Multiple Risk-Sharing: An Analysis of the Policyholder’s Preference

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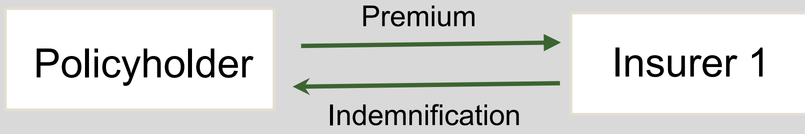
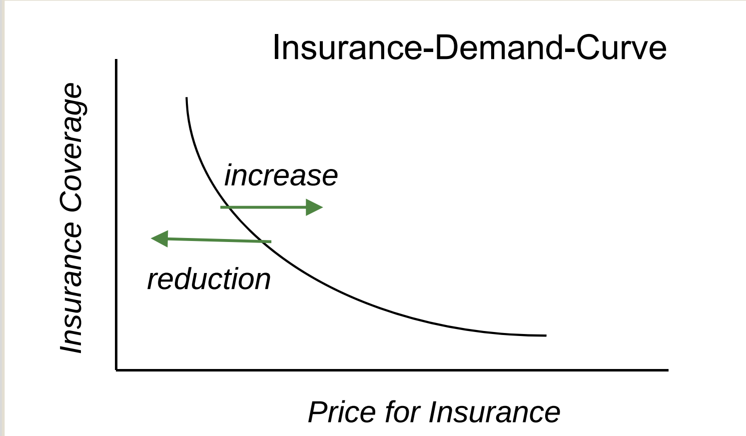
Motivation: Insurance Demand under Default Risk

Bilateral insurance contract under default risk



Indemnification is subject to a default risk of the insurer



What is the optimal coverage level α^* if the insurer's ruin probability is non-zero?

Doherty & Schlesinger (1990):

- Insurance demand is affected by default risk; classic Mossin-theorem does not hold anymore
- Given actuarial fair premium, no unambiguous results whether over- or under-insurance is optimal

Mahul & Wright (2007):

- Given actuarial fair premium, over-insurance is optimal iff the insurer's recovery rate is above a threshold

Motivation: Insurance Demand under Default Risk

Multiple risk-sharing in, e.g., industrial insurance, reinsurance,…

Policyholder

Premium

Indemnification

Insurer 1

Insurer 2

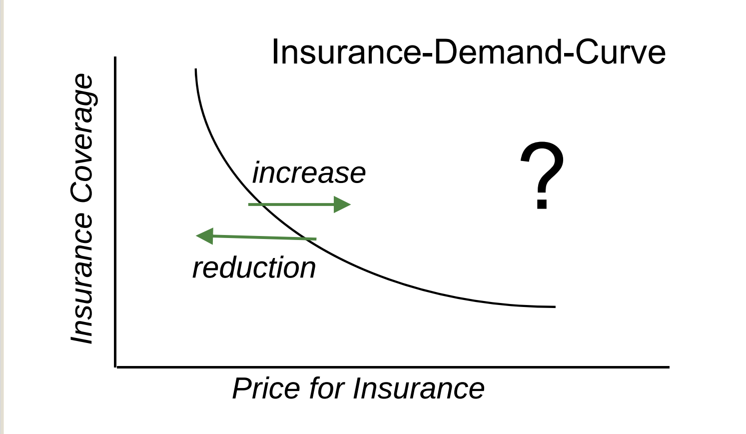
Insurer 3

Insurer 4

Pro rata sharing of risk and premium

Indemnification is subject to a default risk of the insurers

Question 1: How is the insurance demand affected if default risk can be diversified by multiple risk-sharing?



Motivation: Insurance Demand under Default Risk

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Premium →

← Indemnification

Insurer 1 | Insurer 2

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Pro rata sharing of risk and premium

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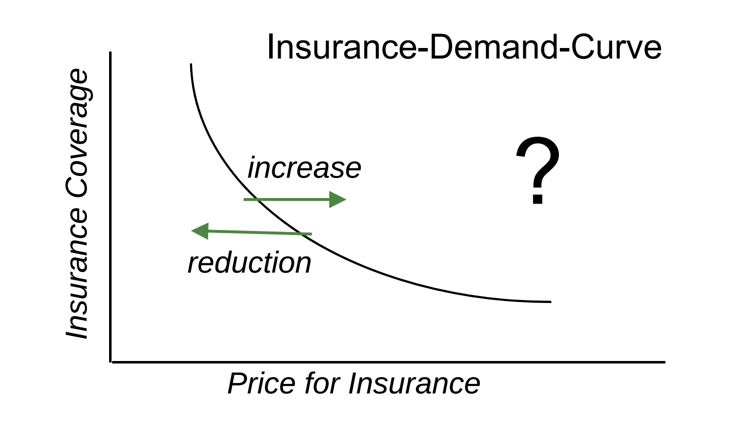
Question 2: How is the insurance demand affected if the fee for a risk-management measure increases?



