

Intergenerational Discounting and Inequality

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- ▶ Policies today affect present and future generations:
 - ▶ Typically, some gain, other lose from the policy.
- ▶ Examples:
 - ▶ Mitigation of climate change;
 - ▶ Public debt reduction;
 - ▶ Research and development for technological change;
 - ▶ Pension reform.

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- ▶ What policies are better? The answers depend on two aspects of **intergenerational justice**:
 - ▶ The attitude toward earlier vs. later generations—*discounting*;
 - ▶ The attitude toward better-off vs. worse-off generations—*inequality*.

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- ▶ What policies are better? The answers depend on two aspects of **intergenerational justice**:
 - ▶ The attitude toward earlier vs. later generations—*discounting*;
 - ▶ The attitude toward better-off vs. worse-off generations—*inequality*.
- ▶ *But*: The set of attitudes we typically adopt for economic policy evaluation and analysis is quite narrow.

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Focus of the paper

- ▶ We develop a new, large family of theories of intergenerational justice.
- ▶ We show that **discounting and inequality** attitudes are generally independent of each other and **can be disentangled**.

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Focus of the paper

- ▶ We develop a new, large family of theories of intergenerational justice.
- ▶ We show that **discounting and inequality** attitudes are generally independent of each other and **can be disentangled**.
- ▶ This **orthogonality property** holds across a wide variety of cases, and provides significantly more general (tractable) criteria that might be more appropriate for policy assessment and might better match individuals' views.

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Focus of the paper

- ▶ A feature of the most common criteria is that they rely on exponentially-discounted utilitarianism.

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Focus of the paper

- ▶ A feature of the most common criteria is that they rely on exponentially-discounted utilitarianism.
- ▶ The **advantage of our separation** is that we can present a wide variety of cases that go beyond exponential discounting and additive separability.
- ▶ Examples:
 - ▶ Non-constant social impatience;
 - ▶ Positional attitudes to inequality between generations.

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1. We develop “**modular**” theories of intergenerational justice:

- 1.1 *Module 1* exclusively deals with **discounting**;
- 1.2 *Module 2* exclusively deals with **inequality**;
- 1.3 Any theory is fully described by these two modules;
- 1.4 Many new theories emerge;
- 1.5 The most common theories fit our approach.
 - ▶ Exponentially-discounted utilitarianism as special case.

Our contribution

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1. We develop “**modular**” theories of intergenerational justice:
 - 1.1 *Module 1* exclusively deals with **discounting**;
 - 1.2 *Module 2* exclusively deals with **inequality**;
 - 1.3 Any theory is fully described by these two modules;
 - 1.4 Many new theories emerge;
 - 1.5 The most common theories fit our approach.
2. We **axiomatically characterize** the class of modular theories of intergenerational justice.

- ▶ The relationship between discounting and inequality remains unclear in the literature (Stern, 2007; Nordhaus, 2007; Dasgupta, 2008).
- ▶ The literature has focused on **discounting attitudes**:
 - ▶ Pareto efficiency and anonymity cannot be combined (Diamond, 1965); efficiency implies some sort of discounting.
- ▶ Our perspective on **inequality attitudes** is borrowed from the leaky bucket experiment popularized by Okun (1975).

Technical remarks

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Agenda

- ▶ Disentanglement;
- ▶ Insights;
- ▶ Richness and characterization;
- ▶ Conclusion.

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A consumption stream

- We study the problem of ranking bounded **consumption streams**

$$c : T \rightarrow \mathbb{R}_+$$

in infinite continuous time $T \equiv [0, \infty)$. The set of all consumption streams is C .

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A consumption stream

- ▶ We study the problem of ranking bounded **consumption streams**

$$c : T \rightarrow \mathbb{R}_+$$

in infinite continuous time $T \equiv [0, \infty)$. The set of all consumption streams is C .

- ▶ A theory of intergenerational justice is represented by a **social welfare function**

$$W : C \rightarrow \mathbb{R}.$$

We assume that the social welfare function is continuous.

