Universität St.Gallen

Choice Under Fundamental Uncertainty: The Case of Aggregate Consumption

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Introduction and Motivation

The Point of Departure

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> *Economists are struggling to forecast how many people who left the workforce in 2020 will eventually return [...] They are also grappling with doubts over when consumers will shift their spending back to services, easing the upward pressure on goods prices caused by bunged-up supply chains.* ***Economic data have become harder to interpret. If retail sales fall, for example, does it reflect economic weakening, or a welcome return to normal patterns of consumption?***
>
> – Economist(2022)

Fundamental Uncertainty (FU)

FU surrounds many questions relevant for economic decision-makers:

- How will AI affect the productivity of labor and capital in the coming 10 years?
- Is the current (U.S.) inflation transitory or permanent? (esp. if asked a year ago)
- Are we in a stagflationary period?

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Decision-makers need to be able to operate under these uncertain conditions.

Fundamental Uncertainty

The Key Questions

How can we model an economy where decision-makers are (partially) ignorant about the data-generating processes?

Is such a model able to replicate real-world patterns in say aggregate consumption?

The Economic Setting

We model a representative agent (RA) maximising lifetime utility

- RA confronted with stochastic stream of labour income at the aggregate level of the economy
- **RA does not know the data-generating process (DGP) of labour income**
- RA's **only** information about the DGP of income is **past experience**
- RA decides consumption (and assets), forms income expectation
- Benchmark: Full information rational expectations (FIRE)
- We simulate c_t, a_t decisions out of the benchmark (FIRE model) and out of the model of fundamentally uncertain agents (FU model) based on empirical income (real US GDP)
- We compare simulated with empirical patterns (correlations, ...)

Let us begin with: Full Information Rational Expectations (FIRE)

Agents choose consumption as to maximise their lifetime utility:

$$\max_{(c_t, c_{t+1}, c_{t+2}, \dots) \in \mathbb{B}_t} \mathbb{E}_t \sum_{h=0}^{\infty} \beta^h u(c_{t+h}) \text{ s.t. standard constraints} \tag{1}$$

The assets evolve according to:

$$(a_t + y_t - c_t)(1 + r) = a_{t+1} \tag{2}$$

Income evolves according to some true DGP (e.g. an AR(1) process) denoted by $f^*(y)$.

Solution (policy function) can be derived from Bellman equation for c_t:

$$V_t(a_t, y_t; f^*(y)) = \max_{c_t} \left\{ u(c_t) + \beta \cdot \mathbb{E}_{t; f^*(y)} \left[V_{t+1}(a_{t+1}, y_{t+1}; f^*(y)) \right] \right\} \tag{3}$$

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This problem cannot be solved without information about the true DGP. What is $E_t V_{t+1}$?

The Model

A Decision-Making Problem Feasible under FU

Our Proposal:

$$V_t(a_t, y_t; \widehat{f}(y)) = \max_{c_t} \left\{ u(c_t) + \omega \cdot V^{FU} \left(a_{t+1} + \mathbb{E}_{t;\hat{f}(y)} \left[y_{t+1} \right] \right) \right\} \tag{4}$$

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Key differences vis-a-vis the FIRE agent's problem:

- The V^{FU} value function is independent of any models about DGP after $t+1$. V^{FU} is a valuation index for estimated next period financial resources.

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- Agents use a "mental model" of the DGP of income, $\widehat{f}(y)$, to make a point forecast for income in $t+1$. Certainty equivalence in V^{FU} results. $\mathbb{E}_{\hat{f}(y),t}\left[y_{t+1}\right] = y^*_{t+1|t}$

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- ω is a weight of importance of the future for the agent.

Interpreting the FU Decision Problem

$$V_t(a_t, y_t, \widehat{f}(y)) = \max_{c_t} \left\{ \log(c_t) + \omega \cdot \log\left(a_{t+1} + y^*_{t+1|t}\right) \right\}$$

This is de-facto a two-period optimisation problem.

Agents face a trade-off between consumption today and their experience-based estimate of the value of resources available in the future.

Alternative way to think about it: Agent anticipates bequests of resources to their own future self. V^{FU} is a valuation index for the estimated bequested resources.

The Expectations under Fundamental Uncertainty: $\mathbb{E}_{t;\hat{f}(y)} = y^*_{t+1|t}$

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The model is able to accommodate a wide range of different forecasts or forecasting rules:

- **Heuristic rule**: simple extrapolation, constant forecasts
- **Empirically measured**: University of Michigan Index of Consumer Expectations, re-scaled
- **Learnt from the experience**: historical analogies (with k−nearest neighbors), regression, ...

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Example of Historical Analogies:

> *It was clear that the Fed needed to do more – but what? In response to current events, people often reach for historical analogies, and this occasion was no exception. The trick is to choose the right analogy. In August 2007, the analogies that came to mind—both inside and outside the Fed—were October 1987, [...], and August 1998, [...]. With help from the Fed, markets had rebounded each time with little evident damage to the economy.*
>
> — Bernanke (2015)

The Solution of FU Agents

$$
\begin{aligned}
V^{FU}(a_t, y_t, \bar{b}_t, \widehat{f}(y)) = \max_{c_t} & \left\{ \log(c_t) + \omega \cdot \log\left(a_{t+1} + y^*_{t+1|t} - \bar{b}^*_{t+1|t}\right) \right\} \quad s.t. \\
& (a_t + y_t - c_t)(1+r) = a_{t+1} \\
& a_t \geq \bar{b}_t
\end{aligned}
\tag{6}
$$

The optimal consumption under FU becomes:

$$
c_t = \frac{1}{1+\omega}\left[a_t + y_t + \left(y^*_{t+1|t} - \bar{b}^*_{t+1|t}\right)/(1+r)\right] \quad \textit{if borrowing-constraint not binding} \tag{7}
$$

Expectations? Propagate with a factor of $((1+\omega)(1+r))^{-1}$ into c_t

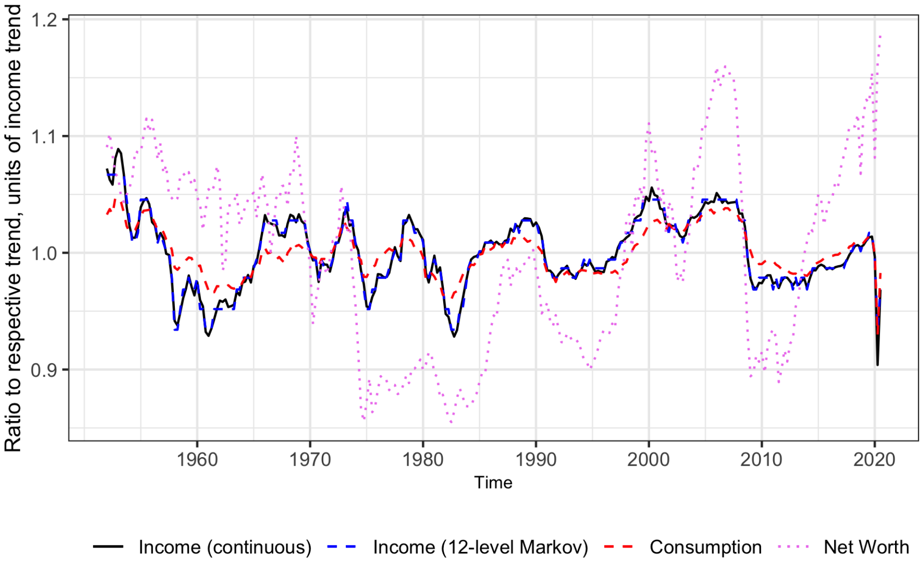
Results

Empirical Series (Income, Consumption, Assets)

