

Intergenerational Discounting and Inequality

Frikk Nesje and Paolo G. Piacquadio

University of Padova

April 10, 2026



Introduction

Motivation
Contribution

Disentanglement

Social welfare
Discounting and equivalent
time representation
The aggregator

Characterization

Existence and uniqueness

Implications

Richness
Efficiency–equity
representation
Generalized Ramsey rule

Comparisons

Impatience and discounting
Inequality aversion and
aggregation

Conclusion

Appendix

Motivation

- ▶ Policies enacted today change the assignment of **resources across generations**.
- ▶ Examples:
 - ▶ Mitigation of climate change;
 - ▶ Public debt reduction;
 - ▶ Research and development for technological change;
 - ▶ Pension reform.

Introduction

Motivation

Contribution

Disentanglement

Social welfare

Discounting and equivalent
time representation

The aggregator

Characterization

Existence and uniqueness

Implications

Richness

Efficiency–equity
representation

Generalized Ramsey rule

Comparisons

Impatience and discounting

Inequality aversion and
aggregation

Conclusion

Appendix

- ▶ Evaluating such policies requires ethical judgments of **two distinct kinds**:
 - ▶ Trade off earlier vs. later generations—*discounting*;
 - ▶ Trade off better-off vs. worse-off generations—*inequality*.

Introduction

Motivation

Contribution

Disentanglement

Social welfare

Discounting and equivalent
time representation

The aggregator

Characterization

Existence and uniqueness

Implications

Richness

Efficiency–equity
representation

Generalized Ramsey rule

Comparisons

Impatience and discounting

Inequality aversion and
aggregation

Conclusion

Appendix

- ▶ Evaluating such policies requires ethical judgments of **two distinct kinds**:
 - ▶ Trade off earlier vs. later generations—*discounting*;
 - ▶ Trade off better-off vs. worse-off generations—*inequality*.
- ▶ The applied literature proceeds by selecting a small set of familiar criteria and conducting sensitivity analysis over a few parameters (Stern, 2007).
- ▶ This restricts attention to a **narrow subset of discounting and inequality attitudes**.

Introduction

Motivation

Contribution

Disentanglement

Social welfare

Discounting and equivalent
time representation

The aggregator

Characterization

Existence and uniqueness

Implications

Richness

Efficiency–equity
representation

Generalized Ramsey rule

Comparisons

Impatience and discounting

Inequality aversion and
aggregation

Conclusion

Appendix

Focus of the paper

- ▶ Our contribution is to **make the underlying ethical choices explicit**.
- ▶ We **characterize the full class of criteria** in which discounting and inequality can be specified independently, compared transparently, and combined in tractable ways.

Introduction

Motivation

Contribution

Disentanglement

Social welfare

Discounting and equivalent
time representation

The aggregator

Characterization

Existence and uniqueness

Implications

Richness

Efficiency–equity
representation

Generalized Ramsey rule

Comparisons

Impatience and discounting

Inequality aversion and
aggregation

Conclusion

Appendix

Our contribution

Introduction

Motivation

Contribution

Disentanglement

Social welfare

Discounting and equivalent
time representation

The aggregator

Characterization

Existence and uniqueness

Implications

Richness

Efficiency–equity
representation

Generalized Ramsey rule

Comparisons

Impatience and discounting

Inequality aversion and
aggregation

Conclusion

Appendix

1. We propose “**modular**” theories of intergenerational justice:
 - 1.1 *Module 1* exclusively deals with **discounting**;
 - 1.2 *Module 2* exclusively deals with **inequality**;
 - 1.3 Any theory is fully described by these two modules;
 - 1.4 It reveals a wealth of new criteria.
2. We **axiomatically characterize** the class of modular theories of intergenerational justice (Pareto and “nested time split”); modularity uniquely identified.

Our contribution

3. We offer **meaningful comparisons** of social impatience and inequality aversion across criteria.
4. We derive an **equity-efficiency decomposition** and a **generalized Ramsey rule**.
5. We **extend the framework beyond Pareto**, which connects our approach to the classic literature (Koopmans, 1960; Diamond, 1965).

Introduction

Motivation

Contribution

Disentanglement

Social welfare

Discounting and equivalent
time representation

The aggregator

Characterization

Existence and uniqueness

Implications

Richness

Efficiency–equity
representation

Generalized Ramsey rule

Comparisons

Impatience and discounting

Inequality aversion and
aggregation

Conclusion

Appendix

- ▶ Pure discounting in time-additive models (Stern, 2007);
- ▶ Intergenerational equity (Koopmans, 1960; Diamond, 1965);
- ▶ Disentanglement results (Atkinson, 1970; Epstein & Zin, 1989);
- ▶ Estimation of discounting in time-additive models (Attema et al. 2016);
- ▶ Probabilistic sophistication (Machina & Schmeidler, 1992);
- ▶ Behavioral restrictions on agents (Loewenstein & Prelec, 1992).

Introduction

Motivation

Contribution

Disentanglement

Social welfare

Discounting and equivalent
time representation

The aggregator

Characterization

Existence and uniqueness

Implications

Richness

Efficiency–equity
representation

Generalized Ramsey rule

Comparisons

Impatience and discounting

Inequality aversion and
aggregation

Conclusion

Appendix

Agenda

- ▶ Disentanglement;
- ▶ Characterization;
- ▶ Implications;
- ▶ If time is left:
 - ▶ comparative normative attitudes;
 - ▶ Generalizations;
- ▶ Conclusion.

Introduction

Motivation

Contribution

Disentanglement

Social welfare

Discounting and equivalent
time representation

The aggregator

Characterization

Existence and uniqueness

Implications

Richness

Efficiency–equity
representation

Generalized Ramsey rule

Comparisons

Impatience and discounting

Inequality aversion and
aggregation

Conclusion

Appendix

Disentanglement

Introduction

Motivation

Contribution

Disentanglement

Social welfare

Discounting and equivalent
time representation

The aggregator

Characterization

Existence and uniqueness

Implications

Richness

Efficiency–equity
representation

Generalized Ramsey rule

Comparisons

Impatience and discounting

Inequality aversion and
aggregation

Conclusion

Appendix

A consumption stream

- We study the problem of ranking bounded **consumption streams**

$$c : T \rightarrow \mathbb{R}_+$$

in infinite continuous time $T \equiv [0, \infty)$. The set of all consumption streams is C .