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Change of perspective

- The central step of our analysis is to *transform* consumption streams in “calendar time” T to consumption streams in “equivalent time” $I \equiv [0, 1)$.

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Change of perspective

- ▶ The central step of our analysis is to *transform* consumption streams in “calendar time” T to consumption streams in “equivalent time” $I \equiv [0, 1)$.
 - ▶ The duration of each generation is *expanded/ compressed* up to the point where all consumption has the **same value**.

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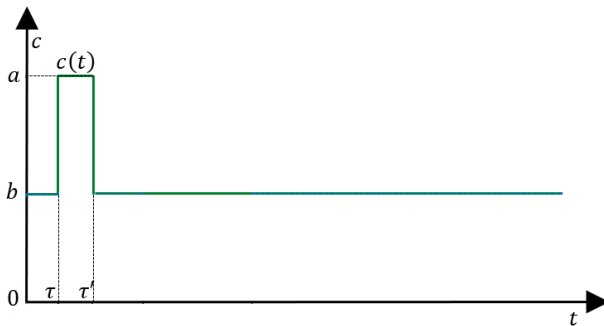
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Discounting and
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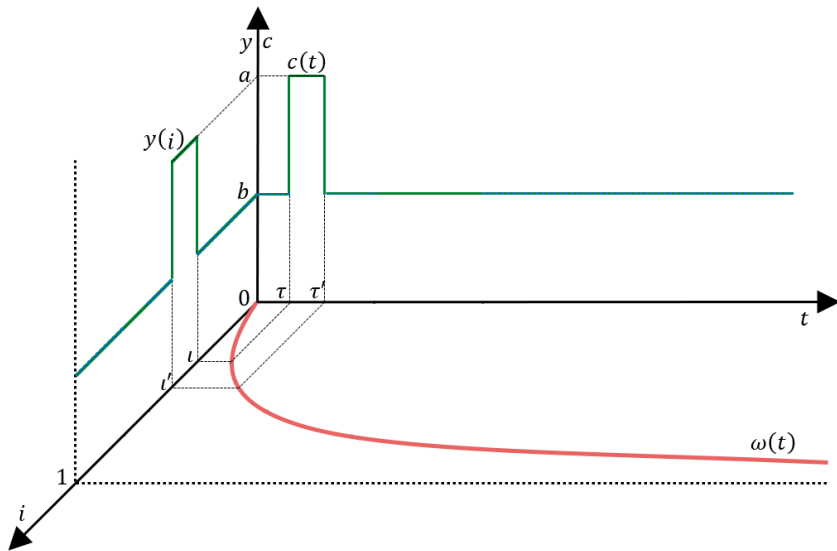
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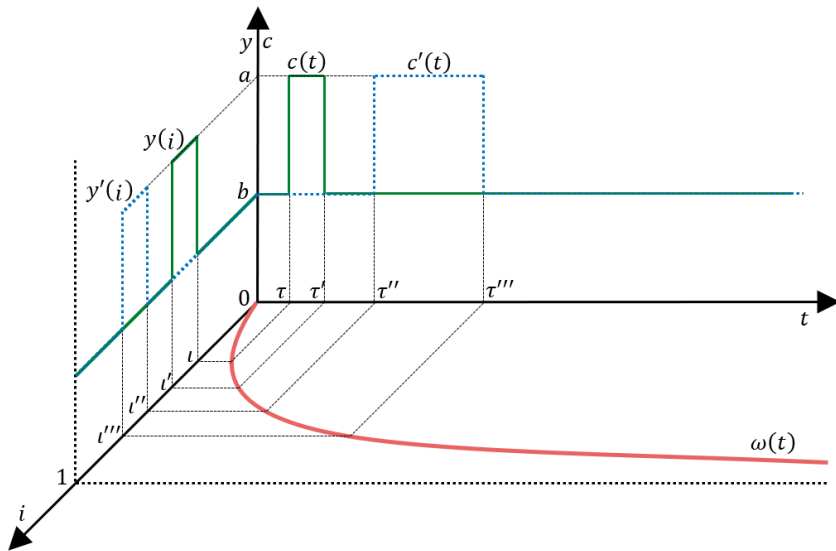
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 - ▶ The duration of each generation is *expanded/ compressed* up to the point where all consumption has the **same value**.