- Data: Real U.S. GDP, Real Personal Consumption Expenditures, Net Worth of Households (all quarterly, from FRED)
- Start: Q1 1952, End: Q2 2021
- Trend estimated as cubic smoothing spline with three knots
- Represent y_t, c_t and a_t as ratios of realisations to their respective trend
- For example: $y_t = 1$ means US Real GDP was equal to its estimated trend
- Another example: $c_t = 0.95$ means that US Real PCE were 5% below its trend that quarter
- For RE optimization: y_t modelled to be Markov with 12 states. Transition probabilities estimated by simulating a long series from an estimated AR(1) process of y_t.

The representative agent observes and decides on the "cyclical component" of income.

Empirical Series (Income, Consumption, Assets)



Correlations Between Simulated and Empirical Series

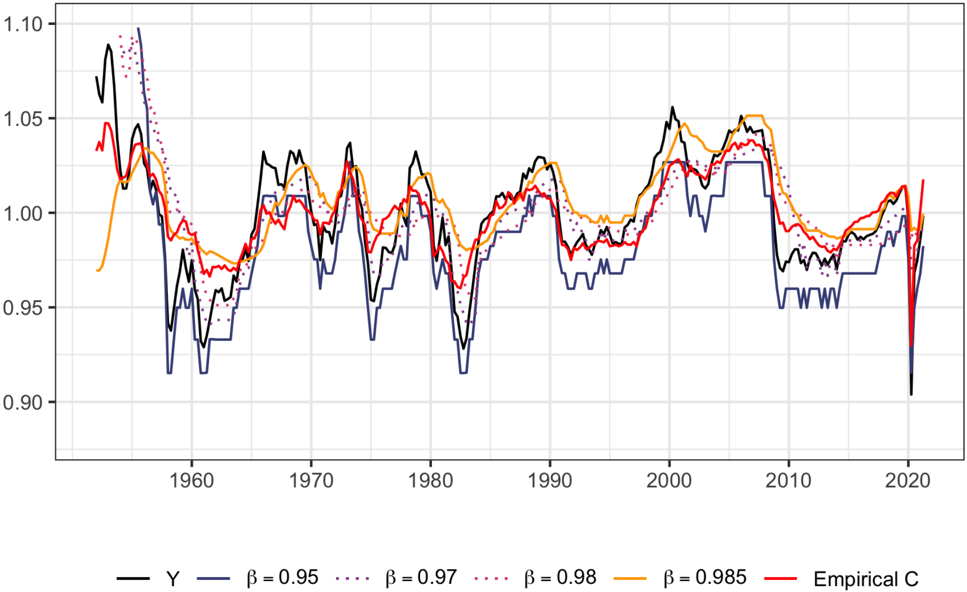
Panel A: FU model

	Historical Analogies		*Constant forcast*		*Simple Extrapolation*		*UoM Survey*	
Omega	Cons	Assets	Cons	Assets	Cons	Assets	Cons	Assets
1.0	0.856	0.047	0.848	0.535	0.872	-0.532	0.792	-0.15
2.0	**0.857**	**0.527**	**0.821**	**0.530**	**0.869**	**0.529**	**0.794**	**0.124**
3.0	0.846	0.525	0.798	0.516	0.857	0.516	0.783	0.247
4.0	0.833	0.496	0.777	0.496	0.843	0.496	0.769	0.288
6.0	0.804	0.446	0.741	0.447	0.814	0.447	0.719	0.261

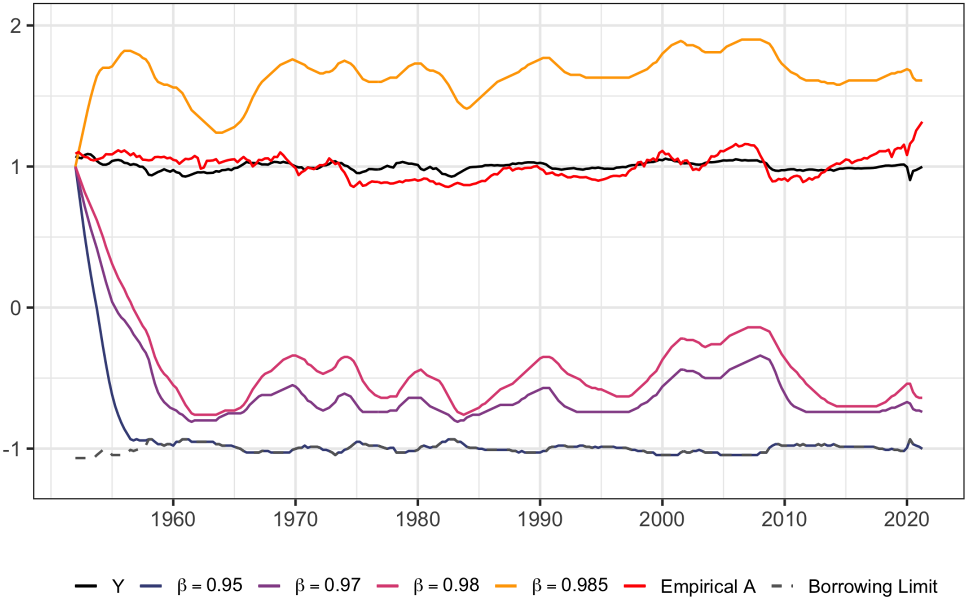
Panel B: FIRE model

Discount factor	Consumption	Assets
0.950	**0.862**	**-0.538**
0.970	0.757	0.396
0.980	**0.694**	**0.251**
0.985	0.785	0.311

