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# Optimization of hydro storage systems and indifference pricing of power contracts

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# Outlook

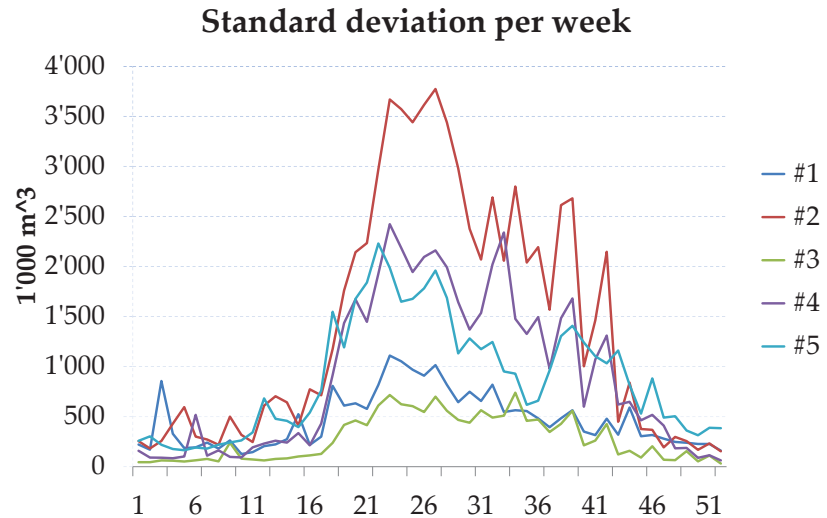
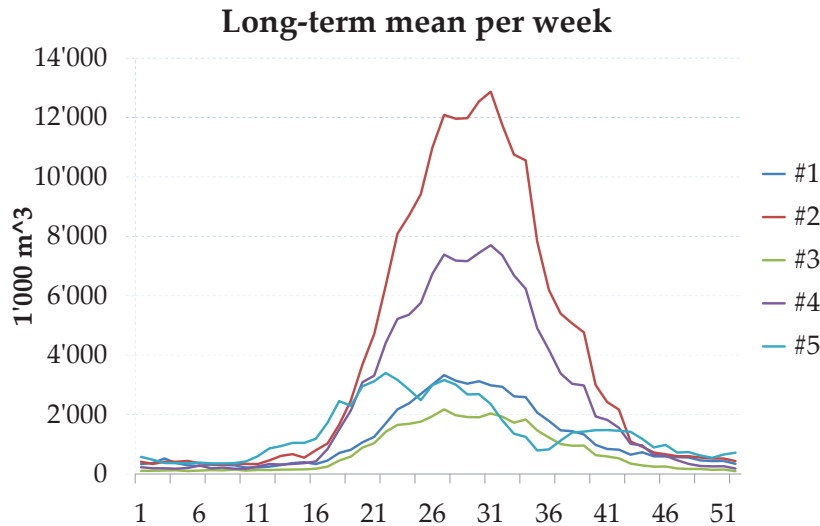
- Producer owns **pumped-storage hydropower plants** (PSHPs). These consist of large (seasonal) connected reservoirs
- We must decide on a production schedule
- We aim at a **mid-term planning model for hydropower production** based on **multistage stochastic programming**
  - Stochastic inflows; stochastic electricity prices
  - Produced electricity is sold at the spot market
- Valuation of non-standard power contracts with indifference pricing (work in progress)
  - Stochastic load profiles
  - Indifference price: entering the contracts with individual consumers or selling only at the spot market

## Inflow data

- Daily inflows are given for **a system with five connected reservoirs** (sample period: 26 years)
- Data are aggregated to weekly inflows
- The inflows show a characteristic seasonal pattern:
  - Winters are dry
  - Inflows are higher in the middle of the year
- Thus, **a seasonal adjustment will be performed**

## Seasonality

- Let  $f_i(\tau)$  be the inflow in reservoir  $i = 1, \dots, 5$  at time  $\tau = 1, \dots, T$
- Let  $\bar{f}_i(w')$  be the long-term mean of inflows,  $w' = 1, \dots, 52$ , and  $\hat{\sigma}_i(w')$  the standard deviation of inflows after seasonal adjustment for all reservoirs over one year
- Allocate each time point to one week  $w'$ :  $\mathcal{T}(w') := \{\tau = 1, \dots, T \mid w(\tau) = w'\}$ 
  - Long-term seasonal adjustment:  $f_i^{adj}(\tau) = f_i(\tau) - \bar{f}_i(w(\tau))$
  - Correct for time-varying volatility:  $f_i^{std}(\tau) = \frac{f_i^{adj}(\tau)}{\hat{\sigma}_i(w(\tau))}$