Introduction

Motivation
Contribution

Disentanglement

Social welfare

Discounting and equivalent
time representation
The aggregator

Characterization

Existence and uniqueness

Implications

Richness
Efficiency–equity
representation
Generalized Ramsey rule

Comparisons

Impatience and discounting
Inequality aversion and
aggregation

Conclusion

Appendix

A consumption stream

- ▶ We study the problem of ranking bounded **consumption streams**

$$c : T \rightarrow \mathbb{R}_+$$

in infinite continuous time $T \equiv [0, \infty)$. The set of all consumption streams is C .

- ▶ A theory of intergenerational justice is represented by a **social welfare function**

$$W : C \rightarrow \mathbb{R}.$$

The social welfare function satisfies a weak continuity requirement (weak measure continuity).

Change of perspective

- ▶ The central step of our analysis is to *transform* consumption streams in “calendar time” T to consumption streams in “equivalent time” $I \equiv [0, 1)$.

Introduction

Motivation
Contribution

Disentanglement

Social welfare

Discounting and equivalent
time representation

The aggregator

Characterization

Existence and uniqueness

Implications

Richness

Efficiency–equity
representation

Generalized Ramsey rule

Comparisons

Impatience and discounting

Inequality aversion and
aggregation

Conclusion

Appendix

Change of perspective

- ▶ The central step of our analysis is to *transform* consumption streams in “calendar time” T to consumption streams in “equivalent time” $I \equiv [0, 1)$.
 - ▶ The duration of calendar time is **stretched/compressed** up to the point where all consumption has “equal value”.
 - ▶ Instead of comparing consumption over calendar time by their present value, we compare them by their duration in equivalent time.