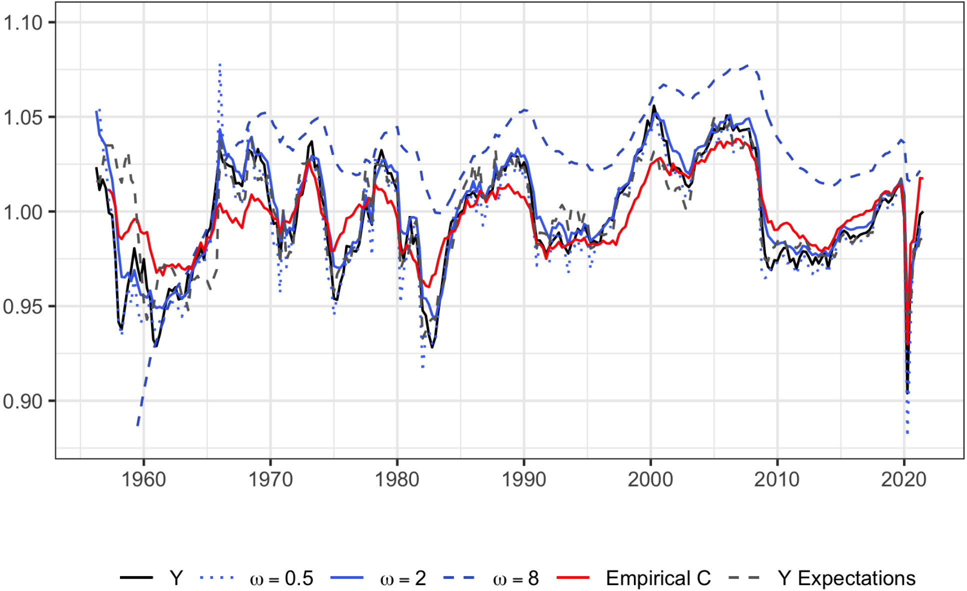
Consumption: Simulated for Benchmark (FIRE) and Empirical



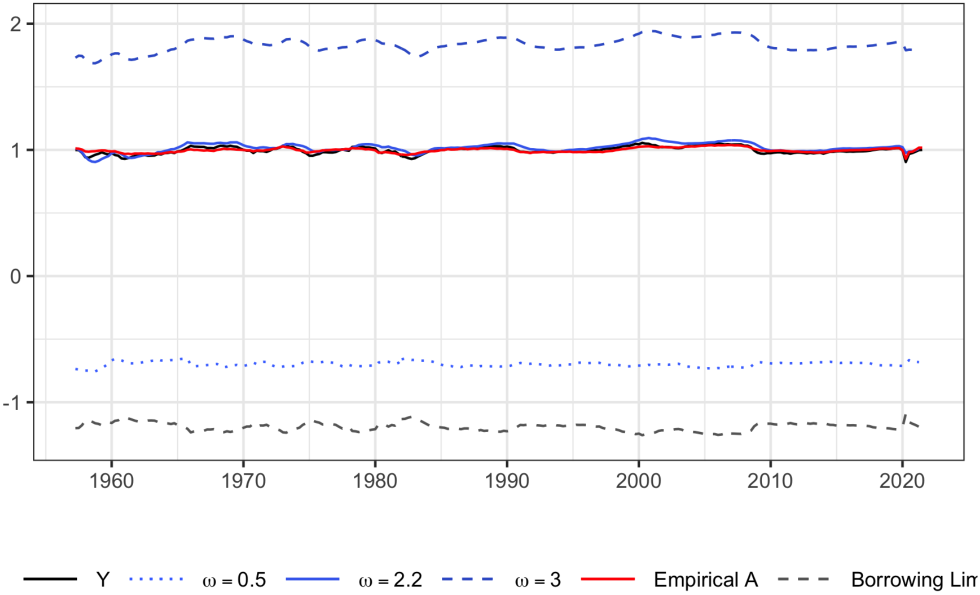
Assets: Simulated for Benchmark (FIRE) and Empirical



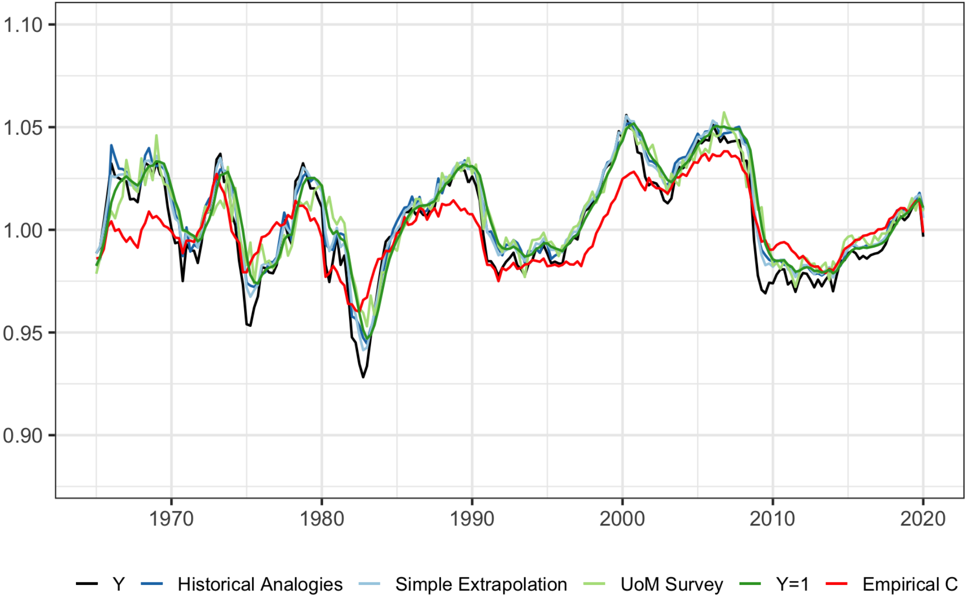
Consumption: Simulated for FU (Expect. from Historical Analogies)



Assets: Simulated for FU (Expectations from Historical Analogies)



Consumption: Simulated for FU; Various Expectations



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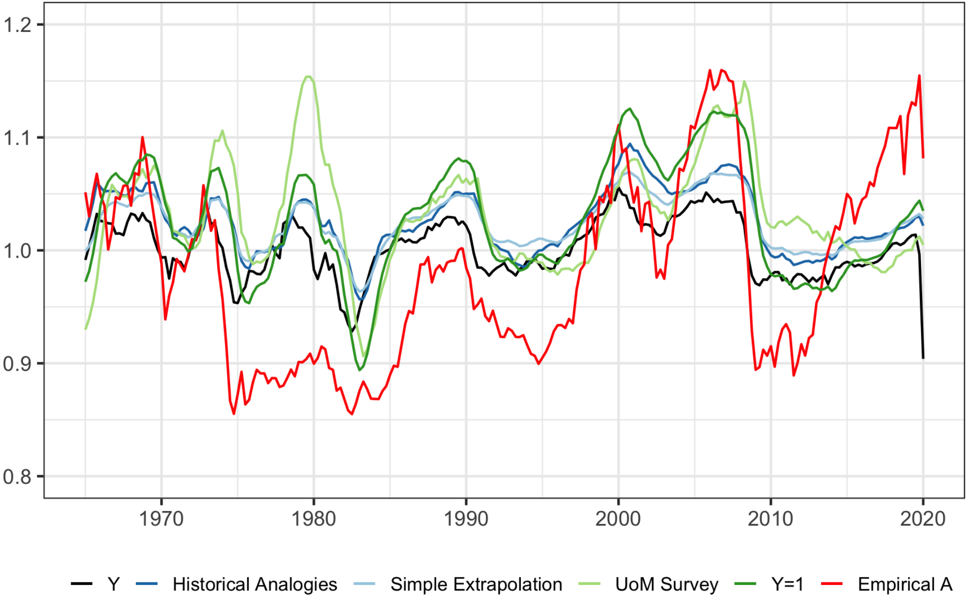

Conclusion

Our Goal Was...