## Principal component analysis

- A principal component analysis is performed on the stochastic component of inflows
- According to the eigenvalue criterion, only the first factor should be considered
- Then 75% of the variation can be explained; with 2(3) factors we explain about 86(93) percent of the variation

Table 1: Principal component analysis

| Component | Eigenvalue | % of variance | Cumulative % |
|-----------|------------|---------------|--------------|
| 1         | 3.744      | 74.9          | 74.9         |
| 2         | 0.573      | 11.5          | 86.3         |
| 3         | 0.371      | 7.4           | 93.8         |
| 4         | 0.279      | 5.6           | 99.3         |
| 5         | 0.033      | 0.7           | 100.0        |

## Spot model for electricity prices

The formulation of the regime-switching model reads:

$$X_t = \begin{cases} \ell_t^L - \xi_t^-, & \text{if the system is in the lower spike regime,} \\ s_t \cdot \exp(r_t), & \text{if the system is in the base regime or} \\ \ell_t^U + \xi_t^+, & \text{if the system is in the upper spike regime.} \end{cases} \quad (1)$$

$$dr_t = -\kappa_\alpha r_t dt + \sigma_\alpha dZ_t \quad (2)$$

$$\ell_t^L = s_t / \exp(a_\gamma^L), \quad \ell_t^U = s_t \cdot \exp(a_\gamma^U). \quad (3)$$

$$\xi_t^- \sim \text{Wei}(\lambda_\beta^-, k_\beta^-), \quad \xi_t^+ \sim \text{Wei}(\lambda_\beta^+, k_\beta^+)$$

$X_t$  spot price in hour  $t$

$s_t$  seasonality shape (deterministic component)

$r_t$  stochastic component of the spot price in the base regime

$\xi_t^+$  upward spike

$\xi_t^-$  downward spike

$\ell_t^U$  upper limit of the base regime in hour  $t$

$\ell_t^L$  lower limit of the base regime in hour  $t$

## Transition probabilities and results

- The probabilities of remaining in the current regime or switching to another one from one hour to the next are modeled by a (time-dependent) transition matrix:

$$\Pi_{\delta} = \begin{pmatrix} 1 - \pi_{\delta}^{LB} & \pi_{\delta}^{LB} & 0 \\ \pi_{\delta}^{BL} & 1 - \pi_{\delta}^{BL} - \pi_{\delta}^{BU} & \pi_{\delta}^{BU} \\ 0 & \pi_{\delta}^{UB} & 1 - \pi_{\delta}^{UB} \end{pmatrix}. \quad (4)$$

where  $\delta := \delta(t)$  is a function that maps  $t$  to a set of indices (seasonality)

- Estimation procedure: **two-step maximum likelihood**

## Estimation results

Estimation sample: Hourly EEX spot prices from 1 Jan 2009 to 31 Dec 2013

| Param.  | $a_{\gamma}^L$ | $a_{\gamma}^U$ | $\kappa_{\alpha}$ | $\sigma_{\alpha}$ | $\lambda_{\beta}^{-}$ | $k_{\beta}^{-}$  | $\lambda_{\beta}^{+}$ | $k_{\beta}^{+}$  | $E(\xi_t^{-})$ | $E(\xi_t^{+})$ |
|---------|----------------|----------------|-------------------|-------------------|-----------------------|------------------|-----------------------|------------------|----------------|----------------|
| Overall | 0.697          | 0.875          | 2.848<br>(0.05)   | 0.566<br>(0.001)  | 0.127<br>(0.005)      | 0.737<br>(0.01)  | 0.09<br>(0.018)       | 0.798<br>(0.101) | 9.493          | 12.681         |
| Summer  | 0.649          | 0.894          | 3.045<br>(0.075)  | 0.555<br>(0.002)  | 0.184<br>(0.008)      | 0.936<br>(0.015) | 0.105<br>(0.042)      | 1.326<br>(0.899) | 5.591          | 8.759          |
| Winter  | 0.89           | 1.054          | 2.538<br>(0.057)  | 0.606<br>(0.002)  | 0.101<br>(0.006)      | 0.646<br>(0.015) | 0.065<br>(0.027)      | 1.134<br>(0.542) | 13.657         | 14.771         |