Introduction

Motivation
Contribution

Disentanglement

Social welfare
Discounting and equivalent
time representation
The aggregator

Characterization

Existence and uniqueness

Implications

Richness
Efficiency–equity
representation
Generalized Ramsey rule

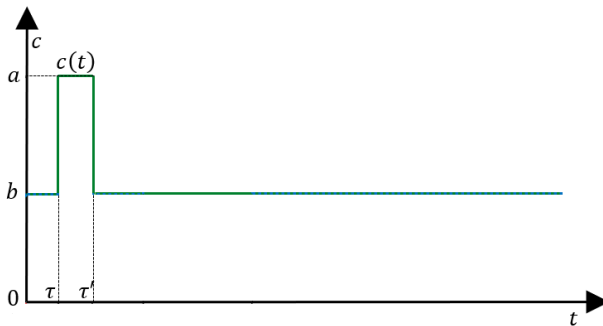
Comparisons

Impatience and discounting
Inequality aversion and
aggregation

Conclusion

Appendix

Change of perspective



Discounting and
Inequality

F. Nesje and
P.G. Piacquadio

Introduction

Motivation
Contribution

Disentanglement

Social welfare

Discounting and equivalent
time representation

The aggregator

Characterization

Existence and uniqueness

Implications

Richness
Efficiency–equity
representation
Generalized Ramsey rule

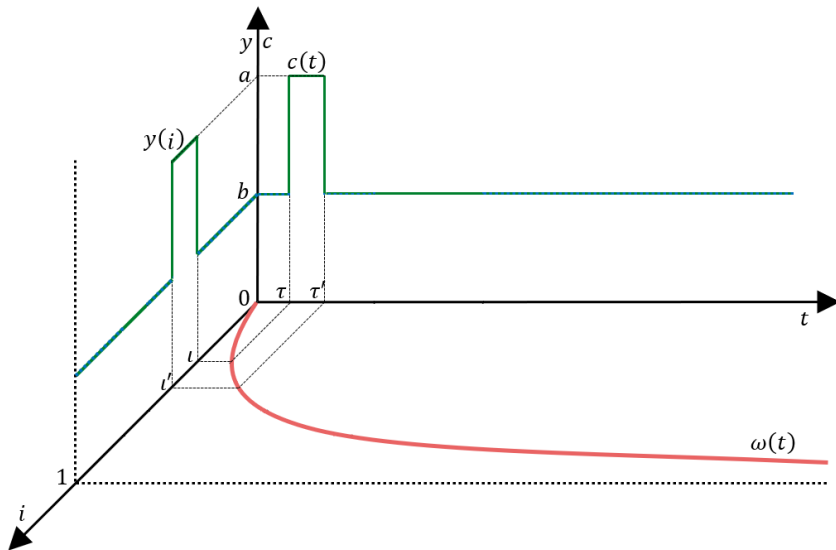
Comparisons

Impatience and discounting
Inequality aversion and
aggregation

Conclusion

Appendix

Change of perspective



Introduction

Motivation
Contribution

Disentanglement

Social welfare
Discounting and equivalent
time representation
The aggregator

Characterization

Existence and uniqueness

Implications

Richness
Efficiency–equity
representation
Generalized Ramsey rule

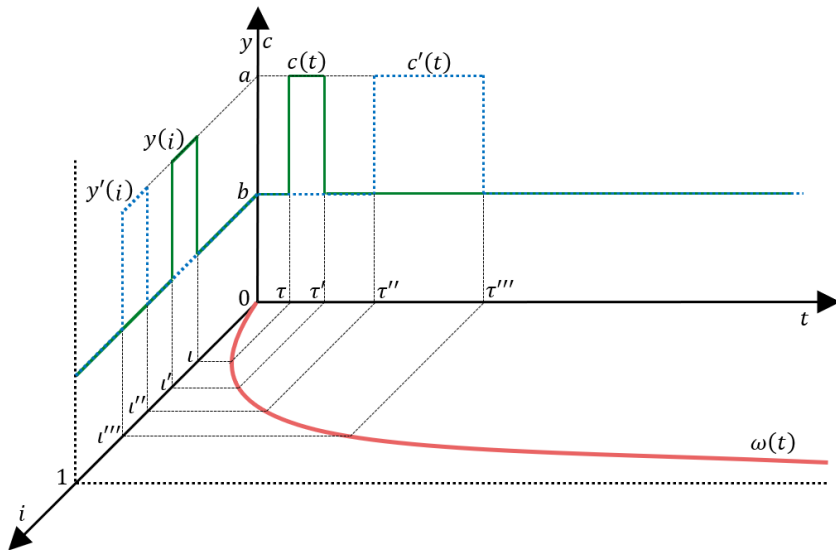
Comparisons

Impatience and discounting
Inequality aversion and
aggregation

Conclusion

Appendix

Change of perspective



Discounting and
Inequality

F. Nesje and
P.G. Piacquadio

Introduction

Motivation
Contribution

Disentanglement

Social welfare

Discounting and equivalent
time representation

The aggregator

Characterization

Existence and uniqueness

Implications

Richness
Efficiency–equity
representation
Generalized Ramsey rule

Comparisons

Impatience and discounting
Inequality aversion and
aggregation

Conclusion

Appendix

Change of perspective

- ▶ The central step of our analysis is to *transform* consumption streams in “calendar time” T to consumption streams in “equivalent time” $I \equiv [0, 1)$.
 - ▶ The duration of calendar time is **stretched/compressed** up to the point where all consumption has “equal value”.
 - ▶ Instead of comparing consumption over calendar time by their present value, we compare them by their duration in equivalent time.
- ▶ *Anticipating the results:* once consumption is reparametrized, inequality aversion can be expressed by a standard law-invariant aggregator.

Introduction

Motivation
Contribution

Disentanglement

Social welfare
Discounting and equivalent
time representation
The aggregator

Characterization

Existence and uniqueness

Implications

Richness
Efficiency–equity
representation
Generalized Ramsey rule

Comparisons

Impatience and discounting
Inequality aversion and
aggregation

Conclusion

Appendix

The time-discounting function

- ▶ We say that consumption streams admit a representation in **equivalent time**, if there exists a strictly increasing, continuous, onto map $\omega : T \rightarrow I$, such that rearrangements on I is a matter of social indifference.

$$W(c) = W(c') \text{ whenever } c = y \circ \omega, c' = y' \circ \omega, y = \Pi y'.$$

- ▶ In other words, all intervals in equivalent time have “equal value”.

Introduction

Motivation

Contribution

Disentanglement

Social welfare

Discounting and equivalent
time representation

The aggregator

Characterization

Existence and uniqueness

Implications

Richness

Efficiency–equity
representation

Generalized Ramsey rule

Comparisons

Impatience and discounting

Inequality aversion and
aggregation

Conclusion

Appendix

The time-discounting function

- ▶ We say that consumption streams admit a representation in **equivalent time**, if there exists a strictly increasing, continuous, onto map $\omega : T \rightarrow I$, such that rearrangements on I is a matter of social indifference.

$$W(c) = W(c') \text{ whenever } c = y \circ \omega, c' = y' \circ \omega, y = \Pi y'.$$

- ▶ In other words, all intervals in equivalent time have “equal value”.
- ▶ We call ω the (cumulative) **time-discounting function**.

Introduction

Motivation

Contribution

Disentanglement

Social welfare

Discounting and equivalent
time representation

The aggregator

Characterization

Existence and uniqueness

Implications

Richness

Efficiency–equity
representation

Generalized Ramsey rule

Comparisons

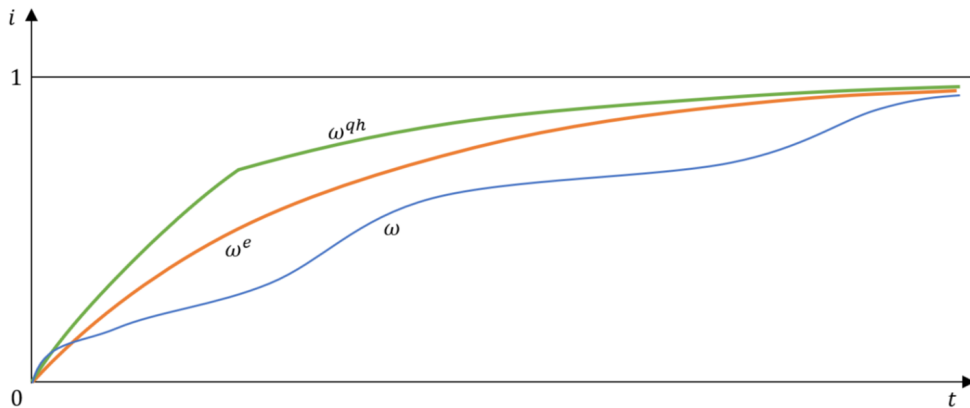
Impatience and discounting

Inequality aversion and
aggregation

Conclusion

Appendix

The time-discounting function



$$\omega^e(t) = 1 - e^{-\rho t}$$

$$\omega^{qh}(t) = \begin{cases} \beta\kappa(1 - e^{-\rho t}), & \text{for } t \in [0, \tau] \\ 1 - \kappa e^{-\rho t}, & \text{for } t > \tau, \end{cases} \text{ with } \kappa = 1/(\beta + (1 - \beta)e^{-\rho\tau})$$

Introduction

Motivation
Contribution

Disentanglement

Social welfare
Discounting and equivalent
time representation
The aggregator

Characterization

Existence and uniqueness

Implications

Richness
Efficiency–equity
representation
Generalized Ramsey rule

Comparisons

Impatience and discounting
Inequality aversion and
aggregation

Conclusion

Appendix

The aggregator

- ▶ How to aggregate consumption streams in equivalent time?
- ▶ Given the social welfare function W and the time-discounting function ω , the **aggregator** $V : Y \rightarrow \mathbb{R}$ immediately follows by setting

$$V(y) = W(y \circ \omega).$$

Introduction

Motivation
Contribution

Disentanglement

Social welfare
Discounting and equivalent
time representation
The aggregator

Characterization

Existence and uniqueness

Implications

Richness
Efficiency–equity
representation
Generalized Ramsey rule

Comparisons

Impatience and discounting
Inequality aversion and
aggregation

Conclusion

Appendix

The aggregator

- ▶ How to aggregate consumption streams in equivalent time?
- ▶ Given the social welfare function W and the time-discounting function ω , the **aggregator** $V : Y \rightarrow \mathbb{R}$ immediately follows by setting

$$V(y) = W(y \circ \omega).$$

- ▶ Note that the aggregator is *law invariant* in equivalent time by definition.
- ▶ Thus, V could be any standard (atemporal) social welfare function.

