA utility, $u(y) = 1 - \exp(-y/R)$ for some $R > 0$, and $\tau(k) = -R \log(1 - k)$ and $\mu_k(f) = R \log \cdot \mathbb{E}[\exp(-f/R)]$ on $k \in (-\infty, 1)$. For this choice function, the risk family μ_k is independent of the index (utility) level k : all variation over the index is embedded within the target function $\tau(k)$.

A related quantity is the decision analytic notion of a maximum purchase price for expected utility maximizers. Specifically, for a decision maker with utility function u and wealth level w , this is defined as

$$\rho(f) = \sup\{b \in \mathbb{R}: \mathbb{E}[u(f-b+w)] \geq u(w)\}.$$

Under the above assumptions on u , this also is an AM, and it is not hard to see that in this case, $\tau(k) = k$, and $\mu_k(f) = -\rho(f)$; i.e., the risk measure family is constant in k . The (negative of the) maximum buying price is therefore a convex risk measure.

EXAMPLE 2 (MAXMIN EUT, CHOQUET UTILITY, AND VARIATIONAL PREFERENCES (WITH CONCAVE UTILITY)). Maxmin expected utility (MEU), developed by Gilboa and Schmeidler (1989), has a similar representation. Here, we have

$$\begin{aligned}\tau(k) &= \inf_{a \in \mathbb{R}}\{a: u(a) \geq k\}, \\ \mu_k(f) &= \inf_{a \in \mathbb{R}}\{a: \inf_{Q \in \mathcal{Q}} \mathbb{E}_Q[u(f+a)] \geq k\} - \tau(k),\end{aligned}$$

where $\mathcal{Q} \subseteq \mathcal{P}$ is a set of σ -additive probability measures on (S, Σ) , and we consider the case when u is nondecreasing and concave. In this case, $F = F_{++}$ again. It is known (Gilboa and Schmeidler 1989) that Choquet expected utility (CEU) (Gilboa 1987, Schmeidler 1989) falls into the class of MEU in the important case when the decision maker is ambiguity averse.

Generalizing the MEU model, Maccheroni et al. (2006) axiomatized a model of *variational preferences*. Here, choice is represented by the function

$$\rho(f) = \inf_{Q \in \mathcal{P}} \{\mathbb{E}_Q[u(f)] + c(Q)\},$$

where u is a differentiable, nondecreasing utility function, and c is a nonnegative convex function on $\mathcal{P}$ with $\inf_{Q \in \mathcal{P}} c(Q) = 0$. In the case of risk aversion, i.e., u is concave, this falls into our setup with $F_{++} = F$.

The generating target function and family of convex risk measures are

$$\begin{aligned}\tau_k(f) &= \inf_{a \in \mathbb{R}}\{a: u(a) \geq k\}, \\ \mu_k(f) &= \inf_{a \in \mathbb{R}}\left\{a: \inf_{Q \in \mathcal{P}} \{\mathbb{E}_Q[u(f+a)] + c(Q)\} \geq k\right\} - \tau(k).\end{aligned}$$

For both MEU and variational preferences, assuming u is invertible, the target function is simply $\tau(k) = u^{-1}(k)$.

EXAMPLE 3 (ACCEPTABILITY, SATISFICING, AND RISKINESS INDICES). Several recent papers have attempted to formalize definitions of risk and related measures of performance. Aumann and Serrano (2008) axiomatize a definition of risk. Their axioms lead to an “index of riskiness” $r(f)$, defined as

$$r(f) = \inf\{a > 0: \mathbb{E}[e^{-f/a}] \leq 1\},$$

and has the interpretation of being the smallest risk tolerance level for a CARA decision maker such that at that level the decision maker would accept the act f . The reciprocal $1/r(f)$ yields the function

$$\rho(f) = \sup\{k > 0: k^{-1} \log \mathbb{E}[e^{-kf}] \leq 0\},$$

where $k^{-1} \log \mathbb{E}[\exp(-kf)]$ is the *entropic risk measure* at level $k > 0$. This is a convex risk measure (e.g., Föllmer and Schied 2004) and therefore is an AM. Foster and Hart (2009) derive an “operational” definition of risk that has a similar representation with logarithmic utility replacing the exponential.

Both of these definitions of risk fall into the class of *satisficing measures* (Brown and Sim 2009), which take the form of a function:

$$\rho(f) = \sup\{k > 0: \mu_k(f - \tau) \leq 0\},$$

where $\{\mu_k\}_{k>0}$ is a family of convex risk measures and τ is a constant act. With μ_k the entropic risk

measure, we obtain the *entropic satisficing measure*. The *acceptability indices* of Cherny and Madan (2009) fall into this framework with $\tau = 0$ when the risk measures μ_k are also positive homogeneous (or “coherent” according to the definition of Artzner et al. 1999).⁴ In all of these cases, diversification is always (weakly) preferred, so $F_{--} = \emptyset$.

EXAMPLE 4 (ENTROPIC STRONG ASPIRATION MEASURE). Note that for any act with $\mathbb{E}[f] < 0$, we have $\mathbb{E}[e^{-kf}] > 1$ for all $k > 0$. This implies that the economic index of riskiness of Aumann and Serrano (2008) is $+\infty$ for all acts with negative expected value. Therefore, one cannot distinguish among any acts with negative expected value using economic index of riskiness; any act with negative expected value is “infinitely risky” regardless of any other considerations according to this definition. Under mild assumptions on the family of risk measures, the same is true with the satisficing measures of Brown and Sim (2009): all acts with expected value lower than the target have $\rho(f) = -\infty$ and therefore satisficing measures do not provide a useful way to choose among acts expected to fall below the target.

As an example of a target-oriented choice function that does not have this difficulty, consider again the family of risk measures

$$\mu_k(f) = \frac{1}{k} \log \mathbb{E}[\exp(-kf)] \quad k \neq 0,$$

with $\lim_{k \downarrow 0} \mu_k(f) = \lim_{k \uparrow 0} \mu_k(f) = \mathbb{E}[-f]$, which is a family of nondecreasing risk measures that are convex for $k > 0$ and concave on $k < 0$. Let τ be a constant act and consider the induced AM

$$\rho(f) = \sup \left\{ k: \frac{1}{k} \log \mathbb{E}[\exp(-k(f - \tau))] \leq 0 \right\}.$$

This is a family of AMs, parameterized by τ , that we call the *entropic strong aspiration measures* (ESAM). In contrast to the above examples, the concentration-favoring set is not empty. In fact, here the diversification favoring set consists of those acts with expected value greater than the target, and the concentration favoring set contains those acts with expected value lower than the target. With ESAM, one can distinguish between acts with expected values below the target.

For example, if f is normally distributed, then we have $\mu_k(f) = -\mathbb{E}[f] + k \sigma^2(f)/2$, where $\sigma^2(f)$ is the variance of f . Therefore, μ_k rewards (i.e., has less “risk”) greater variance on $k < 0$. In this case, we have $\rho(f) = 2\mathbb{E}[f - \tau]/\sigma^2(f)$. Among acts with a given expected value above the target, one prefers smaller

variance (risk aversion). On the other hand, among acts with a given expected value below the target, one prefers larger variance (risk seeking): the intuition is that larger variance gives one better hopes of attaining the target.

Figure 5 provides a simple illustration of ESAM on the set of acts in two states with payoffs bounded between -1 and $+1$ for two levels of the target. In this example, we have a given probability measure with $\mathbb{P}(s_1) = 0.25$ and $\mathbb{P}(s_2) = 0.75$. Note that for ESAM for any act f with $f \geq \tau$ ($f < \tau$), we have $\rho(f) = +\infty$ ($\rho(f) = -\infty$), and such acts are all maximally (minimally) preferred. Thus, acts that are guaranteed to attain at least the target are most preferable, and those that are guaranteed to fall short of the target are least preferable. As τ grows, always attaining at least the target becomes a more stringent requirement, and the set of maximally preferred acts shrinks. Also of note is that the split between diversification favoring and concentration favoring is precisely along the hyperplane of acts f such that $\mathbb{E}_p[f] = \tau$. This turns out to be a general property of all strong aspiration measures, of which ESAM is one example. We now define this subclass of aspiration measures.