To propose a model of decision-making under FU that

- Stays true to the standard intertemporal optimisation and utility maximisation as much as possible
- Enables the agent to operate with minimal information about the DGP
- Is cognitively and psychologically plausible

We have seen that:

- Forward-looking decisions under FU are possible and can be modelled.
- Even with very limited knowledge about DGPs, agents' decisions can match several first-order facts about consumption and assets quite well and with substantial degree of flexibility.

Much Work Remains...

- General equilibrium (decisions of firms!)
- Further intuition about the model (meaning of risk aversion, importance of expectations, ...)
- Are and if so in what ways are FU agents more "robust" than RE agents?
- Endogenous business cycles

Thank you for your attention

Appendix

Correctness of Income Forecasts

Panel A: FU model				
	Historical Analogies	*Constant forcast*	*Simple Extrapolation*	*UoM Survey*
RMSE	0.193	0.392	0.163	0.469
Panel B: RE model				
RMSE		0.155		

Deriving A Problem Solvable Under Fundamental Uncertainty (FU)

$$V_t(a_t, y_t, f^*(y)) = \max_{c_t} \left\{ u(c_t) + \beta \cdot \mathbb{E}_{f^*(y),t} \left[V_{t+1}(a_{t+1}, y_{t+1}, f^*(y)) \right] \right\}$$

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Approximation

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Simplification

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This can be solved under FU!

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This is an "as-if" two-period optimisation problem. Agents face a trade-off between consumption today and their experience-based estimate of the value of resources available in the future.

Alternative way to think about it: Agent anticipates bequests of resources to their own future self. V^{FU} is a valuation index for the estimated bequested resources.

A Decision-Making Problem Solvable under FU

Our Proposal:

$$V_t(a_t, y_t; \widehat{f}(y)) = \max_{c_t} \left\{ u(c_t) + \gamma \cdot V^{FU} \left(a_{t+1} + \mathbb{E}_{t;\hat{f}(y)} \left[y_{t+1} \right] \right) \right\}$$

V^{FU} and $\widehat{f}(y)$ are approximations of V_{t+1} and $f^*(y)$ that are feasible under unknown true DGP.

This is an "as-if" two-period optimisation problem. Agents face a trade-off between consumption today and their experience-based estimate of the value of resources available in the future.

Alternative way to think about it: Agent anticipates bequests of resources to their own future self. V^{FU} is a valuation index for the estimated bequested resources.

V^{FU} and $\widehat{f}(y)$ are both based on agent's *experience* of the DGP of income.
They are *mental models* of attainable utility from financial resources and of the DGP of income.

The Value Function under Fundamental Uncertainty: V^{FU}

$$V_t(a_t, y_t; \widehat{f}(y)) = \max_{c_t} \left\{ u(c_t) + \gamma \cdot V^{FU} \left(a_{t+1} + \mathbb{E}_{\hat{f}(y),t} \left[y_{t+1} \right] \right) \right\}$$

The following was simplified vis-a-vis the RE case:

- The value function is now independent of the characteristics of the true economic system ("DGP independent"). No need to estimate multiple value functions.
- The argument of the FU value function is a measure of estimated resources available tomorrow (sum of financial resources – assets – and expected income).
- Certainty equivalence in V^{FU}. Confer next slide about expectations.
- Functional form for V^{FU}: logarithm. Could be more general like CRRA. Reasonable to model decreasing marginal value of anticipated bequests.

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- Functional form for V^{FU}: logarithm. Could be more general like CRRA. Reasonable to model decreasing marginal value of anticipated bequests.

Note the difference in the "discount factor". γ is a measure of subjective relative importance of the future for the agent.

The Expectations under Fundamental Uncertainty: $\mathbb{E}_{t;\hat{f}(y)}$

$$V_t(a_t, y_t; \widehat{f}(y)) = \max_{c_t} \left\{ u(c_t) + \gamma \cdot V^{FU} \left(a_{t+1} + \mathbb{E}_{\hat{f}(y),t} [y_{t+1}] \right) \right\}$$

We model agents as making *point forecasts* of next period labour income: $\mathbb{E}_{\hat{f}(y),t} [y_{t+1}] = y^*_{t+1|t}$

Note: Making point forecasts instead of probabilistic ones implies certainty equivalence in V^{FU}.

The model is able to accommodate a wide range of different forecasts or forecasting rules:

- **Heuristic**, **measured** or **learning** based forecasts of income are possible
- Example of heuristic forecasting rule ("this time is different"): simple extrapolation ($y^*_{t+1|t} = y_t$), constant forecasts ($y^*_{t+1|t} = 1$)
- Example of measured expectations: University of Michigan Index of Consumer Expectations, re-scaled
- Proposed learnt forecasting rule: historical analogies (via kNN)

The Solution of FU Agents

$$
\begin{aligned}
V^{FU}\left(a_t, y_t, \bar{b}_t, \widehat{f}(y)\right) = \max_{c_t} & \left\{\log\left(c_t\right) + \gamma \cdot \log\left(a_{t+1} + y^*_{t+1|t} - \bar{b}^*_{t+1|t}\right)\right\} \quad s.t. \\
& \left(a_t + y_t - c_t\right)\left(1+r\right) = a_{t+1} \\
& a_t \geq \bar{b}_t
\end{aligned} \tag{8}
$$

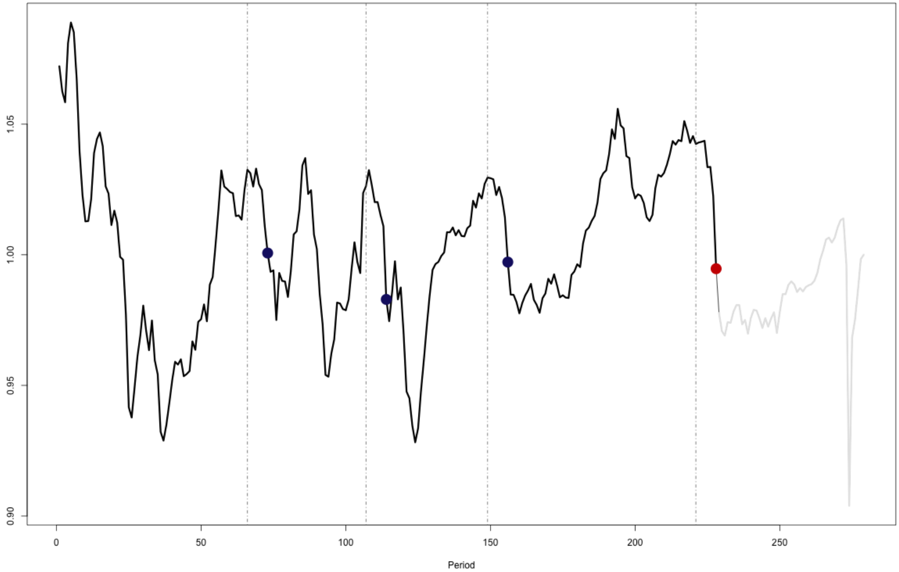
The optimal consumption under FU becomes

$$
c_t = \frac{1}{1+\gamma}\left[a_t + y_t + \left(y^*_{t+1|t} - \bar{b}^*_{t+1|t}\right)/\left(1+r\right)\right] \quad \textit{if borrowing-constraint not binding} \tag{9}
$$

Expectations? Propagate with a factor of $\left(\left(1+\gamma\right)\left(1+r\right)\right)^{-1}$ into c_t

Surprising? finding: The value function substantially more relevant than income expectations in determining the patterns of simulated consumption and assets.