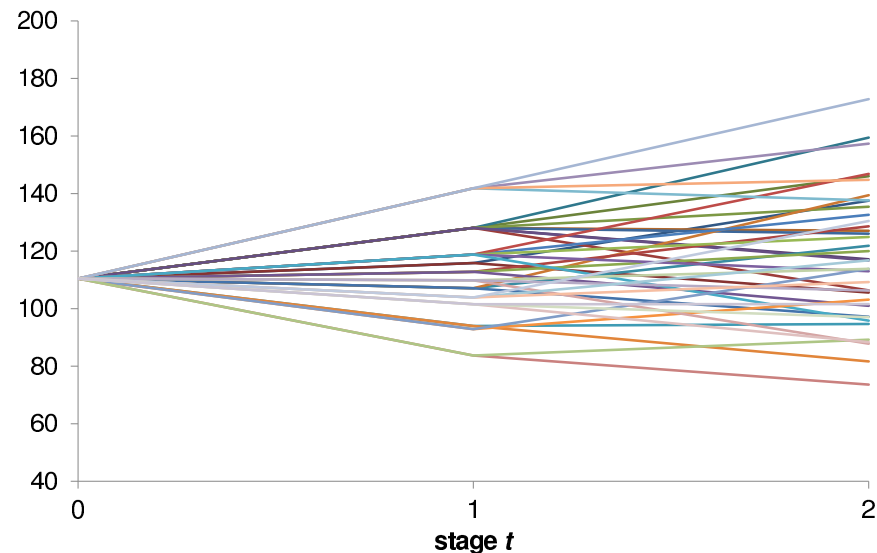
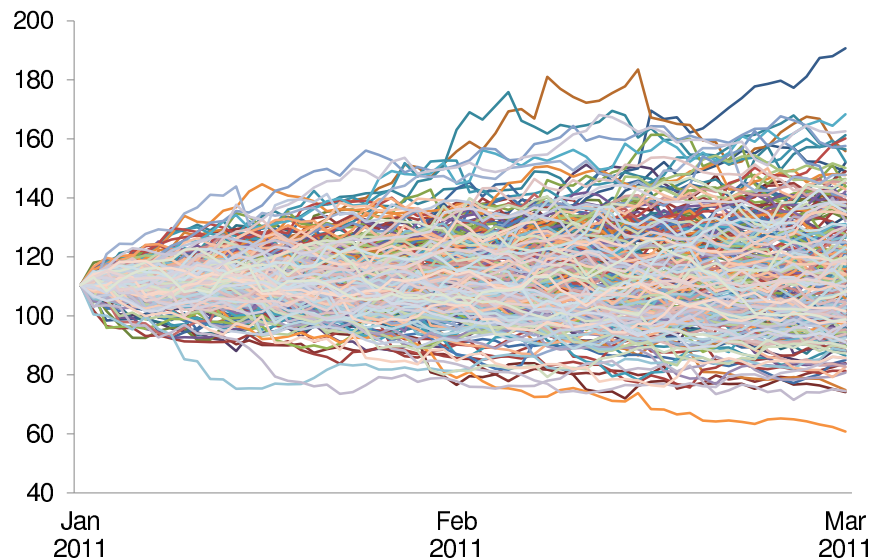
Table 2: Estimation results#1 (standard errors in parenthesis).  $E(\xi_t^{+/-})$  shows the expected value of upwards/downwards spike magnitudes

| Param.      | $\pi_{\delta}^{BL/LB}$ | $\pi_{\delta}^{BU/UB}$ | #base | #lower | #upper | loglike |
|-------------|------------------------|------------------------|-------|--------|--------|---------|
| Overall     | 1.053%<br>24.061%      | 0.078%<br>73.333%      | 41937 | 1837   | 45     | 27156   |
| Summer Base | 1.631%<br>28.800%      | 0.015%<br>66.666%      | 21130 | 826    | 4      | 27338   |
| Summer Peak | 0.193%<br>22.368%      | 0.013%<br>100%         |       |        |        |         |
| Winter Base | 1.309%<br>24.386%      | 0.007%<br>100%         | 21064 | 788    | 7      |         |
| Winter Peak | 0.078%<br>12.631%      | 0.064%<br>80%          |       |        |        |         |