3. Strongly Aspirational Preferences

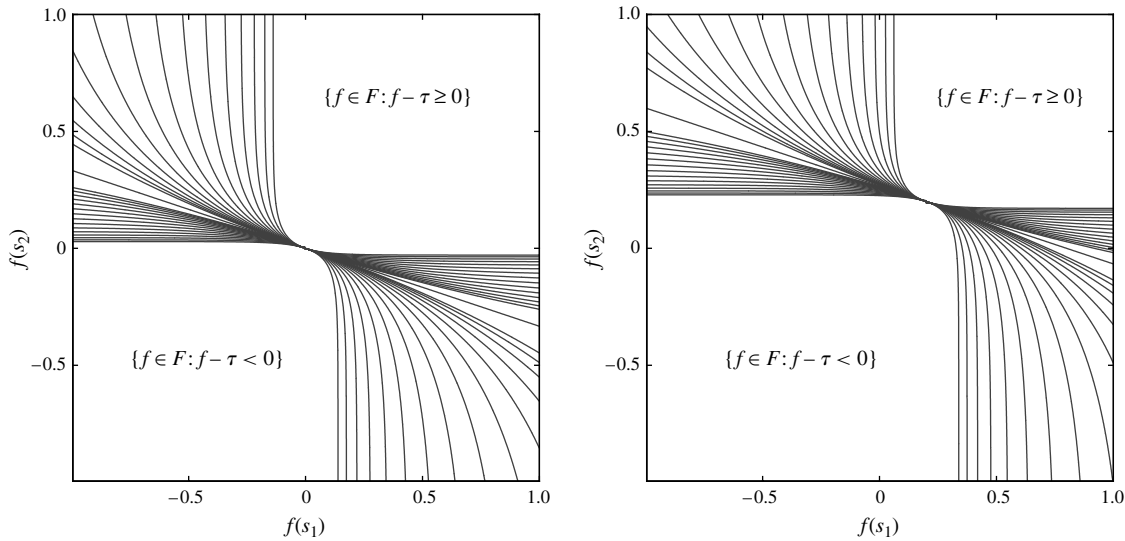
The representation of Theorem 1 provides an interpretation of aspirational preferences in terms of risk of beating a target. Motivated by the empirical evidence on the importance of aspiration levels in choice under uncertainty, we now consider a special case of these preferences when the decision maker is especially fixated on the target. In particular, we consider a bounded target function. The bounds can be interpreted as decision maker aspiration levels: a minimal (reservation) level and a maximal (satiation) level.

To place this in the context of the general framework, note that an aspiration measure $\rho: F \rightarrow \bar{\mathbb{R}}$ naturally defines two target acts.⁵ Let $\tau_u = \inf\{a \in \mathbb{R}: \rho(a) = \infty\}$ and $\tau_l = \inf\{a \in \mathbb{R}: \rho(a) > -\infty\}$, with $\inf \emptyset = \infty$, and consider the case when both are finite. We let τ_u and τ_l denote the constant acts with these values. Because ρ is nondecreasing, $\tau_l \leq \tau_u$. Moreover, for all $f \in F$ with $f \geq \tau_u$, we have $\rho(f) = \infty$, and for all $f \in F$ with $f < \tau_l$, we have $\rho(f) = -\infty$. Consequently, all acts in $\{f \in F: f \geq \tau_u\}$ are “fully satisfactory,” and all acts in $\{f \in F: f < \tau_l\}$ are “fully unsatisfactory.” We define this formally.

⁵ In what follows, we assume without loss of generality that $\sup_{f \in F} \rho(f) = \infty$ and $\inf_{f \in F} \rho(f) = -\infty$. Indeed, if $\sup_{f \in F} \rho(f) = \rho_u$ and $\inf_{f \in F} \rho(f) = \rho_l$, where $\rho_u, \rho_l \in \mathbb{R}$, then one can define a strictly increasing and continuous transformation T such that $T \circ \rho$ satisfy the above conditions. Because T is strictly increasing and continuous, $T \circ \rho$ describes the same preference relation as ρ and also maintains all its properties as given in Theorem 1.

⁴ It is worth mentioning that this definition of a coherent risk measure is distinct from the notion of “coherent probabilities.”

Figure 5 Indifference Curves for ESAM with Two Levels of the Target: $\tau = 0$ (Left) and $\tau = 0.2$ (Right)



Note. Acts in the upper right (lower left) always attain (fall below) the target and thus have $\rho(f) = +\infty$ ($-\infty$).

DEFINITION 4. An aspiration measure $\rho: F \rightarrow \bar{\mathbb{R}}$ is called a *strong aspiration measure* if $\tau_u = \inf\{a \in \mathbb{R}: \rho(a) = \infty\}$ and $\tau_l = \inf\{a \in \mathbb{R}: \rho(a) > -\infty\}$ are finite. We say such a decision maker has *strongly aspirational preferences*.

First, note that by Equation (2) the target function implied by a SAM satisfies $\tau_l = \inf_{k \in \mathbb{R}} \tau(k)$ and $\tau_u = \sup_{k \in \mathbb{R}} \tau(k)$. Hence, when $\tau_l = \tau_u = \tau$ for some constant act τ , the target function is constant and corresponds to the fixed target τ (as is the case in ESAM in Example 4). Strongly aspirational preferences capture the focus of target-driven decision making in the following way: namely, achieving aspiration levels is a central goal of the decision maker, and acts that always attain the satiation level (never attain the reservation level) are most (least) highly valued.

Because the aim of this section is also to characterize F_{++} and F_{--} in case of SAM, we assume that $F_{++} \neq \emptyset$ and $F_{--} \neq \emptyset$. Then we can, without loss of generality, assume $\hat{k} = 0$ in Theorem 1 and make the following identifications:⁶

$$\begin{aligned} F_0 &= \{f \in F: \rho(f) = 0\}, \\ F_{++} &= \{f \in F: \rho(f) > 0\}, \\ F_{--} &= \{f \in F: \rho(f) < 0\}. \end{aligned} \quad (5)$$

In what follows, then, the sign of $\rho(f)$ flags whether f is in the diversification favoring set or the concentration favoring set.

3.1. Examples of Strong Aspiration Measures

We now provide some examples of SAMs in addition to Example 4 described earlier. In Example 5, we focus

on the case of a SAM with $\tau_u = \tau_l = 0$, which will be also considered in our applications of SAM in §4. Example 6 is an example with $\tau_u \neq \tau_l$.

EXAMPLE 5 (CONDITIONAL VALUE-AT-RISK (CVAR) SAM). We assume that (S, Σ) is endowed with a probability measure $\mathbb{P}$ and expectations are taken with respect to $\mathbb{P}$. The family

$$\mu_k(f) = \begin{cases} \text{CVaR}_{e^{-k}}(f) & \text{if } k > 0, \\ -\text{CVaR}_{e^k}(-f) & \text{if } k < 0, \end{cases}$$

where

$$\text{CVaR}_\epsilon(f) = \inf_{\nu \in \mathbb{R}} \left\{ \nu + \frac{1}{\epsilon} \mathbb{E}[(-f - \nu)^+] \right\}$$

is a family of nondecreasing and risk measures that are coherent (convex and positive homogeneous; see Artzner et al. 1999) on $k > 0$. The SAM given by this symmetric family is

$$\rho(f) = \begin{cases} \sup\{k > 0: \text{CVaR}_{e^{-k}}(f) \leq 0\} & \text{if } \mathbb{E}[f] \geq 0, \\ \sup\{k < 0: \text{CVaR}_{e^k}(-f) \geq 0\} & \text{otherwise,} \end{cases}$$

which we call the CVAR SAM. A variant of this measure (without the concentration favoring part and scaled to be on $(0, 1]$) is defined in Brown and Sim (2009). When f is normally distributed under $\mathbb{P}$, we have

$$\begin{aligned} \rho(f) &= \begin{cases} \sup\left\{k > 0: \frac{\phi(\Phi^{-1}(e^{-k}))}{e^{-k}} \sigma(f) \leq \mathbb{E}[f]\right\} & \text{if } \mathbb{E}[f] \geq 0, \\ \sup\left\{k < 0: \frac{\phi(\Phi^{-1}(e^k))}{e^k} \sigma(f) \leq -\mathbb{E}[f]\right\} & \text{otherwise,} \end{cases} \\ &= \begin{cases} \sup\left\{k > 0: \frac{\phi(\Phi^{-1}(e^{-k}))}{e^{-k}} \sigma(f) \leq \mathbb{E}[f]\right\} & \text{if } \mathbb{E}[f] \geq 0, \\ \sup\left\{k < 0: \frac{\phi(\Phi^{-1}(e^k))}{e^k} \sigma(f) \leq -\mathbb{E}[f]\right\} & \text{otherwise,} \end{cases} \end{aligned}$$

⁶ Because $F_{++} \neq \emptyset$ and $F_{--} \neq \emptyset$, then $\hat{k}$ in Theorem 1 is finite and we can shift ρ by the constant $\hat{k}$.

where ϕ and Φ are the standard normal density and cumulative distribution functions, respectively. The SAM in this case is a monotonic transformation of the ratio $\mathbb{E}[f]/\sigma(f)$, similar to the Sharpe ratio.

EXAMPLE 6 (GENERAL SYMMETRIC SAM). A well-known dual representation (e.g., Föllmer and Schied 2002) shows that one can express convex risk measures as

$$\mu(f) = \sup_{\mathbb{Q} \in \mathcal{P}} \{-\mathbb{E}_{\mathbb{Q}}[f] - \alpha(\mathbb{Q})\},$$

where $\mathcal{P}$ is the space of probability measures on (S, Σ) , and α is a convex function with $\inf_{\mathbb{Q} \in \mathcal{P}} \alpha(\mathbb{Q}) = 0$. The entropic risk measure is generated in this way with $\alpha(\mathbb{Q})$ proportional to the relative entropy from $\mathbb{Q}$ to some fixed prior $\mathbb{P}$; CVaR $_{\epsilon}$ is generated with $\alpha(\mathbb{Q})$ as the indicator function on the set of measures $\mathcal{Q} = \{\mathbb{Q}: \mathbb{Q} \leq \mathbb{P}/\epsilon\}$. This dual representation for convex risk measures can be used to generate more general classes of SAMs.