Default-Free Risk-Management Measure (RMM)

Provides replacing payment if one/several (re)insurers fail

Examples: Letter of Credit, Guarantees, Credit Default Swap, Guarantee Fund, Deposits

← Fee

Indemnification →

Policyholder

Premium →

← Indemnification

Basic Insurance Coverage (multiple with default risk)

Insurer 1 | Insurer 2

Insurer 3 | Insurer 4

Pro rata sharing of risk and premium

Model I: Insurance Coverage and External RMM

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Fee ←

→ Indemnification

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Basic Insurance Coverage (multiple with default risk)

Insurer 1 Insurer 2

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Pro rata sharing of risk and premium

Assuming that the number of insurers n is exogenously given, the policyholder decides on two variables:

- The quantity of insurance coverage denoted by e_I (at the premium of $c_I(e_I)$)
- The quantity of the risk-management measure denoted by e_R (at the fee of $c_R(e_R)$)

The policyholder can take three positions at the total costs of $c_I(e_I) + c_R(e_R)$:

$e_I = e_R$ means a perfect hedging → the policyholder eliminates the default risk entirely

$e_I > e_R$ means an under-hedging → policyholder accepts a remaining default risk

$e_I < e_R$ means an over-hedging → policyholder bets on the default of (re)insurers

Model II: Wealth States and Utility

(1) Binary loss event: 0 with probability $1-p, l>0$ with probability p

(2) Multiple risk-sharing: insurer $i=1,\ldots,n$ gets $\frac{1}{n}c_I(e_I)$ of the premium and pays $\frac{1}{n}e_I$ in a loss event

(3) The insurers' defaults are assumed to be stochastically independent

(4) Each insurer fails to compensate its share with probability q (ruin probability = qp, i.i.d-assumption)

(5) The insurer's default-to-liability ratio equals $0\leq\tau\leq 1$, i.e., $(1-\tau)\frac{1}{n}e_I$ are still paid in a default event

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State	Probability	Final Wealth
Loss event and n defaults	pq^n	$W_{L,n} \coloneqq w - c_I(e_I) - c_R(e_R) - l + e_I - \tau(e_I - e_R)$
⋮	⋮	⋮
Loss event and k defaults	$p\binom{n}{k}q^k(1-q)^{n-k}$	$W_{L,k} \coloneqq w - c_I(e_I) - c_R(e_R) - l + e_I - \frac{\tau k}{n}(e_I - e_R)$
⋮	⋮	⋮
Loss event and 0 defaults	$p(1-q)^n$	$W_{L,0} \coloneqq w - c_I(e_I) - c_R(e_R) - l + e_I$
No loss	$1-p$	$W_{NL} \coloneqq w - c_I(e_I) - c_R(e_R)$

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The policyholder possesses a vNM utility function u with $u'>0$ and $u''<0$. The policyholder's objective:

$$\max_{e_I,e_R} U_n = \max_{e_I,e_R}\left\{(1-p)u(W_{NL}) + p\sum_{k=0}^{n}\binom{n}{k}q^k(1-q)^{n-k}u\left(W_{L,k}\right)\right\}$$

Model III: Premium/Fee Principle

Expected value calculus:

Payoff from insurance contract: $E[I_n] = e_I p(1-\tau q)$,
Payoff from risk-management measure: $E[R_n] = e_R pq\tau$.