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- ▶ The central step of our analysis is to *transform* consumption streams in “calendar time” T to consumption streams in “equivalent time” $I \equiv [0, 1)$.
 - ▶ The duration of each generation is *expanded/ compressed* up to the point where all consumption has the **same value**.
- ▶ *Anticipating the results:* since each instant of consumption in equivalent time has the same value, we can apply any anonymous inequality averse aggregation of consumption.

The time-discounting function

- ▶ We say that consumption streams admit a representation in **equivalent time**, if there exists a strictly increasing and bounded function $\omega : T \rightarrow I$, such that permuting the transformed consumption is a matter of social indifference.

$$W(c) = W(c') \text{ whenever } c = y(\omega), c' = y'(\omega), y = \Pi y'.$$

- ▶ In other words, all intervals in equivalent time have the same value.

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The time-discounting function

- ▶ We say that consumption streams admit a representation in **equivalent time**, if there exists a strictly increasing and bounded function $\omega : T \rightarrow I$, such that permuting the transformed consumption is a matter of social indifference.

$$W(c) = W(c') \text{ whenever } c = y(\omega), c' = y'(\omega), y = \Pi y'.$$

- ▶ In other words, all intervals in equivalent time have the same value.
- ▶ We call ω the **time-discounting function**.

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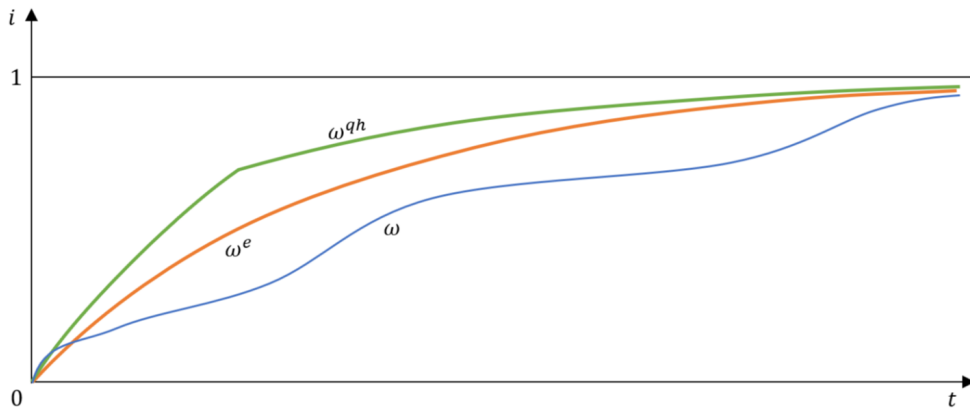
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The time-discounting function



$$\omega^e(t) = 1 - e^{-\rho t}$$

$$\omega^{qh}(t) = \begin{cases} \beta \kappa (1 - e^{-\rho t}), & \text{for } t \in [0, \tau] \\ 1 - \kappa e^{-\rho t}, & \text{for } t > \tau, \end{cases} \text{ with } \kappa = 1/(\beta + (1 - \beta)e^{-\rho \tau})$$

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The aggregator function

- ▶ How to aggregate consumption streams in equivalent time?
- ▶ Given the social welfare function W and the time-discounting function ω , the **aggregator function** $V : Y \rightarrow \mathbb{R}$ immediately follows by setting

$$V(y) = W(y(\omega)).$$

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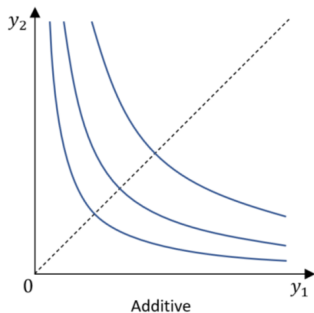
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- ▶ How to aggregate consumption streams in equivalent time?
- ▶ Given the social welfare function W and the time-discounting function ω , the **aggregator function** $V : Y \rightarrow \mathbb{R}$ immediately follows by setting