Introduction

Motivation
Contribution

Disentanglement

Social welfare
Discounting and equivalent
time representation
The aggregator

Characterization

Existence and uniqueness

Implications

Richness
Efficiency–equity
representation
Generalized Ramsey rule

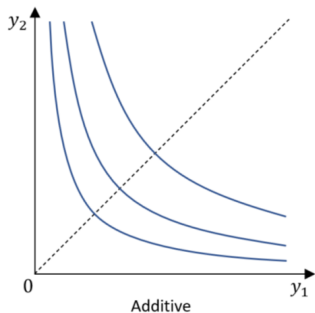
Comparisons

Impatience and discounting
Inequality aversion and
aggregation

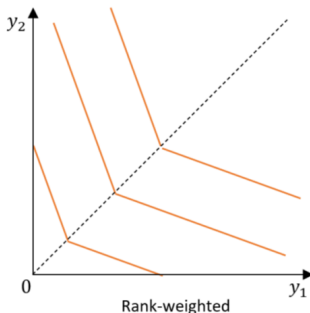
Conclusion

Appendix

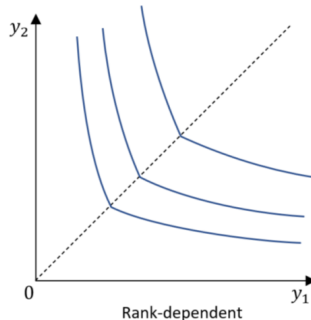
The aggregator



$$V^a = \int_0^1 \frac{(y_i)^{1-\eta}}{1-\eta} di$$



$$V^{rw} = (\gamma + 1) \int_0^1 (1 - r(i))^\gamma y_i di$$



$$V^{rd} = (\gamma + 1) \int_0^1 (1 - r(i))^\gamma \frac{(y_i)^{1-\eta}}{1-\eta} di$$

Introduction

Motivation
Contribution

Disentanglement

Social welfare
Discounting and equivalent
time representation

The aggregator

Characterization

Existence and uniqueness

Implications

Richness
Efficiency–equity
representation
Generalized Ramsey rule

Comparisons

Impatience and discounting
Inequality aversion and
aggregation

Conclusion

Appendix

The modular social welfare function

Discounting and
Inequality

F. Nesje and
P.G. Piacquadio

- ▶ We say that W is **modular** if consumption streams can be represented in equivalent time and, thus, there exist an ω and V induced by W .
- ▶ Importantly, the most common welfare criteria adopted in the literature are modular.

Example formulas

Introduction

Motivation
Contribution

Disentanglement

Social welfare
Discounting and equivalent
time representation
The aggregator

Characterization

Existence and uniqueness

Implications

Richness
Efficiency–equity
representation
Generalized Ramsey rule

Comparisons

Impatience and discounting
Inequality aversion and
aggregation

Conclusion

Appendix

Characterization

Introduction

Motivation
Contribution

Disentanglement

Social welfare
Discounting and equivalent
time representation
The aggregator

Characterization

Existence and uniqueness

Implications

Richness
Efficiency–equity
representation
Generalized Ramsey rule

Comparisons

Impatience and discounting
Inequality aversion and
aggregation

Conclusion

Appendix

Nested time split

- ▶ We axiomatically characterize the disentanglement between intergenerational discounting and inequality (all modular theories of justice) by imposing Pareto and the novel axiom of **nested time split**.

Introduction

Motivation
Contribution

Disentanglement

Social welfare
Discounting and equivalent
time representation
The aggregator

Characterization

Existence and uniqueness

Implications

Richness
Efficiency–equity
representation
Generalized Ramsey rule

Comparisons

Impatience and discounting
Inequality aversion and
aggregation

Conclusion

Appendix

Nested time split

- ▶ We axiomatically characterize the disentanglement between intergenerational discounting and inequality (all modular theories of justice) by imposing Pareto and the novel axiom of **nested time split**.
- ▶ It requires that **any finite calendar time interval** can be divided into two sub-intervals of “equal value”.
- ▶ A social welfare function W satisfies **nested time split** if for every nondegenerate $A = [\underline{t}, \bar{t}] \subseteq T$ with $\bar{t} < \infty$ there exists $t^* \in (\underline{t}, \bar{t})$ with $T_1 = [\underline{t}, t^*)$, $T_2 = [t^*, \bar{t})$ such that, for each consumption stream $c \in C$ and each pair of consumption levels $\underline{c}, \bar{c} \in \mathbb{R}_+$,

$$W(\underline{c}_{T_1}, \bar{c}_{T_2}, c_{T/A}) = W(\bar{c}_{T_2}, \underline{c}_{T_1}, c_{T/A}).$$

Introduction

Motivation
Contribution

Disentanglement

Social welfare
Discounting and equivalent
time representation
The aggregator

Characterization

Existence and uniqueness

Implications

Richness
Efficiency–equity
representation
Generalized Ramsey rule

Comparisons

Impatience and discounting
Inequality aversion and
aggregation

Conclusion

Appendix

Nested time split

- ▶ **Theorem 1.** Assume W is Paretian. W is modular iff W satisfies nested time split.
- ▶ **Corollary 1.** Assume W is Paretian. If W satisfies nested time split, then ω and V are unique.

Introduction

Motivation
Contribution

Disentanglement

Social welfare
Discounting and equivalent
time representation
The aggregator

Characterization

Existence and uniqueness

Implications

Richness
Efficiency–equity
representation
Generalized Ramsey rule

Comparisons

Impatience and discounting
Inequality aversion and
aggregation

Conclusion

Appendix

Nested time split

- ▶ **Theorem 1.** Assume W is Paretian. W is modular iff W satisfies nested time split.
- ▶ **Corollary 1.** Assume W is Paretian. If W satisfies nested time split, then ω and V are unique.
- ▶ *Intuitively:* Pareto rules out intervals of calendar time that are ethically irrelevant.
- ▶ Nested time split then forces the relative weight of adjacent sub-intervals to be independent of the rest of the consumption stream.