Table 3: Estimation results#2 (standard errors in parenthesis)

## Generating scenarios

- A scenario tree approximates the underlying stochastic process for the risk factors
- The Wasserstein distance is employed to identify the best scenario approximation
- We generate scenarios in weekly steps for
  - the stochastic component of the spot price process for the base regime
  - two principal components of the inflow model
- An initial tree is generated with the procedure in Hochreiter/Pflug (2007); it is improved using the stochastic approximation algorithm in Pflug/Pichler (2015) (*implementation ongoing*)



## Time aggregation: price level approach

- The valuation of non-standard power contracts requires an hourly time resolution
- But: The computational effort would explode since scenario trees grow exponentially with the number of stages
- Thus, an **aggregation technique** is required!
- We apply the price level approach (a.k.a. occupation times):
  - Given the realizations of electricity price levels at weekly steps, the distributions of the stochastic price component at each hour of the upcoming week is considered
  - These distributions are approximated by discrete ones
  - Then the deterministic components are added
  - The outcomes of all hours are collected in one sample, then their number is reduced with an aggregation technique

## Optimization model prerequisites

- Notation:

$\mathcal{I} = \{1, \dots, I\}$  set of reservoirs

$\mathcal{J} = \{1, \dots, J\}$  set of connections between reservoirs

$\mathcal{T} \subset \mathcal{J}$  set of turbines

$\mathcal{P} \subset \mathcal{J}$  set of pumps

$\mathcal{W} \subset \mathcal{J}$  set of spillways

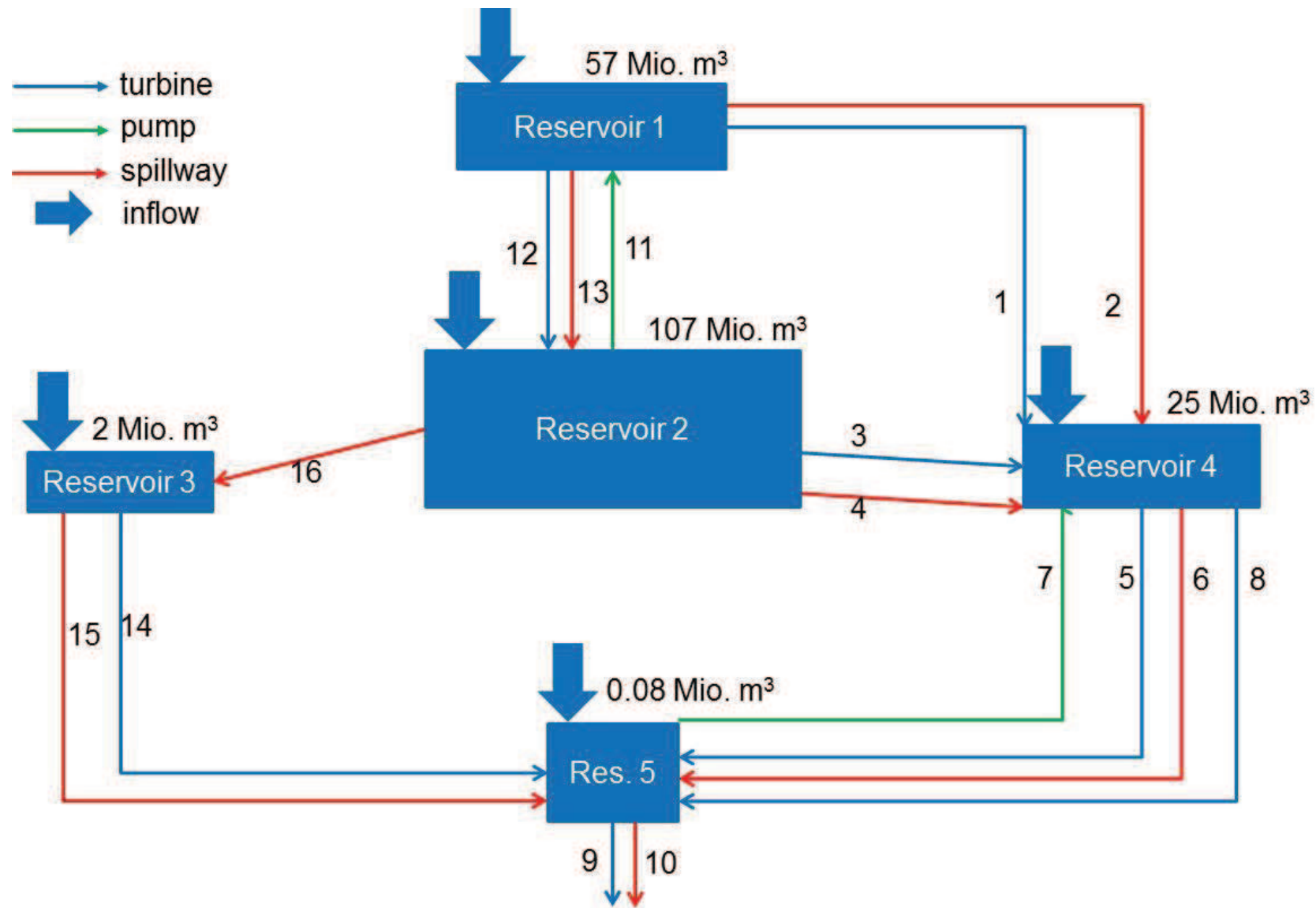
$\mathcal{L} = \{1, \dots, L\}$  price level index set