Specifically, let $\{\alpha_k\}_{k \in \mathbb{R}}$ be a family of functions on $\mathcal{P}$ such that α_k is convex for $k > 0$, $\alpha_k(\mathbb{Q})$ is nonincreasing in k , and $\alpha_k(\mathbb{Q}) = -\alpha_{-k}(\mathbb{Q})$ for all $\mathbb{Q} \in \mathcal{P}$. Additionally, $\sup_{k > 0} \inf_{\mathbb{Q} \in \mathcal{P}} \alpha_k(\mathbb{Q}) = 0$ and $\tau_u = -\inf_{k > 0} \inf_{\mathbb{Q} \in \mathcal{P}} \alpha_k(\mathbb{Q}) < \infty$. For $k > 0$, we define the constant acts $\tau(k) = -\inf_{\mathbb{Q} \in \mathcal{P}} \alpha_k(\mathbb{Q}) \leq \tau_u$ and the family of convex risk measures

$$\mu_k(f) = \sup_{\mathbb{Q} \in \mathcal{P}} \{-\mathbb{E}_{\mathbb{Q}}[f] - \alpha_k(\mathbb{Q})\} - \tau(k).$$

For all $k < 0$, we set

$$\begin{aligned} \tau(k) &= -\tau(-k) \geq -\tau_u = \tau_l, \\ \mu_k(f) &= -\mu_{-k}(-f) = -\sup_{\mathbb{Q} \in \mathcal{P}} \{\mathbb{E}_{\mathbb{Q}}[f] + \alpha_k(\mathbb{Q})\} - \tau(k). \end{aligned}$$

For $k < 0$, μ_k is a concave risk measure. Moreover, the target function $\tau(k)$ is nondecreasing in k and $\mu_k(f - \tau(k))$ is nondecreasing in k for all f . Therefore, this target function and the family of risk measures $\{\mu_k\}$ define a SAM.

3.2. Properties and Interpretation of the Partition

In an AP model, the partition is part of the preference structure and is specified by the decision maker. Describing the partition in the general AP case may be a complex task. In the special case of a decision maker with strongly aspirational preferences using a given probability measure, however, the structure of the partition simplifies. We now discuss this.

When the decision maker has a choice function ρ that is consistent according to some probability measure $\mathbb{P}$, which we may view as a “subjective prior,” we say that ρ is *law-invariant with respect to $\mathbb{P}$* . Formally, ρ is law-invariant with respect to $\mathbb{P}$ if the decision maker is indifferent between any f, g with

the same distribution under $\mathbb{P}$. It turns out that law invariance of an aspiration measure connects closely to standard definitions of stochastic dominance (see the appendix for formal results and proofs). Namely, under the assumption of an atomless⁷ distribution $\mathbb{P}$, any aspiration measure preserves first-order stochastic dominance everywhere and second-order stochastic dominance on the diversification-favoring set. Naturally, any law-invariant SAM is a special case of this and inherits these properties.

On atomless probability spaces, law-invariant risk measures also display important boundedness properties relative to the expectation. These properties are helpful in characterizing the partition for SAM. We say that a convex (concave) risk measure μ is bounded from below (above) by the expectation when $\mu(f) \geq \mathbb{E}[-f]$ ($\mu(f) \leq \mathbb{E}[-f]$) for all $f \in F$. Föllmer and Schied (2004) show that law-invariant convex risk measures are bounded from below by the expectation of atomless probability spaces. This implies that concave risk measures are bounded from above by the expectation. Namely, if μ is law-invariant and concave, then $\bar{\mu}(f) = -\mu(-f)$ is law-invariant and convex; thus, $\bar{\mu}(f) \geq \mathbb{E}[-f]$, or equivalently, $\mu(f) \leq \mathbb{E}[-f]$.

If the probability space is not atomless, then it is generally not true that a convex (concave) risk measure is bounded from below (above).⁸ In many cases, however, convex (concave) risk measures are bounded from below (above) even if the probability space is not atomless. This is the case for the convex risk measures of Examples 4 and 5.

PROPOSITION 1. *The underlying families of risk measures in ESAM and CVaR SAM are bounded by the expectation; i.e., $\mu_k(f) \geq \mathbb{E}[-f]$ for $k > 0$ and $\mu_k(f) \leq \mathbb{E}[-f]$ for $k < 0$.*

Boundedness relative to expectation has important implications for the structure of the partition for SAM.

THEOREM 2. *Let $\{\mu_k\}_{k \in \mathbb{R}}$ be a family of risk measures inducing SAM ρ . If, for $k > 0$, μ_k is bounded from below by the expectation and for $k < 0$, μ_k is bounded from above by the expectation, then $\mathbb{E}[f] < \tau_l \Rightarrow \rho(f) \leq 0$ and $\mathbb{E}[f] \geq \tau_u \Rightarrow \rho(f) \geq 0$.*

From the discussion above, we see that Theorem 2 is general in that it applies not only to any law-invariant SAM on an atomless probability space but

⁷ A probability space is said to be atomless when there is no event $A \in \Sigma$ with $\mathbb{P}(A) > 0$ such that $0 < \mathbb{P}(B) < \mathbb{P}(A)$ for some $B \in \Sigma$. Note that the assumption of an atomless probability space is not restrictive; for instance, random variables with discrete outcomes may be generated (as piecewise constant functions) by atomless probability spaces.

⁸ Rockafellar et al. (2006) add boundedness as an additional property on convex risk measures for their class of *deviation measures*.

also to several SAMs on nonatomic spaces (e.g., those in Proposition 1).

The result has a number of noteworthy implications. First, this provides a characterization of the “partition” describing where diversification and concentration are preferred in terms of expected values. Acts that fail to attain τ_l on average cannot be in the diversification favoring set, whereas acts that attain at least τ_u on average cannot be in the concentration favoring set. Conversely, acts that are in the diversification favoring set must satisfy $\mathbb{E}[f] \geq \tau_l$, and acts in the concentration favoring set must satisfy $\mathbb{E}[f] < \tau_u$.

Second, Theorem 2 also has an implication for choice under SAM. In particular, consider $f, g \in F$ with $\mathbb{E}[f] > \tau_u \geq \tau_l > \mathbb{E}[g]$. Note that to compute these expected values, nothing about the structure of the underlying risk measures is required. Theorem 2 implies that any decision maker using SAM (with the underlying risk family satisfying the boundedness properties) will either prefer f to g or be indifferent between the two. Thus, f can be taken as the (weakly) preferred act in all cases. For expected utility maximizers, by contrast, all rankings are possible: the ranking will depend on the specific structure of the decision maker’s utility function, which must therefore first be specified. In such settings, therefore, the decision maker using SAM can simplify his or her decision problem by disregarding acts with expected payoffs lower than τ_l .

Finally, this insight into the partition also provides some prescriptive grounds for favoring concentration in the case of SAM. Indeed, consider a similar model of choice that favors diversification everywhere, i.e., represented by the function $\tilde{\rho}$, with $\tilde{\rho}(f) = \sup\{k: \mu_k(f) \leq 0\}$, where $\{\mu_k\}$ is a family of convex risk measures bounded below by the expectation such that $\tau_u = \tau_l = 0$ (a constant target is not needed but made to simplify the discussion). It is not hard to see that $\mathbb{E}[f] < 0$ implies that any $g \in F$ is weakly preferred to f . Indeed, because $\mu_k(f) \geq -\mathbb{E}[f] > 0$ for all k , and $\tilde{\rho}(f) = -\infty$ by convention, such acts are minimally preferred over F .

To be useful for decision makers with aggressive targets, the presence of concentration-favoring preferences is crucial in the SAM model. On an intuitive level, it does not seem unreasonable for decision makers to “roll the dice” and concentrate resources, rather than diversify them, when aspirations are high relative to available choices.

4. Strongly Aspirational Preferences and Some Paradoxes

In this section, we apply choice under SAM to several paradoxes of decision theory. We first look at some paradoxes from Allais (1953) and then Ellsberg (1961),

then move on to a brief discussion of gain-loss separability, which is an issue in prospect theory of Kahneman and Tversky (1979) and in cumulative prospect theory of Tversky and Kahneman (1992).

It should not be surprising that aspirational preferences can accommodate many patterns of choice: after all, the model is general and one can tailor a wide range of choice functions to match various choice behaviors. What we would like to point out is that in everything that follows, we will be exclusively applying a very special case of the model: namely, SAM with constant target ($\tau(k) = \tau$ for all $k \in \mathbb{R}$ and some $\tau \in \mathbb{R}$). More specifically, we define a family of SAM with constant targets by $\rho_\tau(f) = \rho(f - \tau)$, where τ is a constant act and ρ is a SAM with zero target. This family of SAMs is parameterized by τ and we will focus on whether certain choice patterns can be explained across some range of the target parameter. In some cases, the valid range of targets may be quite wide.

In all of the examples except Ellsberg (1961), we will apply ESAM with a fixed target and the given probability distribution; in the case of Ellsberg, we will use a variant of ESAM that accounts for ambiguity. The range of targets consistent with empirically typical choice patterns in the various cases will be problem specific: the range certainly depends on the probabilities and payoff values, for instance. Nonetheless, we are focusing on a small subset of SAMs, and the ability to match choice patterns with ESAM and some target range, albeit problem specific, is in contrast to SEU, which is unable to address the following paradoxes with any utility function.