Generalized modular: relaxing Pareto—slow, zero, or negative discounting

Further axioms: stationarity, time consistency and time invariance

Introduction

Motivation
Contribution

Disentanglement

Social welfare
Discounting and equivalent
time representation
The aggregator

Characterization

Existence and uniqueness

Implications

Richness
Efficiency–equity
representation
Generalized Ramsey rule

Comparisons

Impatience and discounting
Inequality aversion and
aggregation

Conclusion

Appendix

Implications

Introduction

Motivation

Contribution

Disentanglement

Social welfare

Discounting and equivalent
time representation

The aggregator

Characterization

Existence and uniqueness

Implications

Richness

Efficiency–equity
representation

Generalized Ramsey rule

Comparisons

Impatience and discounting

Inequality aversion and
aggregation

Conclusion

Appendix

Some modular theories of intergenerational justice

Discounting and
Inequality

F. Nesje and
P.G. Piacquadio

Introduction

Motivation
Contribution

Disentanglement

Social welfare
Discounting and equivalent
time representation
The aggregator

Characterization

Existence and uniqueness

Implications

Richness
Efficiency–equity
representation
Generalized Ramsey rule

Comparisons

Impatience and discounting
Inequality aversion and
aggregation

Conclusion

Appendix

W	V	Additive V^a	Rank-weighted V^{rw}	Rank-dependent V^{rd}
ω		$\int_0^1 f(y_i) di$	$\int_0^1 g(r(i)) y_i di$	$\int_0^1 g(r(i)) f(y_i) di$
Exponential ω^e	$1 - e^{-\rho t}$	EDU	NEW	
Quasi-hyperbolic ω^{qh}	$\begin{cases} \beta k(1 - e^{-\rho t}) & t \leq \tau \\ 1 - k e^{-\rho t} & t > \tau \end{cases}$			
General ω				

A tractable family: $W^{erd} = (1 + \gamma) \int_0^\infty e^{-\rho t} \left(1 - r(\omega^e(t))\right)^\gamma \frac{(c_t)^{1-\eta}}{1-\eta} dt$

Introduction

Motivation
Contribution

Disentanglement

Social welfare
Discounting and equivalent
time representation
The aggregator

Characterization

Existence and uniqueness

Implications

Richness
Efficiency–equity
representation
Generalized Ramsey rule

Comparisons

Impatience and discounting
Inequality aversion and
aggregation

Conclusion

Appendix

- **Proposition 1.** Assume W is modular, Paretian and scale invariant. Then the efficiency–equity representation satisfies:

$$W(c) = m(y) \cdot E(y), \quad y = c \circ \omega^{-1}.$$

- Here, m is the mean and E is the equity factor (or 1 minus inequality), building on Atkinson (1970).

- ▶ **Proposition 2.** Assume W is modular and Paretian, and ω , V have “right-derivatives”. Then the social discount rate satisfies:

$$SDR_t = \frac{1}{t} \ln(\omega'_+(0)/\omega'_+(t)) + \frac{1}{t} \ln(\mu(0; y)/\mu(\omega(t); y)).$$

- ▶ The first term captures the “pure time preference”, embodied by the time-discounting function.
- ▶ The second term captures “local inequality aversion” ($\mu(i; y)$ is the marginal of V at i given y).

Introduction

Motivation

Contribution

Disentanglement

Social welfare

Discounting and equivalent
time representation

The aggregator

Characterization

Existence and uniqueness

Implications

Richness

Efficiency–equity
representation

Generalized Ramsey rule

Comparisons

Impatience and discounting

Inequality aversion and
aggregation

Conclusion

Appendix

- ▶ **Proposition 2.** Assume W is modular and Paretian, and ω , V have “right-derivatives”. Then the social discount rate satisfies:

$$SDR_t = \frac{1}{t} \ln(\omega'_+(0)/\omega'_+(t)) + \frac{1}{t} \ln(\mu(0; y)/\mu(\omega(t); y)).$$

- ▶ The first term captures the “pure time preference”, embodied by the time-discounting function.
- ▶ The second term captures “local inequality aversion” ($\mu(i; y)$ is the marginal of V at i given y).
- ▶ For parametric rank-dependent families: $SDR_t^{erd} = \rho + \eta g_t^c + \gamma g_t^r$.

Introduction

Motivation
Contribution

Disentanglement

Social welfare
Discounting and equivalent
time representation
The aggregator

Characterization

Existence and uniqueness

Implications

Richness
Efficiency–equity
representation
Generalized Ramsey rule

Comparisons

Impatience and discounting
Inequality aversion and
aggregation

Conclusion

Appendix

Comparisons

Introduction

Motivation

Contribution

Disentanglement

Social welfare

Discounting and equivalent
time representation

The aggregator

Characterization

Existence and uniqueness

Implications

Richness

Efficiency–equity
representation

Generalized Ramsey rule

Comparisons

Impatience and discounting

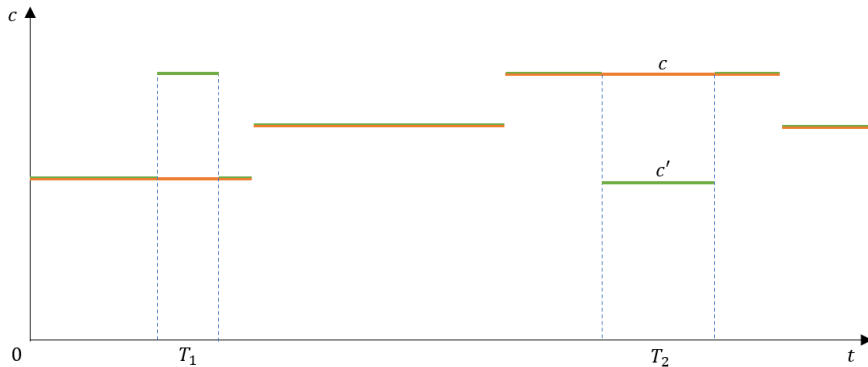
Inequality aversion and
aggregation

Conclusion

Appendix

Social impatience

- Consider c and c' as represented and assume $W(c) = W(c')$. Then W exhibits **social impatience** if T_2 is (weakly) longer than T_1 .



- ▶ Standard definitions of impatience emphasize the preference for advancing benefits and postponing costs (Böhm-Bawerk, 1889; Fisher, 1930; Koopmans, 1960).
- ▶ Our definition emphasizes that discounting is a matter of *stretching/compressing* calendar time.