$\mathcal{N}_t$  set of scenario tree nodes at stage  $t = 0, \dots, T$

- The system topology is described by an incidence matrix  $A$  such that

$$A_{ji} = \begin{cases} 1 & \text{if water flows into reservoir } i \text{ over connection } j, \\ -1 & \text{if water flows out of reservoir } i \text{ over connection } j, \\ 0 & \text{if connection } j \text{ is not related to reservoir } i. \end{cases}$$

## Reservoirs layout



## System parameters

- **System parameters (deterministic):**

$\bar{s}_i$  maximum storage volume (minimum is zero) in  $\text{m}^3$

$g_i$  minimum storage level (end of planning horizon) in  $\text{m}^3$

$\eta_j$  production efficiency in  $\text{MWh}/\text{m}^3$  (positive for turbines, negative for pumps)

$\underline{p}_j, \bar{p}_j$  bounds on consumed/produced power in MW

- **System parameters (stochastic):**

$S_{n\ell}$  spot price at level  $\ell$  in node  $n$  of the scenario tree

$F_{in}$  inflow in reservoir  $i$  in node  $n$  in  $\text{m}^3$

- **Further definitions:**

$\Delta_n$  length of planning period beginning in node  $n$  (in hours)

$\pi_n$  probability of node  $n$

$\pi_{n\ell}$  probability of price level  $\ell$  in node  $n$  (index set:  $\mathcal{L}$ )

$n^-$  predecessor of node  $n > 0$  (root has index 0)

## Optimization model (1)

- For each price level  $\ell$  at each node  $n$  the producer decides how much water  $q_{jn\ell}$  flows over connection  $j$  (in  $\text{m}^3/\text{h}$ )
- This corresponds to the produced power  $p_{jn\ell}$  (in MW) and energy  $e_{jn\ell}$  (in MWh) over the period:

$$p_{jn\ell} = \eta_j q_{jn\ell}, \quad (5)$$

$$e_{jn\ell} = p_{jn\ell} \pi_{n\ell} \Delta_n \quad (6)$$

- Technical constraints for consumed/produced power:

$$\underline{p}_j \leq p_{jn\ell} \leq \bar{p}_j \quad (7)$$

- The stored water  $s_{in}$  in each reservoir at the beginning of the period is restricted by the reservoir capacity:

$$0 \leq s_{in} \leq \bar{s}_i \quad (8)$$

- It starts from an initial value  $s_{i0}$  and develops over time as:

$$s_{in} = \underbrace{s_{in-}}_{\text{stored water in previous week}} + \underbrace{\sum_{j \in \mathcal{J}} A_{ji} \sum_{\ell \in \mathcal{L}} \pi_{n\ell} q_{jn-\ell} \Delta_n}_{\text{expected flows over turbines, pumps, spillways}} + \underbrace{F_{in}}_{\text{natural inflows}} \quad (9)$$