Expected payoffs do not depend on n

But: $Var[I_n] \searrow e_I^2 p(1-p)$ as $n \longrightarrow \infty$ (=variance of a default-free insurance policy)

Assumed premium/fee principle: *Actuarial Fair Premium/Fee* x *Proportional Cost Loading*

Premium for insurance coverage: $c_I(e_I) = E[I_n](1+\lambda_I) = (e_I p - e_I pq\tau\ )(1+\lambda_I) =: e_I \pi_I (1+\lambda_I)$,
Fee for risk-management measure: $c_R(e_R) = E[R_n](1+\lambda_R) = e_R pq\tau(1+\lambda_R) =: e_R \pi_R (1+\lambda_R)$.

- By assumption, λ_I is non-dependent on n
- Assumption might be unmet (especially for large n) due to economies of scale (Mayers & Smith, 1981)
- Premium, fee and expected wealth do not depend on n

→ Multiple risk-sharing has a mean-preserving effect

→ Under this premium principle SOC for the maximization problem $\max\limits_{e_I, e_R} U_n$ is fulfilled

Results I: Demand Effect of Multiple Risk-Sharing

At first, $e_R = 0$ (no risk-management measure at hand)

Policyholder's utility

$$U_n = (1-p)u(W_{NL}) + p\sum_{k=0}^{n}\binom{n}{k} q^k (1-q)^{n-k} u\left(W_{L,k}\right)$$

can be interpreted as Bernstein polynomial. Thereby, one can conclude:

i. $U_{n+1} > U_n$ for all $n \geq 1$, i.e., $U_{n+1}\left(e_{I,n+1}^*\right) > U_n\left(e_{I,n}^*\right)$,

ii. $\lim\limits_{n\to\infty} U_n = (1-p)u(W_{NL}) + pu(w - c_I(e_I) - c_R(e_R) - l + e_I(1-q\tau))$,

iii. $\lim\limits_{n\to\infty} e_{I,n}^* = \frac{e_{I,ND}^*}{(1-q\tau)}$.

- Example for actuarial fair premiums ($\lambda_I = 0$): $\lim\limits_{n\to\infty} e_{I,n}^* = \frac{l}{(1-q\tau)}$; i.e., over-insurance is optimal
- Utility is monotonously increasing in n; is the sequence $e_{I,1}^*, e_{I,2}^*, e_{I,3}^*$, ... monotonously increasing, too?