Introduction

Motivation
Contribution

Disentanglement

Social welfare
Discounting and equivalent
time representation
The aggregator

Characterization

Existence and uniqueness

Implications

Richness
Efficiency–equity
representation
Generalized Ramsey rule

Comparisons

Impatience and discounting
Inequality aversion and
aggregation

Conclusion

Appendix

- ▶ Standard definitions of impatience emphasize the preference for advancing benefits and postponing costs (Böhm-Bawerk, 1889; Fisher, 1930; Koopmans, 1960).
- ▶ Our definition emphasizes that discounting is a matter of *stretching/compressing* calendar time.
- ▶ The definitions are equivalent when assuming *Pareto efficiency*.

Introduction

Motivation
Contribution

Disentanglement

Social welfare
Discounting and equivalent
time representation
The aggregator

Characterization

Existence and uniqueness

Implications

Richness
Efficiency–equity
representation
Generalized Ramsey rule

Comparisons

Impatience and discounting
Inequality aversion and
aggregation

Conclusion

Appendix

- **Proposition 3.** Assume W is modular and Paretian. Then W is social impatient iff the time-discounting function ω is concave.

Introduction

Motivation

Contribution

Disentanglement

Social welfare

Discounting and equivalent
time representation

The aggregator

Characterization

Existence and uniqueness

Implications

Richness

Efficiency–equity
representation

Generalized Ramsey rule

Comparisons

Impatience and discounting

Inequality aversion and
aggregation

Conclusion

Appendix

- ▶ **Proposition 3.** Assume W is modular and Paretian. Then W is social impatient iff the time-discounting function ω is concave.
- ▶ **Proposition 4.** Assume W^* and W are modular and Paretian. Then W^* is more social impatient than W iff the time-discounting function ω^* is more concave than ω .
- ▶ **Corollary 2.** Moreover, if differentiable,

$$\frac{\omega''(t)}{\omega'(t)} \geq \frac{\omega^{*''}(t)}{\omega^{*'}(t)} \text{ for each } t \in T.$$

Introduction

Motivation
Contribution

Disentanglement

Social welfare
Discounting and equivalent
time representation
The aggregator

Characterization

Existence and uniqueness

Implications

Richness
Efficiency–equity
representation
Generalized Ramsey rule

Comparisons

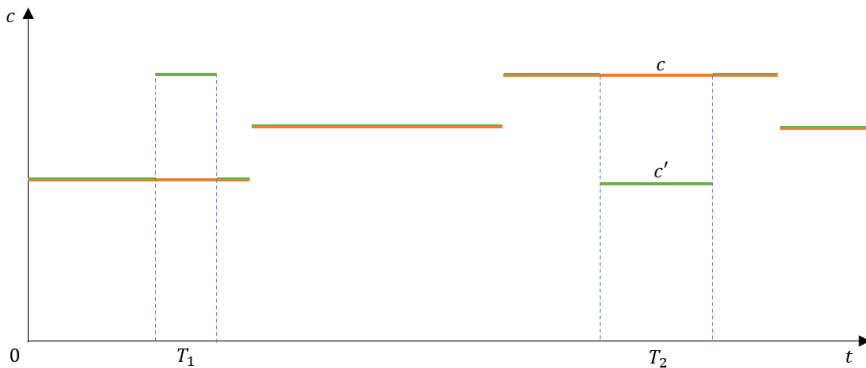
Impatience and discounting
Inequality aversion and
aggregation

Conclusion

Appendix

Inequality aversion

- Assume $W(c) = W(c')$.



Introduction

Motivation
Contribution

Disentanglement

Social welfare
Discounting and equivalent
time representation
The aggregator

Characterization

Existence and uniqueness

Implications

Richness
Efficiency–equity
representation
Generalized Ramsey rule

Comparisons

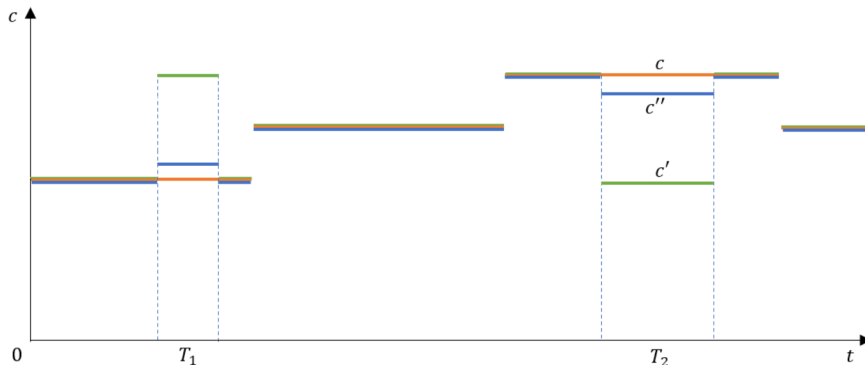
Impatience and discounting
Inequality aversion and
aggregation

Conclusion

Appendix

Inequality aversion

- Assume $W(c) = W(c')$. Then a progressive transfer between disjoint T_1 and T_2 increases social welfare and $W(c'') \geq W(c)$.



Inequality aversion

- ▶ The definition is similar to the Pigou-Dalton transfer principle.
- ▶ However, it first controls for “impatience” by checking indifference.

Introduction

Motivation
Contribution

Disentanglement

Social welfare
Discounting and equivalent
time representation
The aggregator

Characterization

Existence and uniqueness

Implications

Richness
Efficiency–equity
representation
Generalized Ramsey rule

Comparisons

Impatience and discounting
**Inequality aversion and
aggregation**

Conclusion

Appendix

Inequality aversion

Introduction

Motivation
Contribution

Disentanglement

Social welfare
Discounting and equivalent
time representation
The aggregator

Characterization

Existence and uniqueness

Implications

Richness
Efficiency–equity
representation
Generalized Ramsey rule

Comparisons

Impatience and discounting
**Inequality aversion and
aggregation**

Conclusion

Appendix

- ▶ The definition is similar to the Pigou-Dalton transfer principle.
- ▶ However, it first controls for “impatience” by checking indifference.
- ▶ Our definition avoids taking a stand on discounting.
- ▶ The standard Pigou-Dalton principle emerges if there is no discounting.

Inequality aversion

- ▶ **Proposition 5.** Assume W is modular and Paretian. Then W is inequality averse iff the aggregator V is Schur-concave.
 - ▶ Remarks: It works also without differentiability, as needed by rank-dependent criteria (Chew and Mao, 1995).