4.1. Application to Allais

Consider the following two pairs of gambles:

Gamble A: Wins \$500,000 for sure.

Gamble B: 1% chance of 0, 10% chance of winning \$2,500,000, and 89% chance of winning \$500,000, along with

Gamble C: 90% chance of 0, 10% chance of winning \$2,500,000.

Gamble D: 89% chance of 0, 11% chance of winning \$500,000.

The most typical pattern of preferences observed among actual decision makers is to choose A over B and C over D. It is not hard to see that this is inconsistent with traditional expected utility theory with *any* utility function.

In contrast, these choice pairs can in fact be consistent with choice under SAM over a specific range of the target. For instance, let ρ be any law-invariant SAM with constant target function equal to zero and assume that the corresponding family of risk measures satisfies the boundedness properties of Theorem 2. For $\tau \in \mathbb{R}$, define the SAM ρ_τ by

Table 1 Values Attributed to Gambles A, B, C, and D, Described in the Main Text, by ESAM and CVaR SAM for Different Values of the Target τ

τ	Gamble A	Gamble B	Gamble C	Gamble D
ESAM				
55,000	∞	83.7×10^{-6}	1.90×10^{-6}	0
250,000	∞	18.4×10^{-6}	0	-8.36×10^{-6}
500,000	∞	4.80×10^{-6}	-0.60×10^{-6}	$-\infty$
695,000	$-\infty$	0	-0.92×10^{-6}	$-\infty$
2,000,000	$-\infty$	-4.60×10^{-6}	-4.60×10^{-6}	$-\infty$
CVaR SAM				
55,000	∞	4.48864	0.08311	0.00000
250,000	∞	3.91202	0.00000	-1.51413
500,000	∞	0.10259	-0.69315	$-\infty$
695,000	$-\infty$	0.00000	-1.02245	$-\infty$
2,000,000	$-\infty$	-2.01490	-2.07944	$-\infty$

Note. In bold are the preferred gambles in each pair for each target.

$\rho_\tau(f) = \rho(f - \tau)$. Denoting gamble A by f_A , we have, for $\tau \leq \$500,000$, $\rho_\tau(f_A) = \infty$, so f_A is weakly preferred to f_B under the SAM ρ_τ . On the other hand, the expected value of gamble C is \$250,000 and the expected value of gamble D is \$55,000; therefore, using Theorem 2, for $\tau > \$55,000$, $\tau < \$250,000$, we have $\rho_\tau(f_C) \geq 0 \geq \rho_\tau(f_D)$, so the observed pattern above is (weakly) resolved for all SAM ρ_τ when $\tau \in (\$55,000, \$250,000)$. In fact, for ESAM and CVaR SAM, this pattern of preferences will be observed over an even larger range of targets. This is shown in Table 1.

In all cases, gamble A is strictly preferred to gamble B for $\tau \leq \$500,000$, and gamble C is strictly preferred to gamble D for $\tau \in (\tau^*, \$2,000,000)$, for some $0 < \tau^* < \$55,000$. The intuition in the first pair is that gamble A is guaranteed to hit the \$500,000 target; for the second case, as long as the target is not very small, the extra “upside” of \$2,500,000 versus \$500,000 outweighs the small difference in probabilities of zero payoffs. It seems plausible that this type of intuition is perhaps being used by the decision makers who make such choices.

The example above, first pointed out by Allais (1953), is a special case of a more general pattern across a pair of choices. This is typically referred to as the *common consequence effect*. Formally, consider two positive payoffs $x > y > 0$ and two probabilities $q \in (0, 1)$, $p \in (0, 1)$, with $q > p$. As before, we denote the first pair of gambles A and B. Gamble A is a sure payoff of y ; B, on the other hand, pays off x with probability p , y with probability $1 - q$, and zero with probability $q - p$.

The second pair is a pair of all-or-nothing gambles, which we denote C and D. Gamble C pays off x with probability p and zero otherwise; D pays off y with probability q and zero otherwise. We will assume

gamble C beats gamble D in expectation, i.e., $px > qy$, though we could remove this assumption in what follows.

In observed choices, particularly when x is considerably larger than y and $q - p$ is small, real-world decision makers often prefer the “safer” choice among the first two gambles (i.e., the sure payoff of A over the risky payoff B) and the “riskier” choice among the second two gambles (i.e., C over D). The rationale, presumably, is along the lines that A offers a sure payoff, whereas B can result in a zero payoff; for the second pair, though C has a slightly higher chance of paying off nothing, this extra risk may well be worth bearing if the difference $x - y$ is large. It is well known that this pattern of preferences is inconsistent with SEU under any utility function.

The common consequence effect can be explained by SAM over an explicit range of targets; we show a formal result for ESAM.

PROPOSITION 2. Consider the two pairs of gambles, (A, B) and (C, D) , as described above with $px > qy$, and let ρ be ESAM, μ_k denote the entropic risk measure at level k , and $\rho_\tau(f) = \rho(f - \tau)$ for all τ and f . Then for every (x, y, p, q) as above, there exists a target $\tau^* < qy$ such that for all $\tau \in (\tau^*, y]$, $\rho_\tau(f_A) > \rho_\tau(f_B)$ and $\rho_\tau(f_C) > \rho_\tau(f_D)$. Moreover, we have $\tau^* = -\mu_{\rho^*}(f_D)$, where ρ^* is the unique $\rho > 0$ such that $\mu_\rho(f_C) = \mu_\rho(f_D)$.

Applying Proposition 2 to the Allais example at the start of this section, we find $\tau^* \approx \$22,000$. Thus, ESAM with any target in the range $[\$22,000, \$500,000]$ is consistent with the “typical” choice pattern on that example.

In summary, though ESAM is linked to an expected utility representation with an exponential function (CARA utility), the implications for choice may be quite different than those with SEU (which cannot resolve the Allais paradox). Allais (1953) also discusses a similar preference pattern over a pair of choices, called the *common ratio effect*, that violates SEU. SAM over a range of targets is also consistent with the common ratio effect, and results analogous to Proposition 2 can be derived; these results are similar and we omit them.

4.2. Application to Ellsberg

Ellsberg’s (1961) famous experiments provide interesting insights that decisions made under ambiguity can be inconsistent with expected utility theory. We will show that the SAMs can be extended to handle ambiguity and can resolve the classic Ellsberg paradox across a wide range of targets.

To encompass ambiguity in SAM, we confine the probability measure to a family of distributions, $\mathcal{Q}$. The greater the size of the family $\mathcal{Q}$, the greater the degree of ambiguity. If the family is a singleton, i.e., $\mathcal{Q} = \{\mathbb{P}\}$, then the underlying probability measure

is unambiguously specified. Given a family of convex (concave) risk measures $\mu_{\mathbb{P},k}(f)$ ($\bar{\mu}_{\mathbb{Q},k}(f)$), evaluated under the probability measure $\mathbb{Q}$, we can extend this family of risk measures to encompass ambiguity. For $k > 0$, we consider an ambiguity averse risk measure,

$$\mu_k(f) = \sup_{\mathbb{Q} \in \mathcal{Q}} \mu_{\mathbb{Q},k}(f),$$

which retains the convexity of the risk measure. For $k < 0$, the concave counterpart is given by

$$\bar{\mu}_k(f) = \inf_{\mathbb{Q} \in \mathcal{Q}} \bar{\mu}_{\mathbb{Q},k}(f),$$

which corresponds to an ambiguity favoring risk measure. For example, we can extend versions of the CVaR and entropic risk measures in this way to handle ambiguity (see also Föllmer and Knispel 2011).

We note the following about the structure of the partition with such SAMs.

THEOREM 3. *Given a family of risk measures, $\{\mu_{\mathbb{Q},k}\}_{k \in \mathbb{R}}$ (convex for $k > 0$, concave for $k < 0$) with $\mu_{\mathbb{Q},k}(f) \geq \mathbb{E}_{\mathbb{Q}}[-f]$ if $k > 0$ and $\mu_{\mathbb{Q},k}(f) \leq \mathbb{E}_{\mathbb{Q}}[-f]$ if $k < 0$. Let*

$$\mu_k(f) = \begin{cases} \sup_{\mathbb{Q} \in \mathcal{Q}} \mu_{\mathbb{Q},k}(f) & \text{if } k > 0, \\ \inf_{\mathbb{Q} \in \mathcal{Q}} \mu_{\mathbb{Q},k}(f) & \text{if } k < 0 \end{cases}$$

for some $\mathcal{Q} \subseteq \mathcal{P}$ and consider the SAM induced by the family $\{\mu_k\}$ and the target function $\tau(k)$, with $\tau_l = \inf_{k \in \mathbb{R}} \tau(k)$ and $\tau_u = \sup_{k \in \mathbb{R}} \tau(k)$. Then the following implications hold:

$$\exists \mathbb{Q} \in \mathcal{Q}: \mathbb{E}_{\mathbb{Q}}[f] < \tau_l \Rightarrow \rho(f) \leq 0,$$

$$\exists \mathbb{Q} \in \mathcal{Q}: \mathbb{E}_{\mathbb{Q}}[f] \geq \tau_u \Rightarrow \rho(f) \geq 0.$$

Observe that if there exist $\mathbb{Q}_1, \mathbb{Q}_2 \in \mathcal{Q}$ such that $\mathbb{E}_{\mathbb{Q}_1}[f] < \tau_l$ and $\mathbb{E}_{\mathbb{Q}_2}[f] \geq \tau_u$, then f is in the neutral

set; i.e., $\rho(f) = 0$. Thus, as the set of possible distributions $\mathcal{Q}$ grows, the neutral set F_0 , over which the decision maker is indifferent, will become larger. This aligns with the intuition that heightened ambiguity may reduce a decision maker's ability to make distinctions between choices. Figure 6 provides an illustration, in two states, of ESAM with ambiguity for two sets of distributions.