## Optimization model (2)

- At time  $T$ , the stored water volume has to meet the minimum level:

$$s_{in} \geq \bar{g}_i \quad (n \in \mathcal{N}_T) \quad (10)$$

- We start with an initial cash amount  $c_0$ ; for later time points  $n > 0$  cash evolves according to the budget equation:

$$c_n = \underbrace{R_{n-} c_{n-}}_{\substack{\text{cash in previous week} \\ \text{(bearing interest)}}} + \underbrace{\sum_{j \in \mathcal{T} \cup \mathcal{P}} \sum_{\ell \in \mathcal{L}} e_{jn-\ell} S_{n-\ell}}_{\substack{\text{earnings from} \\ \text{energy trading}}} \quad (11)$$

where  $R_n$  is the interest rate in node  $n$  over the period  $\Delta_n$  (equal for borrowing and investing)

- The objective is defined as a **weighted mix of expectation and average value at risk** in view of the available cash at the end of the planning horizon in  $T$ :

$$\max \lambda \mathbb{E}(\text{"cash in } T") + (1 - \lambda) \text{AVaR}(\text{"cash in } T") \quad (12)$$

- $\lambda$  is a parameter for the risk tolerance

## Optimization model overview

- The complete model in the tree-based notation is (with  $\mathcal{N}$  set of all nodes):

$$\max \lambda \sum_{n \in \mathcal{N}_T} \pi_n c_n + (1 - \lambda) z \quad (13)$$

$$\begin{aligned} \text{s.t.} \quad & z \leq y_0 - \frac{1}{\alpha} \sum_{n \in \mathcal{N}_T} \pi_n y_n \\ & n \in \mathcal{N}_T : \quad c_n - y_0 + y_n \geq 0 \\ & n \in \mathcal{N}_T : \quad y_n \geq 0 \\ & j \in \mathcal{J}; \ell \in \mathcal{L}; n \in \mathcal{N} : \quad p_{jnl} = \eta q_{jnl} \\ & j \in \mathcal{J}; \ell \in \mathcal{L}; n \in \mathcal{N} : \quad e_{jnl} = p_{jnl} \pi_{nl} \Delta_n \\ & j \in \mathcal{J}; \ell \in \mathcal{L}; n \in \mathcal{N} : \quad \underline{p}_j \leq p_{jnl} \leq \bar{p}_j \\ & i \in \mathcal{I}; n \in \mathcal{N} \setminus \{0\} : \quad s_{in} = s_{in-} + \sum_{j \in \mathcal{J}} A_{ji} \sum_{\ell \in \mathcal{L}} \pi_{nl} q_{jn-\ell} \Delta_n + F_{in} \\ & i \in \mathcal{I}; n \in \mathcal{N} \setminus \{0\} : \quad 0 \leq s_{in} \leq \bar{s}_i \\ & i \in \mathcal{I}; n \in \mathcal{N}_T : \quad s_{in} \geq \bar{g}_i \\ & n \in \mathcal{N} \setminus \{0\} : \quad c_n = R_{n-} c_{n-} + \sum_{j \in \mathcal{T} \cup \mathcal{P}} \sum_{\ell \in \mathcal{L}} e_{jn-\ell} S_{n-\ell} \end{aligned}$$

## Indifference pricing

- Consider  $M$  delivery contracts  $m \in \mathcal{M} = \{1, \dots, M\}$  of individual consumers
  - Different demand levels  $D_{mnl}$  for different price levels
  - Electricity is delivered at fixed prices  $K_m$  (EUR/MWh)
- **Indifference pricing:** Determine the smallest possible price such that the producer meets the obligations from the contracts; new mix of expectation and AVaR  $\gamma$  should not be worse than without the contract