Approximating $\mathbb{E}_t$



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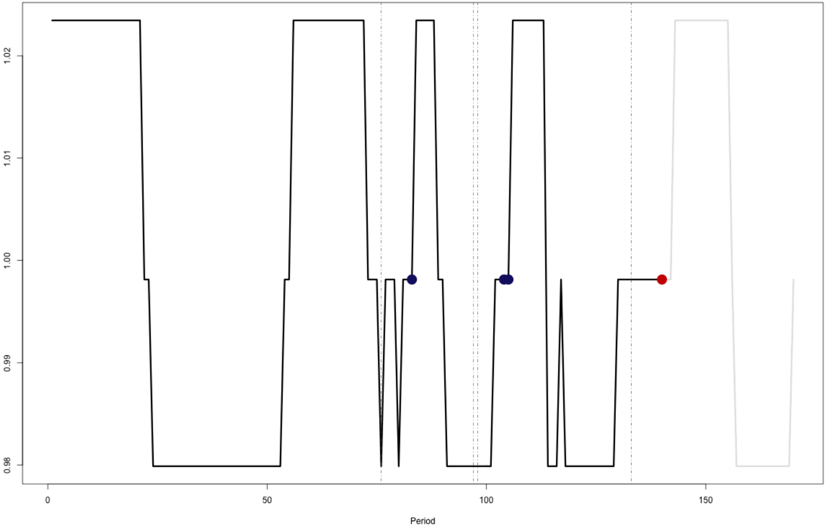

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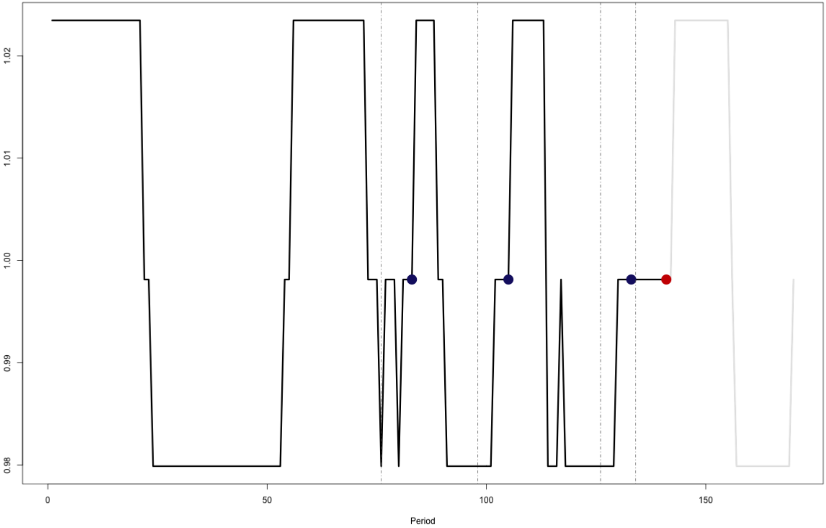

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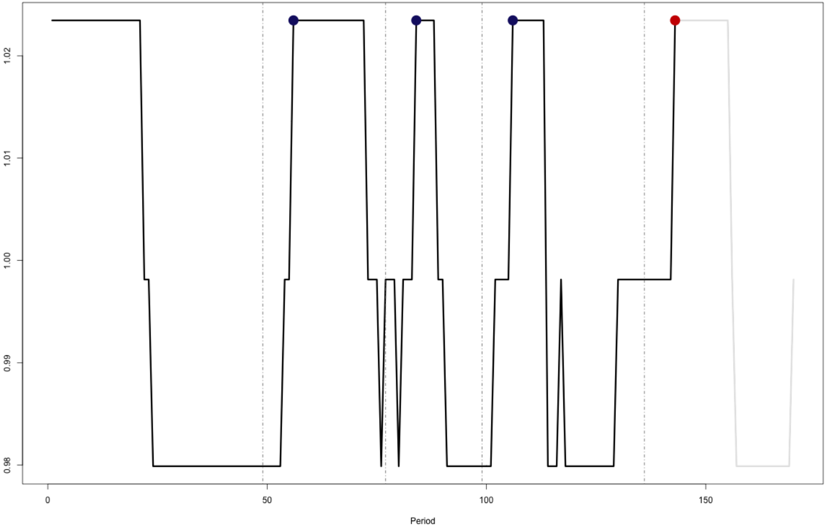

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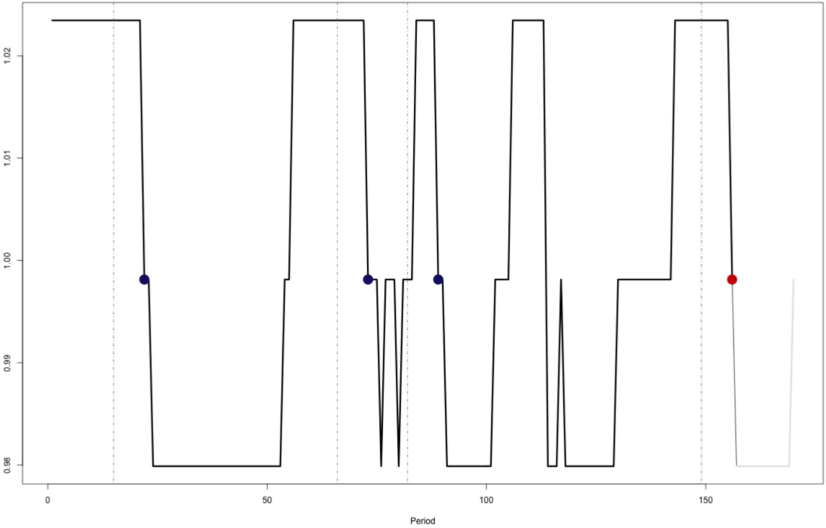

