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Integrating the Attention Mechanism into State Space Models

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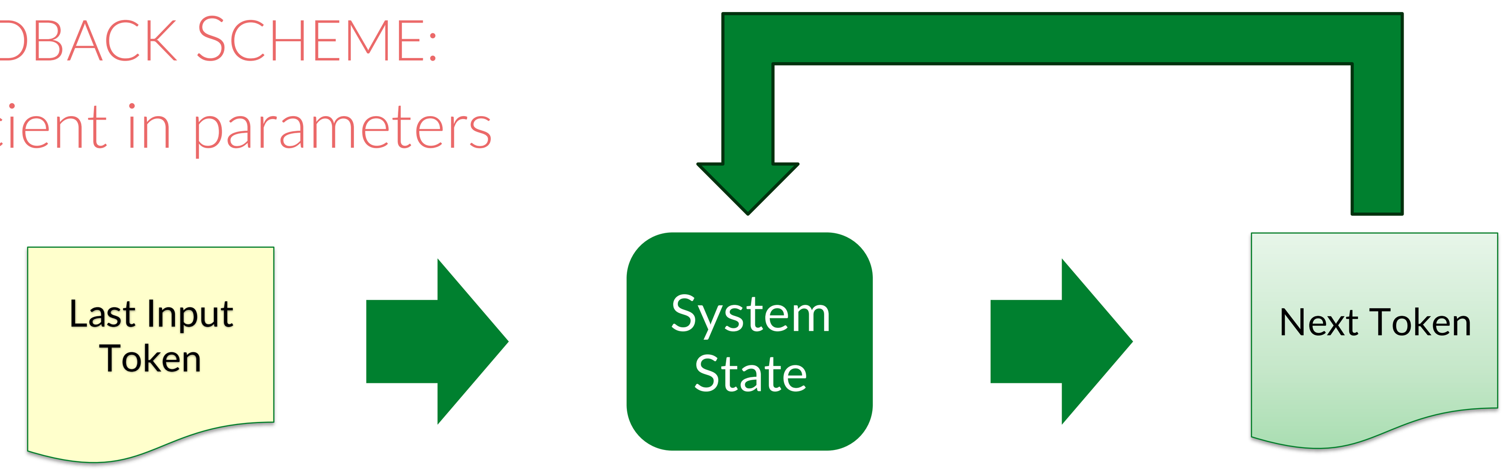
ATTENTION MECHANISM | STATE SPACE MODEL

SIGNAL FLOW

FEEDFORWARD SCHEME: extensive need of parameters

System theory: Finite Impulse Response Model (FIR)

FEEDBACK SCHEME: efficient in parameters



System theory: Infinite Impulse Response Model (IIR)

CONTEXT SOURCE AND REPRESENTATION

CONTEXT SOURCE (Attention Mechanism)

- Complete sequence of token predecessors
- Variable volume growing with the sequence length and position in it
- Quadratic complexity of context relevance

CONTEXT SOURCE (State Space Model)

- Recursively updated state vector
- Constant volume
- Necessity to commit to the state dimension
- Context relevance not yet captured
- Capturing context relevance is the essence of the proposal presented here

WEIGHTING OF THE CONTEXT RELEVANCE IN THE TEXT SEQUENCE

ATTENTION MECHANISM (WITHOUT MULTI-LAYER-PERCEPTRON)

$$y_i = \sum_{j=1}^{i} x_j a_{i,j} W_V W_O$$

context relevance

position independent parameters

STATE SPACE MODEL IN DIAGONAL FORM

$$y_i = \sum_{j=1}^{i} x_i B A^{i-j} C = \sum_{j=1}^{i} x_i \left(\sum_{k=1}^{N} a_k^{i-j} b_k c_k \right)$$

Positional forgetting

position independent parameters

USE OF ATTENTION MECHANISM

CONVENTIONAL CONCEPT

- Relevance determined by attention weights
- They result from the cross-similarities between input embeddings (queries and keys)
- Similarity: $x_i{}' W_Q W_K x_j$ (all i and j)
- Quadratic number of combinations

OUR PROPOSAL

- In addition to positional forgetting: Introducing attention weights
- Resulting from the similarities between input embedding (query) x_i and the actual state vector h_i
- Similarity: $x_i{}' W_S h_i$(only along i)
- Linear complexity

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