4.2.1. Ellsberg's Two-Color Experiment. The set-up for Ellsberg's (1961) two-color experiment is as follows. Box 1 contains 50 red balls and 50 blue balls. Box 2 contains red and blue balls in unknown proportions. In the first test, subjects are given the following two choices:

Gamble A: Win \$100 if ball drawn from box 1 is red.

Gamble B: Win \$100 if ball drawn from box 2 is red.

In the second test, subjects have to decide between the two choices:

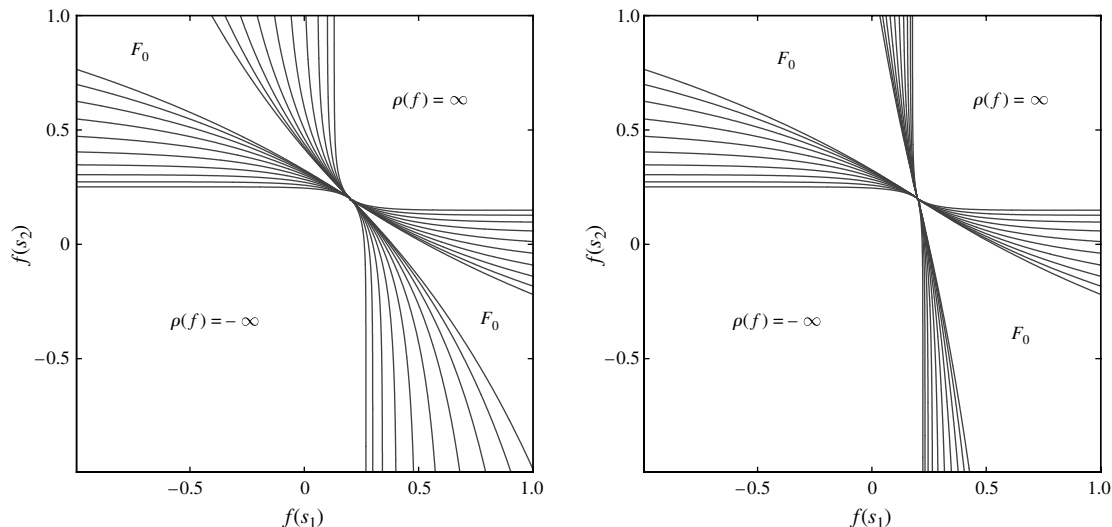
Gamble C: Win \$100 if ball drawn from box 1 is blue.

Gamble D: Win \$100 if ball drawn from box 2 is blue.

In the experimental findings, the majority of subjects is ambiguity averse and strictly prefers gamble A over gamble B and gamble C over gamble D, whereas a smaller portion is actually ambiguity favoring and strictly prefers gamble B over gamble A and gamble D over gamble C. Ellsberg (1961) argues the experimental findings are inconsistent with subjective expected utility theory. The reasoning is as follows: individuals who strictly prefer gamble A over gamble B may perceive that in box 2, red balls are fewer in number than blue ones. In doing so, they should prefer gamble D over gamble C, but this is inconsistent with the experimental findings.

Under Theorem 2, if the corresponding risk measures satisfy the boundedness properties, a SAM on

Figure 6 Indifference Curves for ESAM with Ambiguity for Two Sets of Distributions: $\mathcal{Q} = \{\mathbb{P} \in \mathcal{P}: \mathbb{P}(s_1) \in [.4, .5], \mathbb{P}(s_2) \in [.5, .6]\}$ (Left) and $\mathcal{Q} = \{\mathbb{P} \in \mathcal{P}: \mathbb{P}(s_1) \in [.4, .8], \mathbb{P}(s_2) \in [.2, .6]\}$ (Right)



Notes. In the neutral sets F_0 , we have $\rho(f) = 0$. In both figures, the target is $\tau = 0.2$.

gambles A and C yields nonnegative or nonpositive values when the target is below or above \$50, respectively. Specific SAMs, such as those based on CVaR and entropic risk measures, are strictly positive or negative when the target is below or above \$50, respectively. In contrast, Theorem 3 implies that for any target between \$0 and \$100, these SAMs on gambles B and D, which have unknown distributions, are neutral and thus have a value of zero. Therefore, the preference induced by these SAMs are consistent with the experimental observations.

Clearly, Ellsberg's paradox can also be resolved by several models of aspirational preferences (convex or concave risk measures or by worst-case or best-case expected utility under ambiguity depending on whether the individuals are ambiguity averse or favoring; see, e.g., Föllmer and Schied 2004, Gilboa and Schmeidler 1989). The difference here, however, is that SAMs suggest that the ambiguity preferences depend heavily on the aspiration levels of the subjects. This allows the model to be consistent with some other plausible choice patterns for variations of the Ellsberg experiment. For instance, if the number of red balls in box 1 were known to be much smaller, we expect decision makers would largely prefer gambles B and C. This remains consistent with SAM.

We have applied SAM to other Ellsberg-like examples. In the interests of brevity, we do not present the results here, but these are available upon request. We addressed the three-color version of Ellsberg paradox and found that this can be resolved using ESAM. Machina (2009) recently provided an Ellsberg example that is challenging for Choquet utility and many other choice models in the literature that can explain the standard Ellsberg paradoxes (see Baillon et al. 2010). We have found, however, that ESAM at least nonstrictly satisfies the choice pattern suggested by Machina.

4.3. Gain-Loss Separability

It is known that both prospect theory (Kahneman and Tversky 1979) and cumulative prospect theory (Tversky and Kahneman 1992) require a strong condition of *gain-loss separability*: namely, if both the gain and the loss portion of one gamble are, individually, favored over the respective gain and loss portions of another gamble, then the same direction of preference must hold for the full ("mixed") gambles themselves. Wu and Markle (2008) have shown systematic violations of gain-loss separability in experimental studies. Because choice under SAM allows for both risk aversion and risk seeking, it is interesting to examine implications for gain-loss separability under SAM.

Specifically, Wu and Markle (2008) consider two gambles, High and Low, each with some probability of a positive payoff and some probability of a negative payoff. For gamble High (Low), the payoffs are

G or L with probability p and $1 - p$ (G' or L' with probability p' and $1 - p'$). In all trials, it is assumed that $G > G' > 0 > L > L'$. In monetary act notation, we denote the two gambles by f_{High} and f_{Low} . For act f , the notation f^+ denotes the act $f^+(s) = \max(f(s), 0)$ for all $s \in S$, i.e., the gain part of the act, and the notation f^- denotes the act $f^-(s) = \min(f(s), 0)$ for all $s \in S$, i.e., the loss part of the act.

Wu and Markle (2008) show violations of gain-loss separability by finding experimental violations of *double matching*. Double matching is the requirement $f_{\text{High}}^+ \sim f_{\text{Low}}^+$ and $f_{\text{High}}^- \sim f_{\text{Low}}^- \Rightarrow f_{\text{High}} \sim f_{\text{Low}}$, and is a necessary requirement for gain-loss separability (thus violations of double matching are even stronger than are violations of gain-loss separability). It is not hard to see that any model of choice that is additively separable across states (e.g., expected utility theory) automatically enforces double matching. Indeed, denoting the utility function by u , $f_{\text{High}}^+ \sim f_{\text{Low}}^+$ implies $pu(G) = p'u(G')$, and $f_{\text{High}}^- \sim f_{\text{Low}}^-$ implies $(1 - p)u(L) = (1 - p')u(L')$; these together must also mean that $\mathbb{E}[u(f_{\text{High}})] = \mathbb{E}[u(f_{\text{Low}})]$.

Choice under SAM, on the other hand, need not satisfy double matching. As one example, consider ESAM with a target of zero. Because $f_{\text{High}}^+ \geq 0$ and $f_{\text{Low}}^+ \geq 0$, then $\rho(f_{\text{High}}^+) = \rho(f_{\text{Low}}^+) = \infty$, so $f_{\text{High}}^+ \sim f_{\text{Low}}^+$. In addition, for the entropic risk measure, $f \leq 0$ with $f(s) < 0$ for some state with nonzero probability implies $\bar{\mu}_k(f) > 0$ for all $k < 0$. This in turn implies that $\rho(f_{\text{High}}^-) = \rho(f_{\text{Low}}^-) = -\infty$, so $f_{\text{High}}^- \sim f_{\text{Low}}^-$. On the other hand, the gambles High and Low are different, so in general $\rho(f_{\text{High}}) \neq \rho(f_{\text{Low}})$, and therefore double matching is violated.