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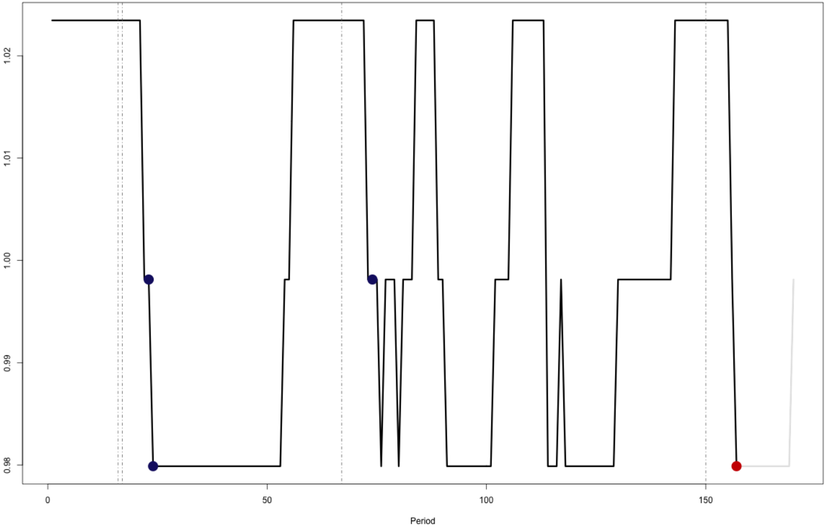

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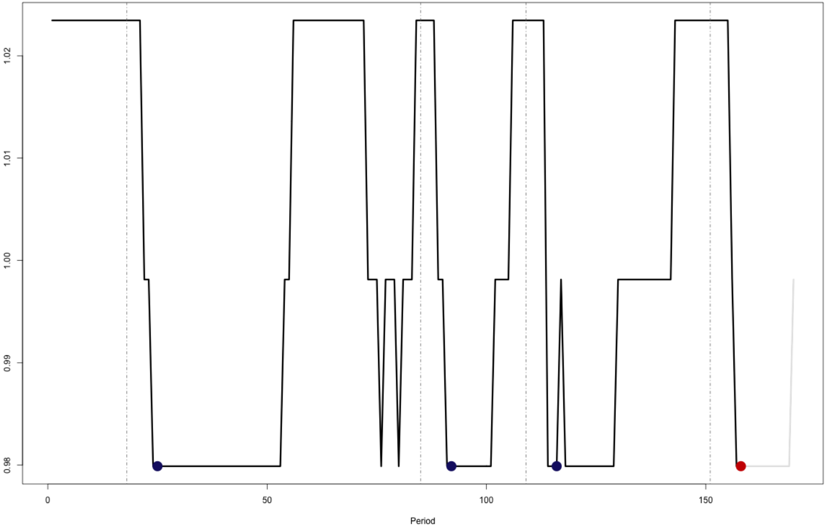

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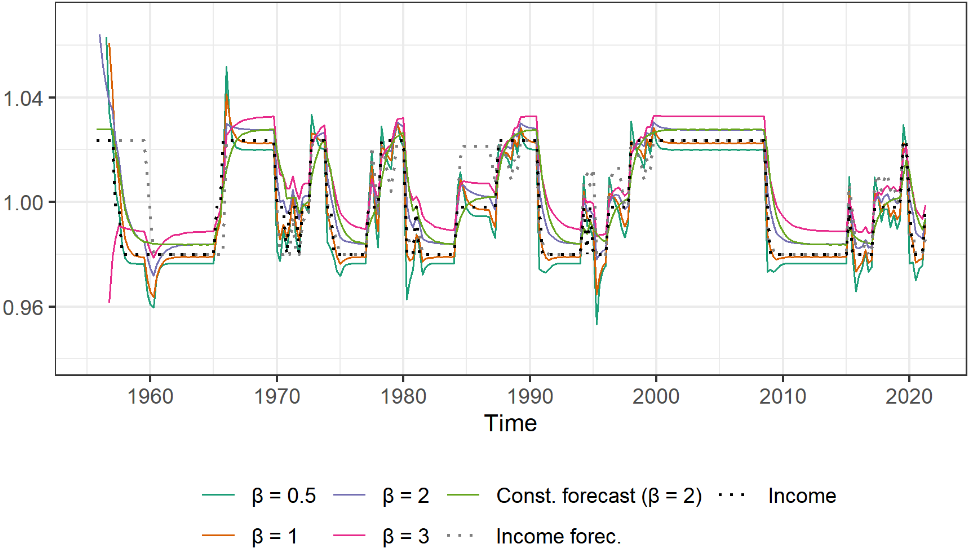

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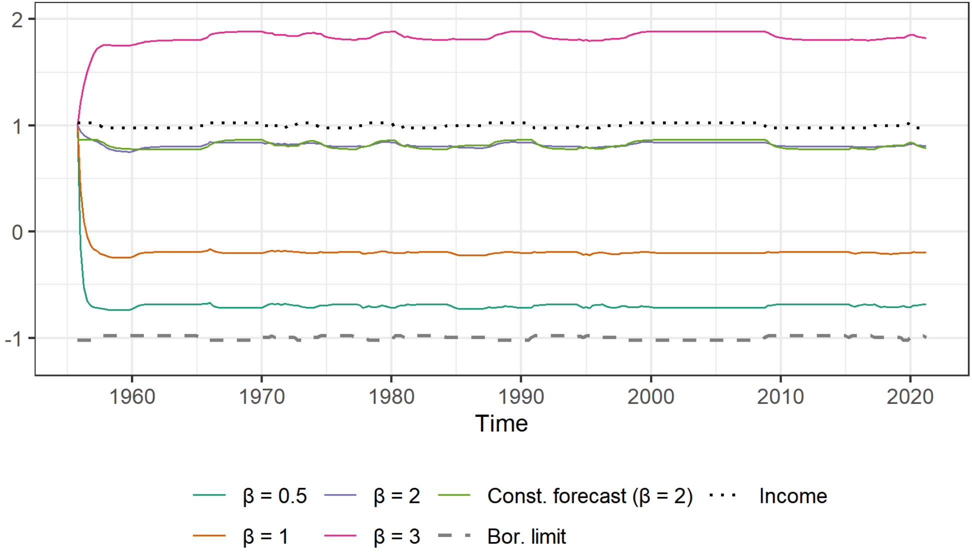

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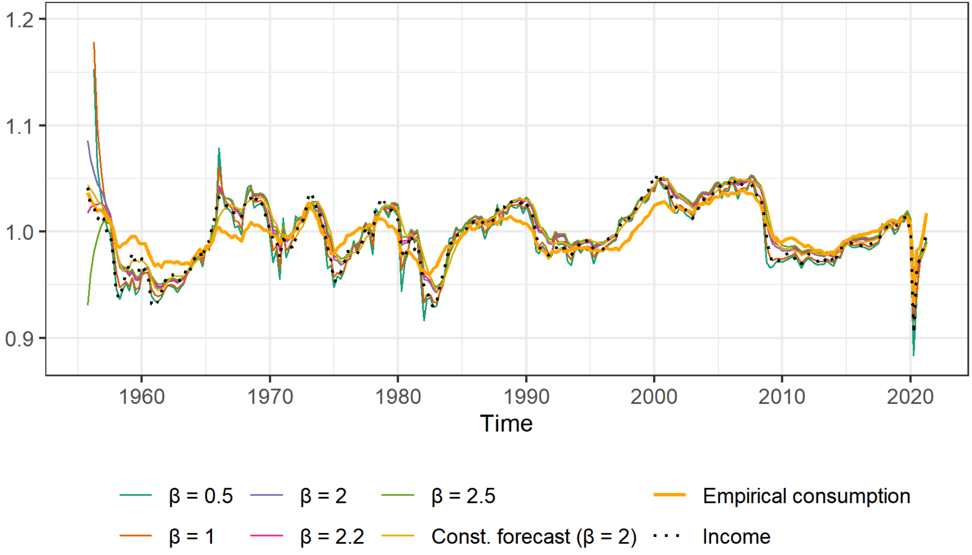