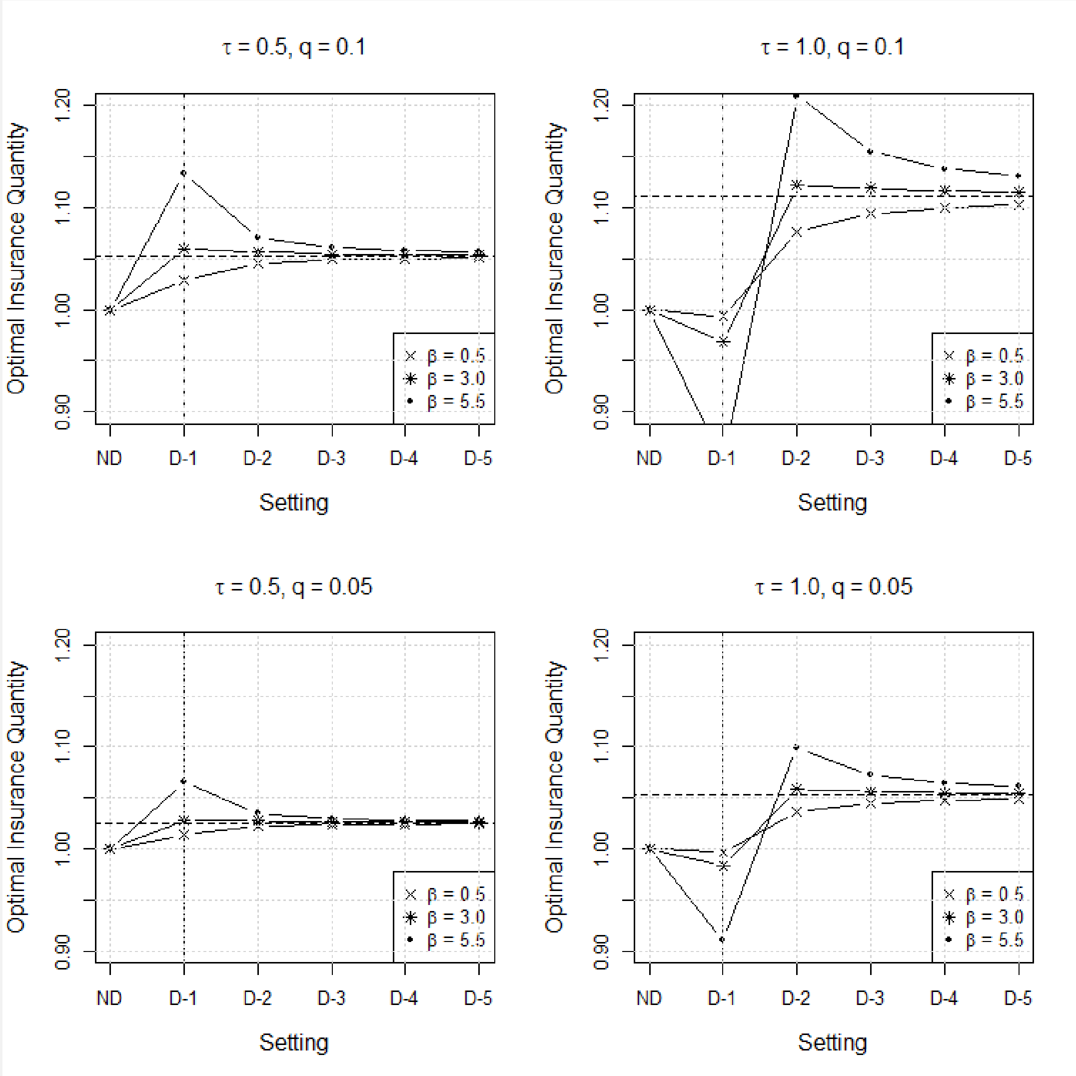
Numeric Example: Multiple Risk-Sharing

Counter-Example:

$$u(x) = -exp(-\beta x).$$

Initial wealth w	Loss probability p	Loss size l	Cost factor λ_I
1.5	0.05	1.0	0.0

- No unambiguous monotonicity for the optimal insurance quantity
- High risk-aversion rather results in a decreasing sequence $e^*_{I,n}$
- For τ close to 1, high risk-aversion implies non-monotonicity for $e^*_{I,n}$
- Fast convergence to $\frac{1}{(1-q\tau)}$
- q does not change results qualitatively but acts as scaling factor



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Results II: Demand Effects of RMM

Now, $e_R \neq 0$ (risk-management measure accessible)

Given the aforementioned model, the policyholder will prefer a perfect hedging (i.e. $e^*_{I,n} = e^*_{R,n}$), iff

$$\lambda_I = \lambda_R.$$

In this case, the optimal quantity equals the optimal quantity of a default-free insurance contract.

→ For $\lambda_I \neq \lambda_R$, policyholder prefers an under(over)-hedged position

Results III: Demand Effects of RMM

i. *Given CARA-preference, an increasing cost loading for the risk-management measure results in a decreasing demand for the risk-management measure, i.e.,*

$$\frac{\partial e_{R,n}^*}{\partial \lambda_R} < 0.$$

ii. *Given CARA-preference, an increasing cost loading for the risk-management measure results in an increasing demand for the insurance coverage (i.e.,* $\partial e_{I,n}^* / \partial \lambda_R > 0$ *) iff*

$$\sum_{k=0}^{n} \binom{n}{k} q^k (1-q)^{n-k} \frac{k}{n} \left(1 - \pi_I - \tau \frac{k}{n}\right) u''(W_{L,k}) < 0\,.$$

A sufficient condition is given by $\tau \leq (1 - p(1+\lambda_I))/(1 - pq(1+\lambda_I))$.

iii. *Given* $e_{I,n}^* = e_{R,n}^*$ *and CARA-preference, an increasing cost loading for the risk-management measure results in an increasing demand for the insurance coverage (i.e.,* $\partial e_{I,n}^* / \partial \lambda_R > 0$ *) iff*

$$n \geq \frac{1-q}{(1-\pi_I)\tau^{-1} - q}.$$

Note: $1 \geq \frac{1-q}{(1-\pi_I)\tau^{-1} - q} \Leftrightarrow \tau \leq (1 - p(1+\lambda_I))/(1 - pq(1+\lambda_I))$.