Table 2 shows application of ESAM with zero target to a set of experiments from Wu and Markle (2008). For comparison, we also show application of the entropic satisficing measure (Brown and Sim 2009; see Example 3) with zero target to these choices. Consistent with empirical observations, ESAM violates double matching in all 34 of the trials; in contrast, entropic satisficing measure is indifferent between the mixed gambles (and thus obeys double matching) in 10 of the 34 trials.

What is perhaps more interesting is that the ESAM seems to match well the preferences of the subject majority over the mixed gambles: namely, the High gamble is preferred in 29 of the 34 trials, whereas a majority of the subjects preferred the High gamble in 27 of the 34 trials. Moreover, of the five cases in which we found $\rho(f_{\text{Low}}) > \rho(f_{\text{High}})$, three corresponded to cases in which a strong majority preferred Low over High (trials 1, 2, and 3; for trial 6, the subjects were nearly evenly split, and ESAM slightly favored Low over High). All told, simple application of ESAM with zero target matched the majority of subjects in 29 of the 34 cases. In contrast, entropic satisficing measure

Table 2 ESAM and Entropic Satisficing Measure Applied to Gambles High and Low in Experiments from Wu and Markle (2008)

Trial	High gamble			Low gamble			% prefer H	<i>n</i> subjects	ESAM results			Entropic satisficing results		
	<i>G</i>	<i>p</i>	<i>L</i>	<i>G'</i>	<i>p'</i>	<i>L'</i>			$\rho(f_{\text{High}})$	$\rho(f_{\text{Low}})$	$f_{\text{High}} > f_{\text{Low}}?$	$\rho(f_{\text{High}})$	$\rho(L)$	$f_{\text{High}} > f_{\text{Low}}?$
1	150	0.3	−25	75	0.8	−60	22.20	81	0.0065	0.0193	Low	0.0065	0.0193	Low
2	1,800	0.05	−200	600	0.3	−250	21.00	81	−0.0007	−0.0001	Low	−∞	−∞	Indiff
3	1,000	0.25	−500	600	0.5	−700	28.30	60	−0.0006	−0.0003	Low	−∞	−∞	Indiff
4	200	0.3	−25	75	0.8	−100	33.30	72	0.0084	0.0095	Low	0.0084	0.0095	Low
5	1,200	0.25	−500	600	0.5	−800	43.10	72	−0.0003	−0.0005	High	−∞	−∞	Indiff
6	750	0.4	−1,000	500	0.6	−1,500	51.40	72	−0.0008	−0.0008	Low	−∞	−∞	Indiff
7	4,200	0.5	−3,000	3,000	0.75	−6,000	51.90	81	0.0001	0.0001	High	0.0001	0.0001	High
8	4,500	0.5	−1,500	3,000	0.75	−3,000	48.30	60	0.0004	0.0004	High	0.0004	0.0004	High
9	4,500	0.5	−3,000	3,000	0.75	−6,000	58.30	60	0.0001	0.0001	High	0.0001	0.0001	High
10	1,000	0.3	−200	400	0.7	−500	51.30	80	0.0013	0.0013	High	0.0013	0.0013	High
12	3,000	0.01	−490	2,000	0.02	−500	59.30	81	−0.0013	−0.0017	High	−∞	−∞	Indiff
11	4,800	0.5	−1,500	3,000	0.75	−3,000	54.20	72	0.0004	0.0004	High	0.0004	0.0004	High
13	2,200	0.4	−600	850	0.75	−1,700	51.70	60	0.0007	0.0003	High	0.0007	0.0003	High
14	2,000	0.2	−1,000	1,700	0.25	−1,100	57.60	59	−0.0005	−0.0005	High	−∞	−∞	Indiff
15	1,500	0.25	−500	600	0.5	−900	51.30	80	0.0000	−0.0006	High	0.0000	−∞	High
16	5,000	0.5	−3,000	3,000	0.75	−6,000	65.00	80	0.0001	0.0001	High	0.0001	0.0001	High
17	1,500	0.4	−1,000	600	0.8	−3,500	58.80	80	0.0000	−0.0002	High	0.0000	−∞	High
18	2,025	0.5	−875	1,800	0.6	−1,000	71.70	60	0.0006	0.0007	Low	0.0006	0.0007	Low
19	600	0.25	−100	125	0.75	−500	57.50	80	0.0018	−0.0013	High	0.0018	−∞	High
20	5,000	0.1	−900	1,400	0.3	−1,700	40.00	60	−0.0002	−0.0007	High	−∞	−∞	Indiff
21	700	0.25	−100	125	0.75	−600	71.30	80	0.0020	−0.0017	High	0.0020	−∞	High
22	700	0.5	−150	350	0.75	−400	63.30	60	0.0042	0.0024	High	0.0042	0.0024	High
23	1,200	0.3	−200	400	0.7	−800	70.00	80	0.0014	0.0002	High	0.0014	0.0002	High
24	5,000	0.5	−2,500	2,500	0.75	−6,000	78.80	80	0.0002	0.0001	High	0.0002	0.0001	High
25	800	0.4	−1,000	500	0.6	−1,600	57.50	80	−0.0007	−0.0008	High	−∞	−∞	Indiff
26	5,000	0.5	−3,000	2,500	0.75	−6,500	71.30	80	0.0001	0.0000	High	0.0001	0.0000	High
27	700	0.25	−100	100	0.75	−800	72.50	80	0.0020	−0.0029	High	0.0020	−∞	High
28	1,500	0.3	−200	400	0.7	−1,000	75.00	80	0.0016	−0.0002	High	0.0016	−∞	High
29	1,600	0.25	−500	600	0.5	−1,100	72.50	80	0.0000	−0.0008	High	0.0000	−∞	High
30	2,000	0.4	−800	600	0.8	−3,500	65.00	80	0.0004	−0.0002	High	0.0004	−∞	High
31	2,000	0.25	−400	600	0.5	−1,100	80.00	80	0.0004	−0.0008	High	0.0004	−∞	High
32	1,500	0.4	−700	300	0.8	−3,500	77.50	80	0.0003	−0.0007	High	0.0003	−∞	High
33	900	0.4	−1,000	500	0.6	−1,800	70.00	80	−0.0006	−0.0009	High	−∞	−∞	Indiff
34	1,000	0.4	−1,000	500	0.6	−2,000	77.50	80	−0.0004	−0.0009	High	−∞	−∞	Indiff

Notes. Values rounded to four decimal points. In bold are the trial cases when each choice function did not match the preferences of the majority of subjects.

matched only in 22 of the 34 cases; these 7 additional cases of mismatch all corresponded to mixed gambles that both had negative value in expectation, and thus the entropic satisficing measure could not distinguish between them.

Although choice under ESAM seems to match the subject behavior well here, more study is required. Nonetheless, there are two noteworthy observations from these results. First, SAM does not treat decompositions of gambles into losses and gains at all the same way that prospect theory (or SEU) does. Second, the ability to distinguish between gambles with expected value below the target is a useful property of SAM that may be important in empirical applications of the model.

5. Optimization of Aspiration Measures

In this section, we discuss the issue of optimization of the AM choice function. The model is amenable to

large-scale optimization, which is important for use in applications with many decision variables. One such example, which we illustrate here, is portfolio choice. Although this example is by no means intended to be a completely realistic model of portfolio choice, it serves the purpose of illustrating the relevant computational issues at hand.

Specifically, given a AM ρ , we consider the problem

$$z^* = \sup\{\rho(f) : f \in F\}, \quad (6)$$

where F is the convex hull $\{\sum_{i=1}^n w_i f_i : \sum_{i=1}^n w_i = 1, w_i \geq 0, \forall i = 1, \dots, n\}$ of n available assets. Here, the “act” f_i corresponds to the uncertain return for asset i .

From a computational perspective, finding a feasible solution in a convex set is relatively easy compared to finding a feasible solution in a nonconvex one. Observe that for $k > \hat{k}$, the acceptance set

$$\mathcal{A}_{\mu_k} = \{f \in F : \rho(f) \geq k\},$$

which can be empty, is convex. If the diversification favoring set is nonempty, i.e., $F_{++} \neq \emptyset$, we can efficiently obtain the optimal solution to problem (6) using the binary search procedure of Brown and Sim (2009). Otherwise, if this set is empty (which only happens if the target τ is sufficiently large), we have $z^* \leq \hat{k}$. In this case, each of the extreme points f_i must either be in F_0 or F_{--} . If one of them, say f_j , is in F_0 , then if the diversification favoring set is empty, $\rho(f_j) = \hat{k}$ attains the highest AM value over F .

Otherwise, all n available assets are in the concentration favoring set. Then, by quasi-convexity of ρ in this set, there exists an extreme point that is optimal. Hence,

$$z^* = \max_{i=1, \dots, n} \{\rho(f_i)\},$$

and we can simply enumerate the AM values for the n assets and choose the largest one in this case. Here, we can exploit the simple structure of this constraint set; searching over all the extreme points of a more general set may be more computationally taxing.