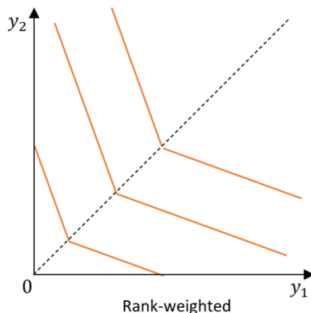
$$V(y) = W(y(\omega)).$$

- ▶ Note that the aggregator function is *anonymous* in equivalent time by definition.
- ▶ Thus, V could be any standard (atemporal) social welfare function.

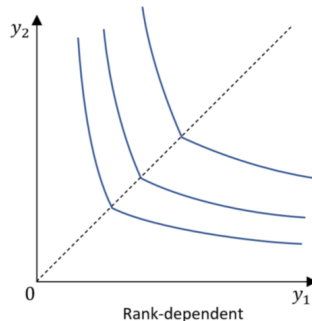
The aggregator function



$$V^a = \int_0^1 \frac{(y_i)^{1-\eta}}{1-\eta} di$$



$$V^{rw} = (\gamma + 1) \int_0^1 (1 - r(i))^\gamma y_i di$$



$$V^{rd} = (\gamma + 1) \int_0^1 (1 - r(i))^\gamma \frac{(y_i)^{1-\eta}}{1-\eta} di$$

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The modular social welfare function

Discounting and
Inequality

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- ▶ We say that W is **modular** if consumption streams can be represented in equivalent time and, thus, there exist an ω and V induced by W .
- ▶ Importantly, the most common welfare criteria adopted in the literature are modular.

Example formulas

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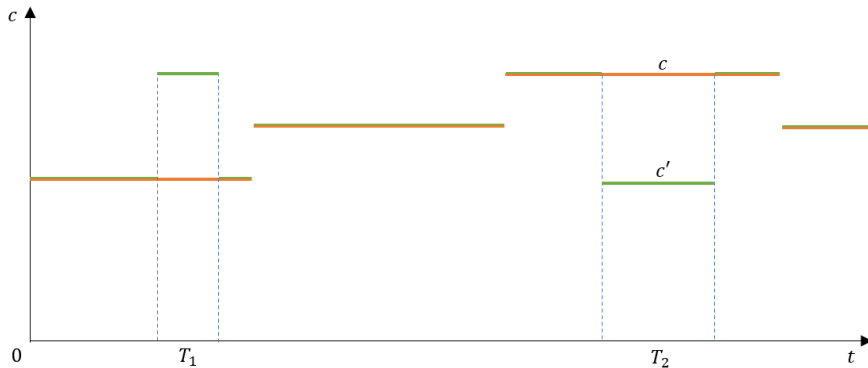
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Social impatience

- Consider c and c' as represented and assume $W(c) = W(c')$. Then W exhibits **social impatience** if T_2 is longer than T_1 .



- ▶ Standard definitions of impatience emphasize the preference for advancing rewards and postponing damages (Böhm-Bawerk, 1889; Fisher, 1930; Koopmans, 1960).
- ▶ Our definition emphasizes that discounting is a matter of *contracting/expanding* time.

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- ▶ Standard definitions of impatience emphasize the preference for advancing rewards and postponing damages (Böhm-Bawerk, 1889; Fisher, 1930; Koopmans, 1960).
- ▶ Our definition emphasizes that discounting is a matter of *contracting/expanding* time.
- ▶ The definitions are equivalent when assuming *Pareto efficiency*.

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- **Proposition 1.** Assume W is modular. Then W is social impatient iff the time-discounting function ω is concave.