$$\min_{K, q, p, e, c, s, h} \sum_{m \in \mathcal{M}} K_m$$

$$\text{s.t. (4) – (10)}$$

$$\text{AVaR-constraints}$$

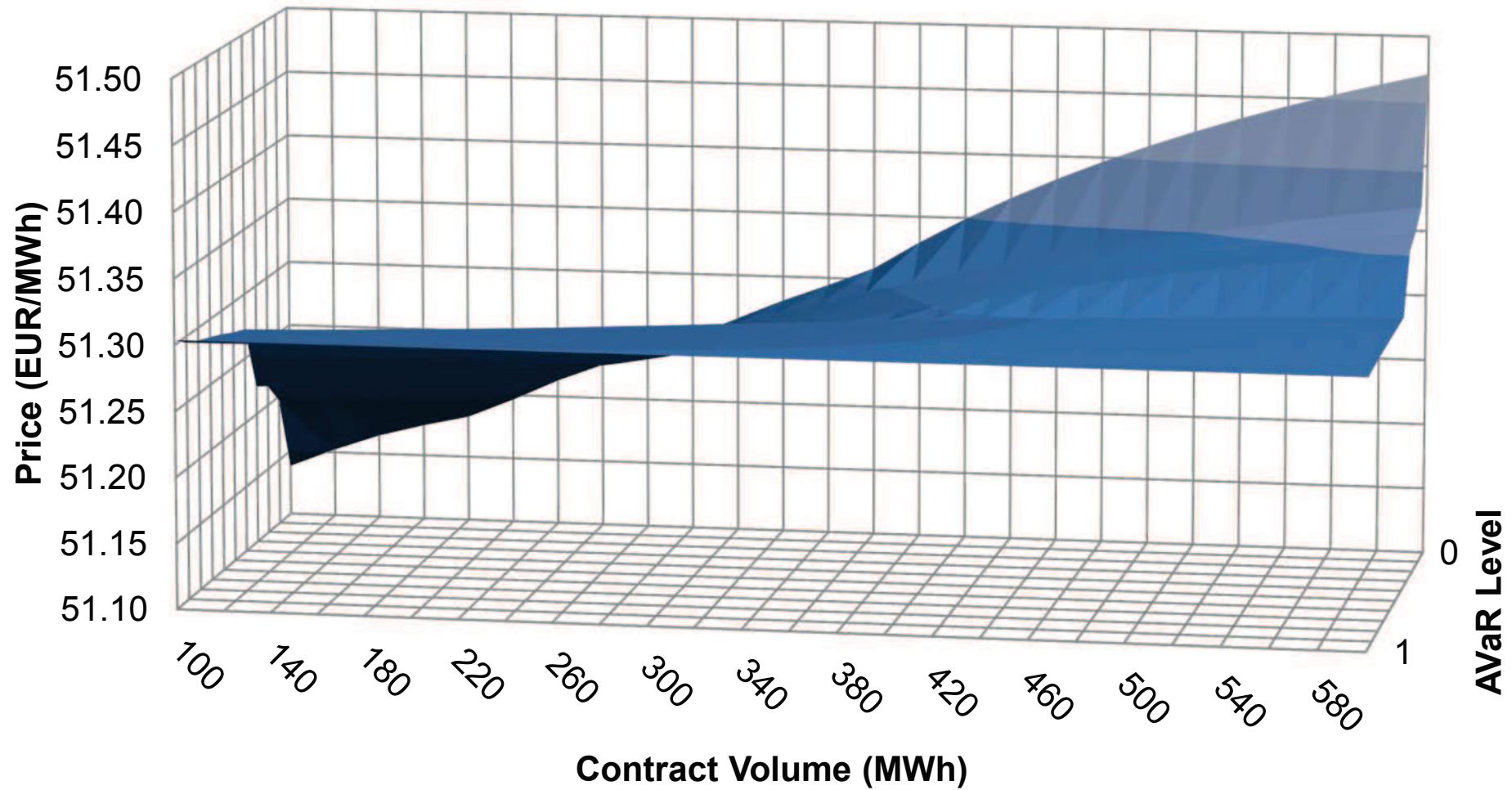
$$\gamma \geq \gamma^*$$

$$\sum_{j \in \mathcal{T} \cup \mathcal{P}} e_{jnl} = h_{nl} + \Delta_n \pi_{nl} \sum_{m \in \mathcal{M}} D_{mnl}$$

$$c_n = R_{n-} - c_{n-} + \sum_{\ell \in \mathcal{L}} S_{n\ell} h_{n\ell} + \Delta_n \sum_{\ell \in \mathcal{L}} \pi_{n\ell} \sum_{m \in \mathcal{M}} K_m D_{mnl}$$

$$s_{in} \geq s_{in}^* \quad (n \in \mathcal{N}_T)$$

## Indifference pricing for a variable load, one contract



## Summary

- Optimization of PSHP operations by multistage stochastic programming with aggregation based on price levels
- Model aims at large reservoirs (seasonal storages)
  - Regime-switching model for electricity spot prices
  - Factor model for inflows, taking into account seasonality
- Valuation of power contracts with indifference pricing
- Open:
  - Modelling stochastic load profiles
  - Extension to several contracts (portfolio effects)
  - Comparison with other approaches