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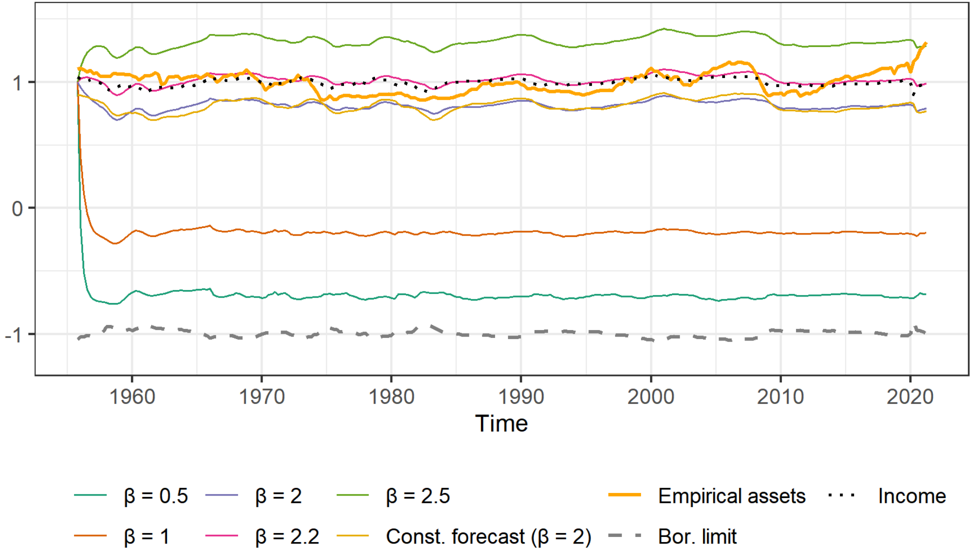

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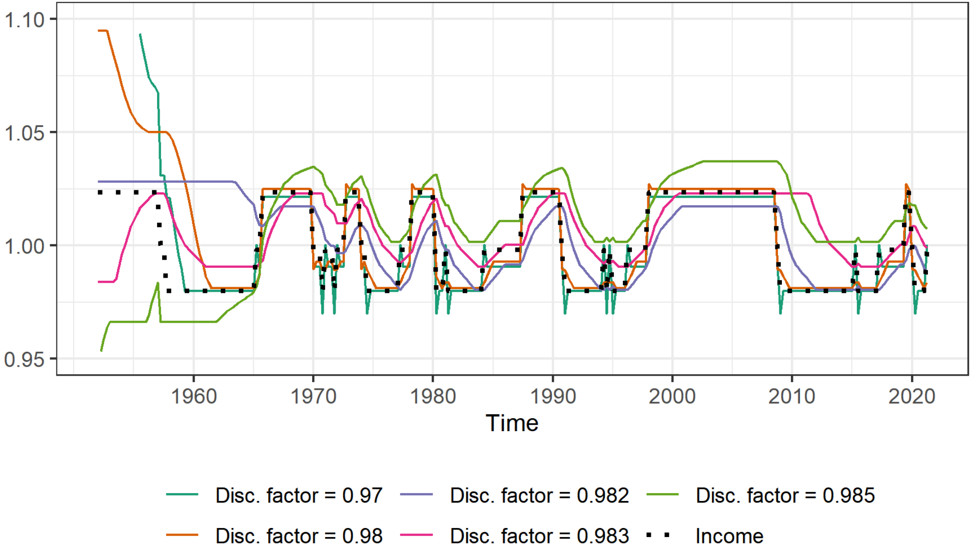

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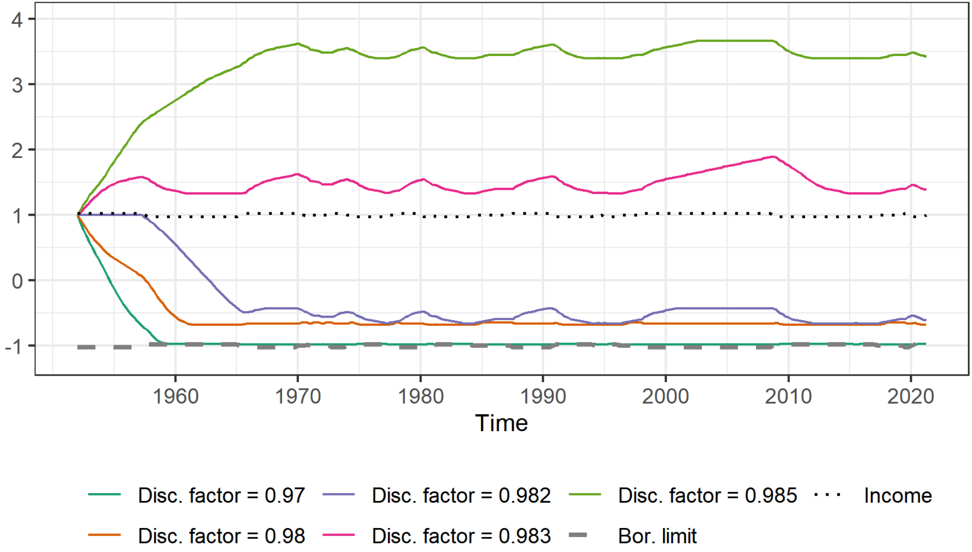

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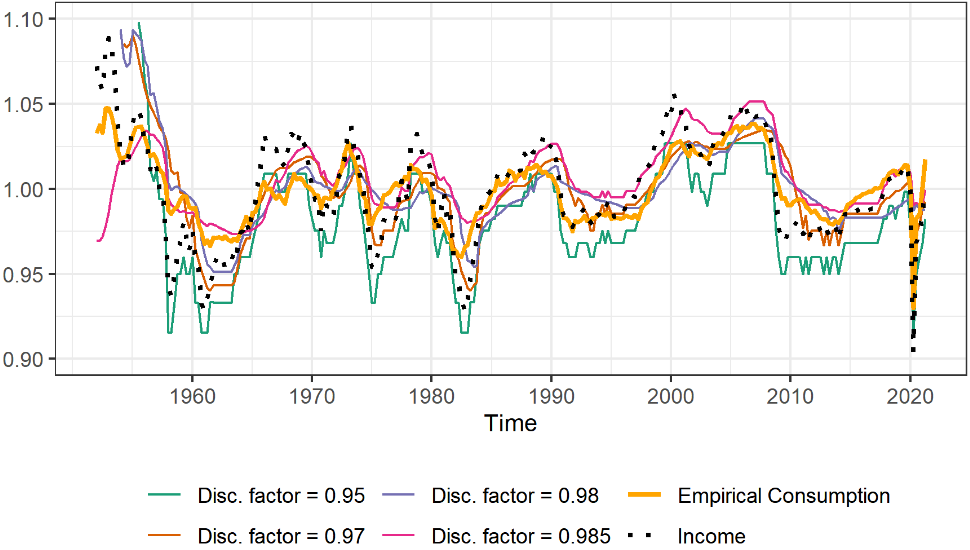