Introduction

Motivation

Contribution

Disentanglement

Social welfare

Discounting and equivalent
time representation

The aggregator

Characterization

Existence and uniqueness

Implications

Richness

Efficiency–equity
representation

Generalized Ramsey rule

Comparisons

Impatience and discounting

Inequality aversion and
aggregation

Conclusion

Appendix

- ▶ **Proposition 5.** Assume W is modular and Paretian. Then W is inequality averse iff the aggregator V is Schur-concave.
 - ▶ Remarks: It works also without differentiability, as needed by rank-dependent criteria (Chew and Mao, 1995).
- ▶ **Proposition 6.** Assume W^* and W are modular and Paretian. Then W^* is more inequality averse than W iff the local inequality aversion (IA) of V^* is always at least as large as that of V .
- ▶ $IA(i, i', y) = \mu(i; y)/\mu(i'; y)$ measures the marginal willingness to transfer income across two disjoint equally-sized intervals of equivalent time. Inequality aversion implies $IA \geq 1$.

Introduction

Motivation

Contribution

Disentanglement

Social welfare

Discounting and equivalent
time representation

The aggregator

Characterization

Existence and uniqueness

Implications

Richness

Efficiency–equity
representation

Generalized Ramsey rule

Comparisons

Impatience and discounting

Inequality aversion and
aggregation

Conclusion

Appendix

Conclusion

Introduction

Motivation

Contribution

Disentanglement

Social welfare

Discounting and equivalent
time representation

The aggregator

Characterization

Existence and uniqueness

Implications

Richness

Efficiency–equity
representation

Generalized Ramsey rule

Comparisons

Impatience and discounting

Inequality aversion and
aggregation

Conclusion

Appendix

Conclusion

- ▶ We propose modular theories of intergenerational justice.
- ▶ Distinctive features:
 - ▶ It builds on and highlights a variety of attitudes towards discounting and inequality;
 - ▶ It is very general and flexible, and enriches the toolbox of economic analysis. At the same time, the parametric specifications of these theories remain suitable for applied work.
- ▶ Estimation, application, and generalization of the modular theories also open many avenues for future research.

Introduction

Motivation
Contribution

Disentanglement

Social welfare
Discounting and equivalent
time representation
The aggregator

Characterization

Existence and uniqueness

Implications

Richness
Efficiency–equity
representation
Generalized Ramsey rule

Comparisons

Impatience and discounting
Inequality aversion and
aggregation

Conclusion

Appendix

Thank you!

E-mail: paolo.piacquadio@unisg.ch.

Introduction

Motivation
Contribution

Disentanglement

Social welfare
Discounting and equivalent
time representation
The aggregator

Characterization

Existence and uniqueness

Implications

Richness
Efficiency–equity
representation
Generalized Ramsey rule

Comparisons

Impatience and discounting
Inequality aversion and
aggregation

Conclusion

Appendix

Appendix

Introduction

Motivation
Contribution

Disentanglement

Social welfare
Discounting and equivalent
time representation
The aggregator

Characterization

Existence and uniqueness

Implications

Richness
Efficiency–equity
representation
Generalized Ramsey rule

Comparisons

Impatience and discounting
Inequality aversion and
aggregation

Conclusion

Appendix

Example formulas

- Consider exponentially-discounted utilitarianism:

$$W = \int_0^{\infty} \omega'(t) f(c(t)) dt = \int_0^{\infty} e^{-\rho t} f(c(t)) dt,$$

with $\rho > 0$ and f increasing concave.

Introduction

Motivation
Contribution

Disentanglement

Social welfare
Discounting and equivalent
time representation
The aggregator

Characterization

Existence and uniqueness

Implications

Richness
Efficiency–equity
representation
Generalized Ramsey rule

Comparisons

Impatience and discounting
Inequality aversion and
aggregation

Conclusion

Appendix

Example formulas

- ▶ Consider exponentially-discounted utilitarianism:

$$W = \int_0^{\infty} \omega'(t) f(c(t)) dt = \int_0^{\infty} e^{-\rho t} f(c(t)) dt,$$

with $\rho > 0$ and f increasing concave.

- ▶ From our characterization:
 - ▶ The time-discounting function is $i = \omega(t) = 1 - e^{-\rho t}$;
 - ▶ The aggregator in equivalent time, $V = \rho^{-1} \int_0^1 f(y(i)) di$, is additive.

Introduction

Motivation
Contribution

Disentanglement

Social welfare
Discounting and equivalent
time representation
The aggregator

Characterization

Existence and uniqueness

Implications

Richness
Efficiency–equity
representation
Generalized Ramsey rule

Comparisons

Impatience and discounting
Inequality aversion and
aggregation

Conclusion

Appendix

Back

Characterization: generalized modular

- ▶ We now explore the family of “modular” theories of intergenerational justice that emerge by satisfying nested time split, while relaxing Pareto efficiency.
- ▶ Let the **generalized time-discounting function** $\bar{\omega} : T \rightarrow \mathbb{R}_+$ be a continuous and strictly increasing transformation function mapping calendar time into the **generalized equivalent time**, now possibly unbounded.

Introduction

Motivation
Contribution

Disentanglement

Social welfare
Discounting and equivalent
time representation
The aggregator

Characterization

Existence and uniqueness

Implications

Richness
Efficiency–equity
representation
Generalized Ramsey rule

Comparisons

Impatience and discounting
Inequality aversion and
aggregation

Conclusion

Appendix

Characterization: generalized modular

- ▶ We now explore the family of “modular” theories of intergenerational justice that emerge by satisfying nested time split, while relaxing Pareto efficiency.
- ▶ Let the **generalized time-discounting function** $\bar{\omega} : T \rightarrow \mathbb{R}_+$ be a continuous and strictly increasing transformation function mapping calendar time into the **generalized equivalent time**, now possibly unbounded.
- ▶ **Theorem 2.** Assume $\bar{W}$ satisfies liminf-restricted dominance. $\bar{W}$ is modular iff $\bar{W}$ satisfies nested time split.