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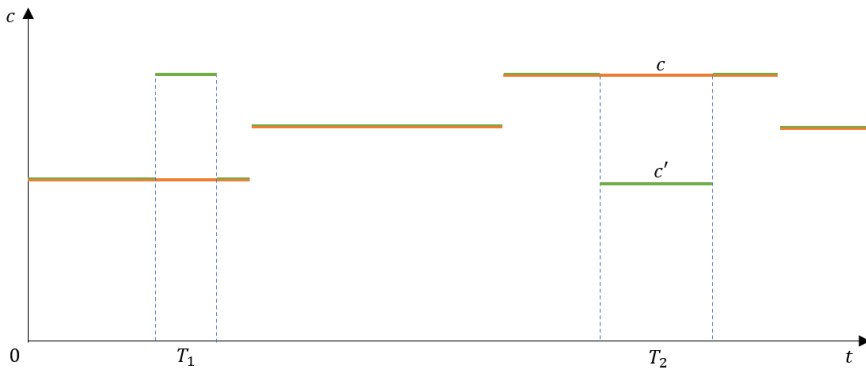
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- ▶ **Proposition 1.** Assume W is modular. Then W is social impatient iff the time-discounting function ω is concave.
- ▶ **Proposition 2.** Assume W^* and W are modular. Then W^* is more social impatient than W iff the time-discounting function ω^* is more concave than ω .
- ▶ **Corollary 1.** If differentiable,

$$\frac{\omega''(t)}{\omega'(t)} \geq \frac{\omega^{*''}(t)}{\omega^{*'}(t)} \text{ for each } t \in T.$$

Inequality aversion

- Assume $W(c) = W(c')$.



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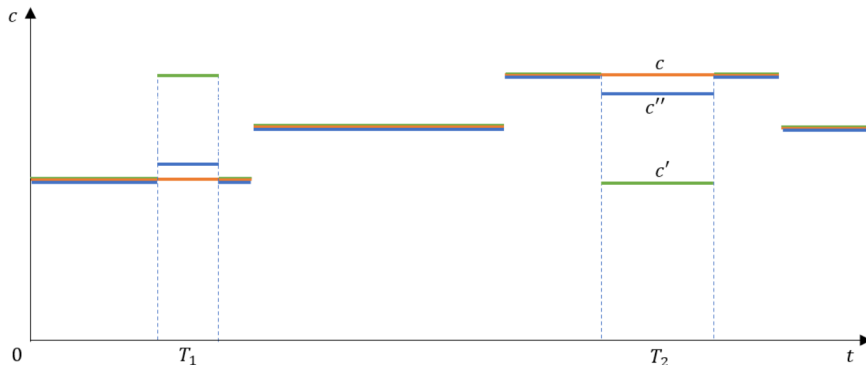
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Inequality aversion

- Assume $W(c) = W(c')$. Then a progressive transfer between T_1 and T_2 increases social welfare and $W(c'') > W(c)$.



Inequality aversion

Discounting and
Inequality

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- ▶ The definition is similar to the Pigou-Dalton transfer principle.
- ▶ However, it first controls for “impatience” by checking indifference.

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- ▶ The definition is similar to the Pigou-Dalton transfer principle.
- ▶ However, it first controls for “impatience” by checking indifference.
- ▶ Our definition avoids taking a stand on discounting.
- ▶ The standard Pigou-Dalton principle emerges if there is no discounting.

- **Proposition 3.** Assume W is modular and Paretian. Then W is inequality averse iff the aggregator function V satisfies SOSD.

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- ▶ **Proposition 3.** Assume W is modular and Paretian. Then W is inequality averse iff the aggregator function V satisfies SOSD.
- ▶ **Proposition 4.** Assume W^* and W are modular and Paretian. Then W^* is more inequality averse than W iff the local inequality aversion (LIA) of V^* is always at least as large as that of V .
- ▶ *LIA* measures the marginal willingness to transfer income across two disjoint equally-sized intervals of equivalent time. Inequality aversion implies $LIA > 0$:
 - ▶ Remarks: It works also without differentiability, as needed by rank-dependent criteria (Chew and Mao, 1995).