We now demonstrate this concretely on an asset allocation problem in which the underlying marginal distributions of asset returns are not known exactly, whereas asset returns are assumed to be independent. Here, independence is assumed for sake of simplicity, but we can extend the results to also incorporate dependence. We thus consider n assets with independent returns f_i , $i = 1, \dots, n$. The exact marginal distribution of f_i is unknown but can be characterized by its support $[\underline{f}_i, \bar{f}_i]$; i.e., the probability that f_i belongs to $[\underline{f}_i, \bar{f}_i]$ is one. Also, the mean of f_i is unknown and lies in $[\underline{\mu}_i, \bar{\mu}_i] \subseteq [\underline{f}_i, \bar{f}_i]$. We then consider the problem

$$\sup \left\{ \rho \left(\sum_{i=1}^n w_i f_i - \tau \right) : \sum_{i=1}^n w_i = 1, w_i \geq 0, i = 1, \dots, n \right\},$$

where τ is a given target return. The decision variables are the n weights w_i for each available asset.

We consider the case of ρ as ESAM. Because the returns are independently distributed, the underlying risk measure is given by

$$\begin{aligned} \mu_k \left(\sum_{i=1}^n w_i f_i - \tau \right) &= \frac{1}{k} \log \sup_{\mathbb{Q} \in \mathcal{Q}} \mathbb{E}_{\mathbb{P}} \left[\exp \left(-k \left(\sum_{i=1}^n w_i f_i - \tau \right) \right) \right] \\ &= \sum_{i=1}^n \frac{1}{k} \log \sup_{\mathbb{Q} \in \mathcal{Q}} \mathbb{E}_{\mathbb{P}} [\exp(-k w_i f_i)] + \tau \end{aligned}$$

for $k > 0$ and $k < 0$, and $\mathcal{Q}$ is the set of probability measures such that for each asset i , f_i possesses a feasible distribution (with the given support $[\underline{f}_i, \bar{f}_i]$ and mean in the corresponding interval $[\underline{\mu}_i, \bar{\mu}_i]$), and returns are independent.

PROPOSITION 3. Let f be an act and $\mathcal{Q}$ be the set of all probability measures such that f has distribution with support $[\underline{f}, \bar{f}]$ and its mean lies in $[\underline{\mu}, \bar{\mu}] \subseteq [\underline{f}, \bar{f}]$. Then

$$\sup_{\mathbb{Q} \in \mathcal{Q}} \mathbb{E}_{\mathbb{Q}} [\exp(-af)] = \begin{cases} p \exp(a \underline{f}) + q \exp(a \bar{f}) & \text{if } a \geq 0, \\ \bar{p} \exp(a \underline{f}) + \bar{q} \exp(a \bar{f}) & \text{otherwise,} \end{cases}$$

where $p = (\bar{f} - \underline{\mu})/(\bar{f} - \underline{f})$, $q = 1 - p$, $\bar{p} = (\bar{f} - \bar{\mu})/(\bar{f} - \underline{f})$ and $\bar{q} = 1 - \bar{p}$.

Given a target τ , Proposition 3 enables us to compute the ESAM with ambiguity for this problem. Here, there is ambiguity in the return distribution. Observe that

$$\rho \left(\sum_{i=1}^n w_i f_i - \tau \right) = \rho \left(\sum_{i=1}^n w_i (f_i - \tau) \right).$$

Hence, if $\rho(f_i - \tau) < 0$ for all i , then all assets are in the concentration favoring set, and it is optimal to invest in the asset with the highest value of $\rho(f_i - \tau)$. Otherwise, we solve the following optimization problem

$$\begin{aligned} \sup \left\{ k: \sum_{i=1}^n \frac{1}{k} \log (p_i \exp(-w_i k \underline{f}_i) \right. \\ \left. + q_i \exp(-w_i k \bar{f}_i)) + \tau \leq 0, \right. \\ \left. \sum_{i=1}^n w_i = 1, k > 0, w_i \geq 0, i = 1, \dots, n \right\}, \end{aligned}$$

where $p_i = (\bar{f}_i - \underline{\mu}_i)/(\bar{f}_i - \underline{f}_i)$, $q_i = 1 - p_i$. The decision variables are the ESAM level k and weights w_i . By replacing k with its reciprocal, we can obtain the optimal portfolio allocation by solving the single convex optimization problem

$$\begin{aligned} \inf \left\{ a: \sum_{i=1}^n a \log (p_i \exp(-w_i \underline{f}_i/a) \right. \\ \left. + q_i \exp(-w_i \bar{f}_i/a)) + \tau \leq 0, \right. \\ \left. \sum_{i=1}^n w_i = 1, a > 0, w_i \geq 0, i = 1, \dots, n \right\}, \end{aligned}$$

which can be solved efficiently, in high dimension, using interior point methods.⁹

We now present a numerical example based on the information presented in Table 3 of the online appendix. Note that the asset returns are defined such that asset 1 is a risk-free asset that pays 2% in all states. Assets 2 to 6 are risky assets, with asset 2 being the one with the smallest downside but least upside and asset 6 being the one with the largest downside

⁹ For this example, it is convenient to use a solver that can explicitly handle the “exponential cone”; here we use the software package ROME (Goh and Sim 2011) to solve our example problem.

but most upside. We solve the optimal asset allocation for various targets as shown in Table 3 of the online appendix. The lowest target corresponds to the risk-free rate. Here, the investor can reach the target for sure by investing in asset 1. As the target increases, the risk-free asset becomes less attractive because it fails to attain the target with certainty. The investor puts some wealth into the risky assets. If the target becomes very high, i.e., the investor is ambitious, then he or she only holds asset 6, the asset with the highest upside potential and a positive probability of beating the target.

This example demonstrates the intuitive idea that if the investor possesses a high target return, then he or she will be willing to take more risk. This pattern is similar to that observed in mutual fund managers during the technology bubble of the 1990s (Dass et al. 2008). Managers with high contractual incentives to rank at the top (i.e., those with a high target) adopted the aggressive and risky strategy to not invest in bubble stocks, because avoiding the herd was the only way to highly outperform the market.

We are not aware of other models that can accommodate differing attitudes toward both risk and ambiguity in a computationally tractable way. Maximizing the probability of beating a target is a highly difficult optimization problem in general. Nemirovski and Shapiro (2006), for instance, show that even computing the distribution of a sum of uniform random variables is NP-hard. Prospect theory may also be quite difficult to use—Chen et al. (2011) show that optimization of the expected value of an S-shaped value function over box constraints is NP-hard. The α -maxmin model (Ghirardato et al. 2004) allows for ambiguity seeking and aversion but results in a choice function that is neither convex nor concave.

Although all these models have important implications both theoretically and descriptively, they may be difficult to use in optimization settings, and computing globally optimal solutions in general can only be done with enumeration across grid-based approximations. On a grid with 1% resolution, this approach on this six-asset example would require computation and comparison of 10^{12} values. By contrast, on a standard desktop machine, optimization of ESAM here takes about one second.

6. Discussion

We have considered the problem of choice over monetary acts and examined the case of a general preference structure over such acts. In addition to monotonicity, the preferences favor diversification, except perhaps on a set of unfavorable acts for which concentration is preferred. We have shown a representation of these aspirational preferences. This states

that we can always express the choice function in terms of a maximum index level at which a measure of risk of beating a target function is acceptable.

This result provides an interpretation of a number of models in this context, including expected utility theory and several generalizations, and perhaps opens the door to new models of choice. One that we then considered here is the special case when the target function is bounded. These strongly aspirational preferences are partly motivated from the perspective of bounded rationality and, though more study is required, seem to have some descriptive potential. This corroborates a body of work that suggests that aspiration levels play a key role in individual decision making.

An application like portfolio choice, where performance is often adjusted relative to a benchmark, may be a natural fit for the SAM model. In addition to considering new classes of choice models in this framework and investigating in more depth the empirical implications of strongly aspirational preferences, exploring use of the model in applications is of interest.

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Appendix

PROOF OF THEOREM 1. Suppose ρ takes the form (1), where $\{\mu_k\}$ is the family of risk measures and $k \rightarrow \tau(k)$ the target function as described. Let $F_{++} = \{f \in F: \rho(f) > \hat{k}\}$, $F_{--} = \{f \in F: \rho(f) < \hat{k}\}$ and $F_0 = \{f \in F: \rho(f) = \hat{k}\}$. We show that ρ is an aspiration measure with partition F_{++}, F_{--}, F_0 .

1. *Upper semicontinuity of ρ :* Upper semicontinuity for ρ is equivalent to $\{f \in F: \rho(f) \geq k\}$ being closed for all k . Let $k \in \mathbb{R}$ and take a sequence $(f_n)_n$ in $\{f \in F: \rho(f) \geq k\}$ such that $f_n \rightarrow f$ as $n \rightarrow \infty$. Because $\rho(f_n) \geq k$, then $\mu_k(f_n - \tau(k)) \leq 0$. Therefore, the sequence $(f_n - \tau(k))_n$ belongs to the acceptance set $\mathcal{A}_{\mu_k}$. Because $\mathcal{A}_{\mu_k}$ is closed, then $f - \tau(k) \in \mathcal{A}_{\mu_k}$, i.e., $\mu_k(f - \tau(k)) \leq 0$. This implies

$\rho(f) \geq k$; i.e., $f \in \{f \in F: \rho(f) \geq k\}$. This proves that $\{f \in F: \rho(f) \geq k\}$ is closed for all k , and thus ρ is upper semicontinuous.