Back

Discounting and
Inequality

F. Nesje and
P.G. Piacquadio

Introduction

Motivation
Contribution

Disentanglement

Social welfare
Discounting and equivalent
time representation
The aggregator

Characterization

Existence and uniqueness

Implications

Richness
Efficiency–equity
representation
Generalized Ramsey rule

Comparisons

Impatience and discounting
Inequality aversion and
aggregation

Conclusion

Appendix

Further axioms: stationarity

- ▶ Next, we look at the relationship between modular theories and **standard axioms** of intergenerational justice.

Introduction

Motivation
Contribution

Disentanglement

Social welfare
Discounting and equivalent
time representation
The aggregator

Characterization

Existence and uniqueness

Implications

Richness
Efficiency–equity
representation
Generalized Ramsey rule

Comparisons

Impatience and discounting
Inequality aversion and
aggregation

Conclusion

Appendix

Further axioms: stationarity

- ▶ Next, we look at the relationship between modular theories and **standard axioms** of intergenerational justice.
- ▶ A social welfare function W satisfies **stationarity** if for each partition of calendar time $T_1, T_2 \subset T$ with $T_1 \cup T_2 = T$ with $T_1 = [0, \bar{t})$, $T_2 = [\bar{t}, \infty)$ and each pair of consumption streams $c, c' \in C$ with $c(t) = c'(t)$ for each $t \in T_1$, $W(c) \geq W(c')$ iff $W(c_{T_2}) \geq W(c'_{T_2})$.
- ▶ **Theorem 3.** Assume W is modular and Paretian. Then W satisfies stationarity iff it is exponentially-discounted utilitarian.
- ▶ **Corollary 3.** Assume W is Paretian. Then W satisfies nested time-split and stationarity iff it is exponentially-discounted utilitarian.

Introduction

Motivation
Contribution

Disentanglement

Social welfare
Discounting and equivalent
time representation
The aggregator

Characterization

Existence and uniqueness

Implications

Richness
Efficiency–equity
representation
Generalized Ramsey rule

Comparisons

Impatience and discounting
Inequality aversion and
aggregation

Conclusion

Appendix

Further axioms: time consistency and time invariance

- ▶ Next, we follow Halevy (2015) and introduce two further properties of intertemporal criteria: **time consistency** and **time invariance**.
- ▶ Rather than imposing restrictions on W , these axioms impose some consistency requirement between social welfare functions at **different calendar times**.

Introduction

Motivation
Contribution

Disentanglement

Social welfare
Discounting and equivalent
time representation
The aggregator

Characterization

Existence and uniqueness

Implications

Richness
Efficiency–equity
representation
Generalized Ramsey rule

Comparisons

Impatience and discounting
Inequality aversion and
aggregation

Conclusion

Appendix

Further axioms: time consistency and time invariance

- ▶ Next, we follow Halevy (2015) and introduce two further properties of intertemporal criteria: **time consistency** and **time invariance**.
- ▶ Rather than imposing restrictions on W , these axioms impose some consistency requirement between social welfare functions at **different calendar times**.
- ▶ **Theorem 4.** Assume $W_{\underline{t}}$, one for each $\underline{t}$, are modular and Paretian. Then $W_{\underline{t}}$, one for each $\underline{t}$, satisfy time consistency iff: i) the time-discounting functions satisfy $\omega_{\underline{t}}(t) = 1 - (1 - \omega(t))/(1 - \omega(\underline{t}))$ for each $\underline{t} \in T$ and each $t \geq \underline{t}$; and ii) the aggregator is additive and satisfies $V_{\underline{t}} = V^a$ for each $\underline{t} \in T$.
- ▶ **Theorem 5.** Assume $W_{\underline{t}}$, one for each $\underline{t}$, are modular and Paretian. Then $W_{\underline{t}}$, one for each $\underline{t}$, satisfy time invariance iff $W_{\underline{t}} = W_{\underline{t}'}$ for each $\underline{t}, \underline{t}' \in T$.

Introduction

Motivation
Contribution

Disentanglement

Social welfare
Discounting and equivalent
time representation
The aggregator

Characterization

Existence and uniqueness

Implications

Richness
Efficiency–equity
representation
Generalized Ramsey rule

Comparisons

Impatience and discounting
Inequality aversion and
aggregation

Conclusion

Appendix

Back

Exponential discounting compresses the far future (1%)

Another perspective on exponential discounting

- ▶ Benefits in years **1–22** are approximately as desirable as the same benefits in years **23–50**.

Discounting and
Inequality

F. Nesje and
P.G. Piacquadio

Introduction

Motivation

Contribution

Disentanglement

Social welfare

Discounting and equivalent
time representation

The aggregator

Characterization

Existence and uniqueness

Implications

Richness

Efficiency–equity
representation

Generalized Ramsey rule

Comparisons

Impatience and discounting

Inequality aversion and
aggregation

Conclusion

Appendix

Exponential discounting compresses the far future (1%)

Another perspective on exponential discounting

- ▶ Benefits in years **1–22** are approximately as desirable as the same benefits in years **23–50**.
- ▶ Benefits in years **1–57** are approximately as desirable as the same benefits in years **58–200**.

Introduction

Motivation
Contribution

Disentanglement

Social welfare
Discounting and equivalent
time representation
The aggregator

Characterization

Existence and uniqueness

Implications

Richness
Efficiency–equity
representation
Generalized Ramsey rule

Comparisons

Impatience and discounting
Inequality aversion and
aggregation

Conclusion

Appendix

Exponential discounting compresses the far future (1%)

Another perspective on exponential discounting

- ▶ Benefits in years **1–22** are approximately as desirable as the same benefits in years **23–50**.
- ▶ Benefits in years **1–57** are approximately as desirable as the same benefits in years **58–200**.
- ▶ Benefits in years **1–70** are approximately as desirable as the same benefits in years **71–1000**.

Takeaway

- ▶ Even at a low 1% rate, very long delayed blocks are needed to match long near-term blocks of the same annual benefit.
- ▶ This illustrates how exponential discounting starts slow, kicks in quickly, and flattens in the far future.

Introduction

Motivation
Contribution

Disentanglement

Social welfare
Discounting and equivalent
time representation
The aggregator

Characterization

Existence and uniqueness

Implications

Richness
Efficiency–equity
representation
Generalized Ramsey rule

Comparisons

Impatience and discounting
Inequality aversion and
aggregation

Conclusion

Appendix