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Some modular theories of intergenerational justice

Discounting and
Inequality

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W	V	Additive V^a	Rank-weighted V^{rw}	Rank-dependent V^{rd}
ω		$\int_0^1 f(y_i) di$	$\int_0^1 g(r(i)) y_i di$	$\int_0^1 g(r(i)) f(y_i) di$
Exponential ω^e	$1 - e^{-\rho t}$	EDU	NEW	
Quasi-hyperbolic ω^{qh}	$\begin{cases} \beta k(1 - e^{-\rho t}) & t \leq \tau \\ 1 - k e^{-\rho t} & t > \tau \end{cases}$			
General ω				

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Decomposition: discounting, efficiency, equity

Discounting and
Inequality

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- **Proposition 5.** Assume W is modular and Paretian. Then, the following decomposition emerges:

$$W = \mu(y) \cdot E(y).$$

- Here, μ is the mean, $y(\omega) = c$, and E is a measure of equity (or 1 minus inequality), building on Atkinson (1970).

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Decomposition: discounting, efficiency, equity

- ▶ **Proposition 5.** Assume W is modular and Paretian. Then, the following decomposition emerges:

$$W = \mu(y) \cdot E(y).$$

- ▶ Here, μ is the mean, $y(\omega) = c$, and E is a measure of equity (or 1 minus inequality), building on Atkinson (1970).
- ▶ Thus, social welfare can be decomposed in two main aspects:
 - ▶ *the time attitude* (ω),
 - ▶ *the efficiency-equity trade-off* ($\mu \cdot E$).

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Appraisal: generalized Ramsey rule

- ▶ **Proposition 6.** Assume W is modular, Paretian and differentiable. Then, the social discount rate follows:

$$SDR_t = \frac{1}{t} \ln(\omega'(0)/\omega'(t)) + \frac{1}{t} \ln(ILIA_t(y)).$$

- ▶ The first term captures the “pure time preference”, embodied by the time-discounting function.
- ▶ The second term captures social attitudes to inequality ($ILIA_t$ is instantaneous LIA).

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Appraisal: generalized Ramsey rule

- ▶ **Proposition 6.** Assume W is modular, Paretian and differentiable. Then, the social discount rate follows:

$$SDR_t = \frac{1}{t} \ln(\omega'(0)/\omega'(t)) + \frac{1}{t} \ln(ILIA_t(y)).$$

- ▶ The first term captures the “pure time preference”, embodied by the time-discounting function.
- ▶ The second term captures social attitudes to inequality ($ILIA_t$ is instantaneous LIA).
- ▶ As a special case, the standard Ramsey rule obtains: $SDR_t = \rho + \eta g_t^C$.

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- ▶ The first term captures the “pure time preference”, embodied by the time-discounting function.
- ▶ The second term captures social attitudes to inequality ($ILIA_t$ is instantaneous LIA).
- ▶ As a special case, the standard Ramsey rule obtains: $SDR_t = \rho + \eta g_t^c$.
- ▶ For parametric rank-dependent families: $SDR_t^{erd} = \rho + \eta g_t^c + \gamma g_t^r$.

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Characterization: nested time split

- We axiomatically characterize the disentanglement between intergenerational discounting and inequality (all modular theories of justice) by imposing the novel axiom of **nested time split**.

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Characterization: nested time split

- ▶ We axiomatically characterize the disentanglement between intergenerational discounting and inequality (all modular theories of justice) by imposing the novel axiom of **nested time split**.
- ▶ It requires that **any interval of calendar time** can be divided into “equally valuable” sub-intervals.
- ▶ *Intuitively*: two sub-intervals are equally valuable when permuting their constant consumption is a matter of social indifference.
- ▶ The axiom also requires that the split is independent of the specific consumption stream considered.