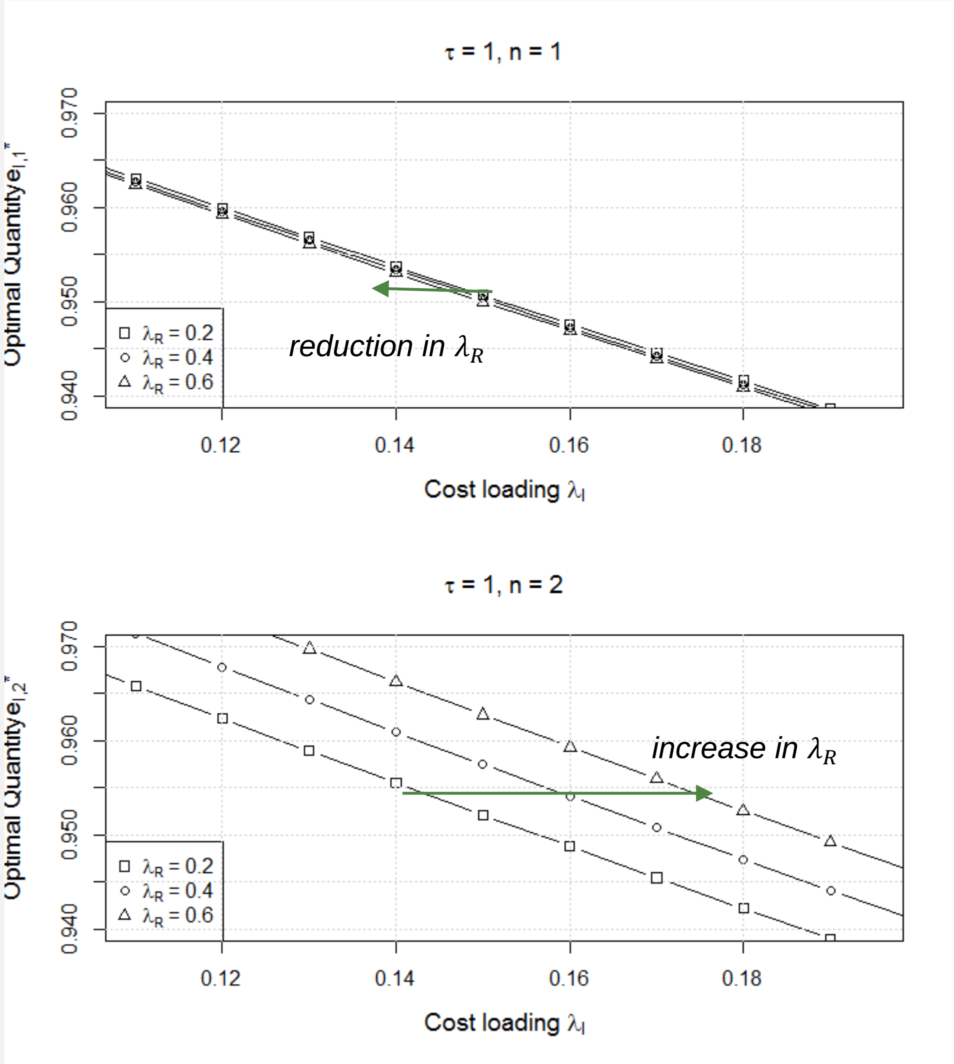
Numeric Example: Demand Effect

Counter-Example:

$$u(x) = -exp(-\beta x).$$

Initial wealth w	Loss probability p	Default probability q	Loss size l	Risk-aversion β
1.5	0.05	0.1	1.0	3.0

- For $n = 1$ the insurance-demand curve is shifted to the left if the RMM becomes more expensive (less insurance is demanded)
- For $n = 2$ the insurance-demand curve is shifted to the right if the RMM becomes more expensive (more insurance is demanded)



Final Comment

Summary:

- In our framework, multiple risk-sharing is a costless measure to diversify default risk
- Better does not always mean more: despite increasing utility, non-monotonous relationship between number of insurers and demanded insurance quantity in multiple risk-sharing possible
- Demand on available risk-management measure depends on difference of cost-loading factors λ_I and λ_R
  → Three possible states: perfect hedging, over-hedging, under-hedging
- Given CARA, insurance coverage serves as substitute for the risk-management measure if default rate is below a threshold
- Multiple risk-sharing lowers this threshold

Open issues:

- Analysis on DARA & IARA
- Easing i.i.d-assumption on insurer default; other multiple risk-sharing apart from quota share

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Thank you

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