2. ρ nondecreasing: Follows clearly from monotonicity of the underlying risk measures.

3. *Mixing*: First, by definition of F_{++} , F_{--} , and F_0 given above, acts in F_{++} are strictly preferred to acts in F_0 , which are strictly preferred to acts in F_{--} .

a. *Quasi-concavity over diversification favoring acts*: Let $f, g \in F_{++}$ and $k^* = \min\{\rho(f), \rho(g)\} > \hat{k}$. Note that $\mu_k(f - \tau(k)) \leq 0$ and $\mu_k(g - \tau(k)) \leq 0$ for all $k \in (\hat{k}, k^*)$. Then, using convexity of μ_k on $k > \hat{k}$, we have

$$\begin{aligned} & \rho(\lambda f + (1 - \lambda)g) \\ &= \sup\{k \in \mathbb{R}: \mu_k(\lambda f + (1 - \lambda)g - \tau(k)) \leq 0\} \\ &\geq \sup\{k \in (\hat{k}, \infty): \mu_k(\lambda f + (1 - \lambda)g - \tau(k)) \leq 0\} \\ &\geq \sup\{k \in (\hat{k}, \infty): \lambda \mu_k(f - \tau(k)) + (1 - \lambda) \mu_k(g - \tau(k)) \leq 0\} \\ &\geq k^* \\ &= \min\{\rho(X), \rho(Y)\}. \end{aligned}$$

b. *Quasi-convexity over concentration favoring acts*: Let $f, g \in F_{--}$, and $k^* = \max\{\rho(f), \rho(g)\} < \hat{k}$. Note that $\mu_k(f - \tau(k)) > 0$ and $\mu_k(g - \tau(k)) > 0$ for all $k > k^*$. Hence, for all $k \in (k^*, \hat{k})$,

$$\begin{aligned} & \mu_k(\lambda f + (1 - \lambda)g - \tau(k)) \\ &\geq \lambda \mu_k(f - \tau(k)) + (1 - \lambda) \mu_k(g - \tau(k)) > 0. \end{aligned}$$

Because $\mu_k(h - \tau(k))$ is nondecreasing in k for all $h \in F$, the above inequality also holds for $k > k^*$. Therefore, we have

$$\begin{aligned} & \rho(\lambda f + (1 - \lambda)g) \\ &= \sup\{k \in \mathbb{R}: \mu_k(\lambda f + (1 - \lambda)g - \tau(k)) \leq 0\} \\ &= \sup\{k \in (-\infty, k^*]: \mu_k(\lambda f + (1 - \lambda)g - \tau(k)) \leq 0\} \\ &\leq k^* \\ &= \max\{\rho(f), \rho(g)\}. \end{aligned}$$

We now show that an aspiration measure takes the form (1) where the family of risk measures $\{\mu_k\}$ is defined in Equation (3) and the target function is defined in Equation (2). Because ρ is nondecreasing, Equations (3) and (2) imply that $\mu_k(f) + \tau(k) = \mu_k(f - \tau(k))$ is nondecreasing in k for all $f \in F$. Moreover, because ρ is nondecreasing, Equation (2) implies that $k \rightarrow \tau(k)$ is nondecreasing. To verify that μ_k is a risk measure with a closed acceptance set, we note the following:

1. *Closed acceptance set*: We show that $\mu_k(f - \tau(k)) \leq 0$ is equivalent to $\rho(f) \geq k$. One direction is trivial; i.e., when $\rho(f) \geq k$, then $\mu_k(f - \tau(k)) \leq 0$. For the other direction, we note that upper semicontinuity for ρ implies upper semicontinuity for $a \rightarrow \rho(f + a)$, for all $f \in F$. Moreover, because $a \rightarrow \rho(a + f)$ is also increasing because ρ is increasing, then it is also right-continuous and the limit of problem (3) is achievable. It follows that when $\mu_k(f - \tau(k)) \leq 0$, then there exists an $a \leq 0$ such that $\rho(a + f) \geq k$. Because ρ is increasing, we also have $\rho(f) \geq k$. We have thus showed

$$\{f \in F: \mu_k(f - \tau(k)) \leq 0\} = \{f \in F: \rho(f) \geq k\}.$$

Because ρ is upper semicontinuous, $\{f \in F: \rho(f) \geq k\}$ is closed and thus so is $\{f \in F: \mu_k(f - \tau(k)) \leq 0\}$. Because $\mathcal{A}_{\mu_k} = \{g \in F: g + \tau(k) \in \{f \in F: \mu_k(f - \tau(k)) \leq 0\}\}$, then also $\mathcal{A}_{\mu_k}$ is closed.

2. *Monotonicity of μ_k* : Clear.

3. *Translation invariance of μ_k* : For all constant acts $x \in F$,

$$\begin{aligned} \mu_k(f + x) &= \inf\{a: \rho(f + x + a) \geq k\} - \tau(k) \\ &= \inf\{a - x: \rho(f + a) \geq k\} - \tau(k) \\ &= \inf\{a: \rho(f + a) \geq k\} - x - \tau(k) \\ &= \mu_k(f) - x. \end{aligned}$$

4. *Normalization of μ_k* : Clear.

5. *Convexity of μ_k on $k > \hat{k}$* : Given $f, g \in F$, because ρ is nondecreasing and the definition of μ_k , we have for all $\epsilon > 0$,

$$\rho(f + \mu_k(f) + \tau(k) + \epsilon) \geq k$$

and

$$\rho(g + \mu_k(g) + \tau(k) + \epsilon) \geq k.$$

Because $k > \hat{k}$, we have $f + \mu_k(f) + \tau(k) + \epsilon, g + \mu_k(g) + \tau(k) + \epsilon \in F_{++}$. For every $\lambda \in [0, 1]$, define

$$a_\lambda = \lambda \mu_k(f) + (1 - \lambda) \mu_k(g) + \tau(k).$$

Then, for all $\epsilon > 0$,

$$\begin{aligned} & \rho(\lambda f + (1 - \lambda)g + a_\lambda + \epsilon) \\ &= \rho(\lambda(f + \mu_k(f) + \tau(k) + \epsilon) + (1 - \lambda)(g + \mu_k(g) + \tau(k) + \epsilon)) \\ &\geq \min\{\rho(f + \mu_k(f) + \tau(k) + \epsilon), \rho(g + \mu_k(g) + \tau(k) + \epsilon)\} \\ &\geq k > \hat{k}. \end{aligned}$$

Then

$$\begin{aligned} \mu_k(\lambda f + (1 - \lambda)g) &= \inf\{a: \rho(\lambda f + (1 - \lambda)g + a) \geq k\} - \tau(k) \\ &\leq a_\lambda - \tau(k) \\ &= \lambda \mu_k(f) + (1 - \lambda) \mu_k(g). \end{aligned}$$

6. *Concavity on $k < \hat{k}$* : Because $\mu_k(f) = \inf\{a: \rho(f + a) \geq k\} - \tau(k)$, it follows that $\rho(f + \mu_k(f) + \tau(k) + a) < k < \hat{k}$ and $\rho(g + \mu_k(g) + \tau(k) + a) < k < \hat{k}$ for all $a < 0$. Therefore, for all $a < 0$, $f + \mu_k(f) + \tau(k) + a \in F_{--}$, and $g + \mu_k(g) + \tau(k) + a \in F_{--}$; hence,

$$\begin{aligned} & \rho(\lambda(f + \mu_k(f) + \tau(k)) + (1 - \lambda)(g + \mu_k(g) + \tau(k)) + a) \\ &\leq \max\{\rho(f + \mu_k(f) + \tau(k) + a), \rho(g + \mu_k(g) + \tau(k) + a)\} < k \end{aligned}$$

for all $\lambda \in [0, 1]$. Therefore,

$$\begin{aligned} & \mu_k(\lambda(f + \mu_k(f) + \tau(k)) + (1 - \lambda)(g + \mu_k(g) + \tau(k))) \\ &= \inf\{a: \rho(\lambda(f + \mu_k(f) + \tau(k)) \\ &\quad + (1 - \lambda)(g + \mu_k(g) + \tau(k)) + a) \geq k\} - \tau(k) \\ &= \inf\{a: \rho(\lambda(f + \mu_k(f) + \tau(k)) \\ &\quad + (1 - \lambda)(g + \mu_k(g) + \tau(k)) + a) \geq k, a \geq 0\} - \tau(k) \\ &\geq -\tau(k). \end{aligned}$$

Concavity then follows from the translation invariance property of μ_k .

Finally, we need to show that

$$\rho(f) = \sup\{k \in \mathbb{R} : \mu_k(f - \tau(k)) \leq 0\}.$$

We have seen in item 1 above that the limit of problem (3) is achievable. Therefore,

$$\begin{aligned} & \sup\{k \in \mathbb{R} : \mu_k(f - \tau(k)) \leq 0\} \\ &= \sup\{k \in \mathbb{R} : \mu_k(f) \leq -\tau(k)\} \\ &= \sup\{k \in \mathbb{R} : \exists a \leq 0 \text{ s.t. } \rho(f + a) \geq k\} \\ &= \sup\{\rho(f + a) : a \leq 0\} \\ &= \rho(f), \end{aligned}$$

which completes the proof. $\square$

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