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Characterization: nested time split

- A social welfare function W satisfies **nested time split** if for each $\overline{T} \subseteq T$ there exists a partition of $\overline{T}$ in two time intervals $T_1, T_2 \subset \overline{T}$ with $T_1 \cup T_2 = \overline{T}$ such that, for each consumption stream $c \in C$ and each pair of consumption levels $\underline{c}, \bar{c} \in \mathbb{R}_+$,

$$W(\underline{c}_{T_1}, \bar{c}_{T_2}, c_{T/\overline{T}}) = W(\bar{c}_{T_2}, \underline{c}_{T_1}, c_{T/\overline{T}}).$$

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- ▶ A social welfare function W satisfies **nested time split** if for each $\bar{T} \subseteq T$ there exists a partition of $\bar{T}$ in two time intervals $T_1, T_2 \subset \bar{T}$ with $T_1 \cup T_2 = \bar{T}$ such that, for each consumption stream $c \in C$ and each pair of consumption levels $\underline{c}, \bar{c} \in \mathbb{R}_+$,

$$W(\underline{c}_{T_1}, \bar{c}_{T_2}, c_{T/\bar{T}}) = W(\bar{c}_{T_2}, \underline{c}_{T_1}, c_{T/\bar{T}}).$$

- ▶ **Theorem 1.** Assume W is Paretian. W is modular iff W satisfies nested time split.
- ▶ **Corollary 2.** Assume W is Paretian. If W satisfies nested time split, ω is unique and V is unique up to an order-preserving transformation.

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- ▶ We propose modular theories of intergenerational justice.
- ▶ Distinctive features:
 - ▶ It builds on and highlights a variety of attitudes towards discounting and inequality;
 - ▶ It explores intergenerational justice through the lens of largely debated theories of *intratemporal* distributional justice;
 - ▶ It is very general and flexible, and enriches the toolbox of economic analysis. At the same time, the parametric specifications of these theories remain suitable for investigation of optimal policies.
- ▶ Estimation, application, and generalization of the modular theories also open many avenues for future research.

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Thank you!

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- Workhorse theories of intergenerational justice concern parameterization of **exponentially-discounted utilitarianism** (Stern, 2007; Nordhaus, 2007):

$$W^{edu} = \int_0^{\infty} e^{-\rho t} u(c_t) dt.$$

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- ▶ Workhorse theories of intergenerational justice concern parameterization of **exponentially-discounted utilitarianism** (Stern, 2007; Nordhaus, 2007):

$$W^{edu} = \int_0^{\infty} e^{-\rho t} u(c_t) dt.$$

- ▶ Our contribution enriches the economic toolbox of theories of intergenerational justice.
- ▶ We provide **tractable criteria** that can better match individuals' views.

- ▶ The conundrum of **combining efficiency with equality** in infinite horizon settings (Diamond, 1965) has limited the focus to discounting.

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- ▶ The conundrum of **combining efficiency with equality** in infinite horizon settings (Diamond, 1965) has limited the focus to discounting.
- ▶ We clarify that, while “anonymity” rules out discounting, discounting and efficiency are **compatible with** a plethora of inequality attitudes.
 - ▶ The key is to impose anonymity in “equivalent time” rather than in “calendar time”.
- ▶ Our approach can accommodate the call for more discussion of inequality attitudes for intergenerational justice (Nesje et al., 2023).

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- ▶ *Additional detail:* efficiency seems to rule out a continuous and complete inequality averse social welfare function (see Asheim, 2010):
 - ▶ Partial solution: rank utility streams.
 - ▶ Weakenings of anonymity, continuity, or completeness (Chichilnisky, 1996; Basu and Mitra, 2003; Asheim et al., 2012).

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- ▶ *Additional detail:* efficiency seems to rule out a continuous and complete inequality averse social welfare function (see Asheim, 2010):
 - ▶ Partial solution: rank utility streams.
 - ▶ Weakenings of anonymity, continuity, or completeness (Chichilnisky, 1996; Basu and Mitra, 2003; Asheim et al., 2012).
- ▶ A recent survey of the criteria that satisfy some form of anonymity in “calendar time” is Fleurbaey and Pivato (2024).
- ▶ An alternative is to reject efficiency and satisfy anonymity in “equivalent time”. Seminal contribution: Zuber and Asheim (2012).

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Example formulas

- Consider exponentially-discounted utilitarianism:

$$W = \int_0^{\infty} \omega'(t) f(c(t)) dt \propto \int_0^{\infty} e^{-\rho t} f(c(t)) dt,$$

with $\rho > 0$ and f increasing concave.

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Example formulas

- ▶ Consider exponentially-discounted utilitarianism:

$$W = \int_0^{\infty} \omega'(t) f(c(t)) dt \propto \int_0^{\infty} e^{-\rho t} f(c(t)) dt,$$

with $\rho > 0$ and f increasing concave.

- ▶ From our characterization:
 - ▶ The time-discounting function is $i = \omega(t) = 1 - e^{-\rho t}$;
 - ▶ The aggregator function in equivalent time, $V = \int_0^1 f(y(i)) di$, is additive.

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Characterization: generalized modular

- ▶ We now explore the family of “modular” theories of intergenerational justice that emerge by satisfying nested time split, while relaxing Pareto efficiency.
- ▶ Let the **generalized time-discounting function** $\bar{\omega} : T \rightarrow \mathbb{R}_+$ be a continuous and strictly increasing transformation function mapping calendar time into the **generalized equivalent time**, now possibly unbounded.

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- ▶ Let the **generalized time-discounting function** $\bar{\omega} : T \rightarrow \mathbb{R}_+$ be a continuous and strictly increasing transformation function mapping calendar time into the **generalized equivalent time**, now possibly unbounded.
- ▶ **Theorem 2.** Assume $\bar{W}$ satisfies inf-restricted dominance. $\bar{W}$ is modular iff $\bar{W}$ satisfies nested time split.
- ▶ **Corollary 3.** Theorem 2 holds also when inf-restricted dominance is substituted for liminf-restricted dominance.

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Discounting and
Inequality

F. Nesje and
P.G. Piacquadio

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Further axioms: stationarity

- ▶ Next, we look at the relationship between modular theories and **standard axioms** of intergenerational justice.

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Further axioms: stationarity

- ▶ Next, we look at the relationship between modular theories and **standard axioms** of intergenerational justice.
- ▶ A social welfare function W satisfies **stationarity** if for each partition of calendar time $T_1, T_2 \subset T$ with $T_1 \cup T_2 = T$ and each pair of consumption streams $c, c' \in C$ with $c(t) = c'(t)$ for each $t \in T_1$, $W(c) \geq W(c')$ iff $W(c_{T_2}) \geq W(c'_{T_2})$.
- ▶ **Theorem 3.** Assume W is modular and Paretian. Then W satisfies stationarity iff it is exponentially-discounted utilitarian.
- ▶ **Corollary 4.** Assume W is Paretian. Then W satisfies nested time-split and stationarity iff it is exponentially-discounted utilitarian.

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Further axioms: time consistency and time invariance

- ▶ Next, we follow Halevy (2015) and introduce two further properties of intertemporal criteria: **time consistency** and **time invariance**.
- ▶ Rather than imposing restrictions on W , these axioms impose some consistency requirement between social welfare functions at **different calendar times**.

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- ▶ Next, we follow Halevy (2015) and introduce two further properties of intertemporal criteria: **time consistency** and **time invariance**.
- ▶ Rather than imposing restrictions on W , these axioms impose some consistency requirement between social welfare functions at **different calendar times**.
- ▶ **Theorem 4.** Assume $W_{\underline{t}}$, one for each $\underline{t}$, are modular and Paretian. Then $W_{\underline{t}}$, one for each $\underline{t}$, satisfy time consistency iff: i) the time-discounting functions satisfy $\omega_{\underline{t}}(t) = 1 - (1 - \omega(t))/(1 - \omega(\underline{t}))$ for each $\underline{t} \in T$ and each $t \geq \underline{t}$; and ii) the aggregator function is additive and satisfies $V_{\underline{t}} = V^a$ for each $\underline{t} \in T$.
- ▶ **Theorem 5.** Assume $W_{\underline{t}}$, one for each $\underline{t}$, are modular and Paretian. Then $W_{\underline{t}}$, one for each $\underline{t}$, satisfy time invariance iff $W_{\underline{t}} = W_{\underline{t}'}$ for each $\underline{t}, \underline{t}' \in T$.

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