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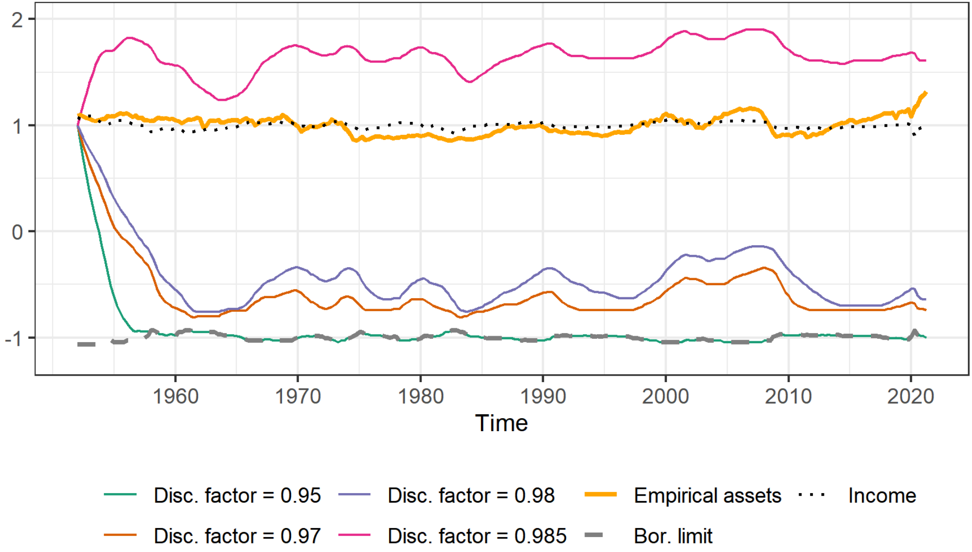

How Rational are the FU Agents?

	Dependent variable:		
	Y, kNN	Y, Simple Extrapolation	Y, RE
	(1)	(2)	(3)
$\mathbb{E}_{t-1}[Y_t]$	0.868***	0.913***	1.016***
	(0.052)	(0.027)	(0.027)
Constant	0.133**	0.087***	−0.016
	(0.052)	(0.027)	(0.027)
Observations	225	225	225
R^2	0.559	0.835	0.861
Note:			*p<0.1; **p<0.05; ***p<0.01

How Rational are FU Agents?

	Dependent variable:		
	(1/Cons_12Y_FU)	Cons_12Y_FU	
	(1)	(2)	(3)
lag(1/Cons_12Y_FU)	1.000***		
	(0.001)		
lag(Cons_12Y_FU)		1.152***	0.574***
		(0.063)	(0.050)
lag(Cons_12Y_FU, n = 2)		−0.069	
		(0.095)	
lag(Cons_12Y_FU, n = 3)		−0.135	
		(0.095)	
lag(Cons_12Y_FU, n = 4)		−0.013	
		(0.062)	
lag(Y_12Y_FU)			0.385***
			(0.050)
Constant		0.064***	0.042***
		(0.018)	(0.016)
Observations	262	259	262
R^2	1.000	0.923	0.933
Note:	*p<0.1; **p<0.05; ***p<0.01		

Hall's RE-PIH Test with our Data

	Dependent variable:
	C_empirical
lag(C_empirical)	0.931***
	(0.022)
Constant	0.069***
	(0.022)
Observations	271
R^2	0.869
Adjusted R^2	0.869
Residual Std. Error	0.007 (df = 269)
F Statistic	1,786.107*** (df = 1; 269)
Note:	*p<0.1; **p<0.05; ***p<0.01

The Point of Departure

In real world situations, households, managers, policy makers, often struggle to understand the true properties of the data generating process (DGP) of the "economic system".

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> *Economists are struggling to forecast how many people who left the workforce in 2020 will eventually return [...] They are also grappling with doubts over when consumers will shift their spending back to services, easing the upward pressure on goods prices caused by bunged-up supply chains. Economic data have become harder to interpret.* ***If retail sales fall, for example, does it reflect economic weakening, or a welcome return to normal patterns of consumption?***
>
> – Economist(2022)

Other Examples

Or other questions such as

- How will the US-China relationship evolve in the coming 10 years and how would it affect wages and capital returns in, say, the US?
- How will "artificial intelligence" affect the productivity of labor and capital in the coming 10 years?
- Is the current (U.S.) inflation transitory or permanent? (esp. if asked a year ago)
- Are we in a stagflationary period?

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Such questions are often answered by "sense-making" about how the economic system and the relevant DGP works.

Sense-Making

The world economy

Stagflation sensation

The Economist,
Oct 9th, 2021

WASHINGTON, DC

Soaring energy prices and a faltering recovery invite comparisons with the 1970s. But the past is not the best guide to the present

IT IS NEARLY half a century since the Organisation of the Petroleum Exporting Countries imposed an oil embargo on America, turning a modest inflation problem into a protracted bout of soaring prices and economic misery. But the stagflation of the 1970s is back on economists' minds today, as they confront strengthening inflation and disappointing economic activity. The voices warning of unsettling echoes with the past are influential ones, including Larry Summers and Kenneth Rogoff of Harvard University and Mohamed El-Erian of Cambridge University and previously of PIMCO, a bond-fund manager.

ment in America is sputtering. Meanwhile, after a decade of sluggishness, price pressures are intensifying (see chart 1 on next page). Inflation has risen above central-bank targets across most of the world, and exceeds 3% in Britain and the euro area and 5% in America.

The economic picture is not as bad as the situation during the 1970s (see chart 2 on next page). But what worries stagflationists is less the precise figures than the fact that an array of forces threatens to keep inflation high even as growth slows—and that these look eerily similar to the factors behind the stagflation of the 1970s.

One parallel is that the world economy is once again weathering energy- and food-price shocks. Global food prices have risen by roughly a third over the past year. Gas and coal prices are close to record levels in Asia and Europe (see next story). Stocks of both fuels are disconcertingly low in big economies such as China and India; power cuts, already a problem in China, may spread. Rising energy costs will exert more upward pressure on inflation and further darken the economic mood worldwide.

Other costs are rising too: shipping rates have soared, because of a shift in consumer spending towards goods and covid-related backlogs at ports. Workers are enjoying greater bargaining power this year, as firms facing surging demand struggle to attract sufficient labour. Unions in Germany, for instance, are demanding higher pay; some workers are going on strike.

Stagflationists see another similarity with the past in the current policy environment. They fret that macroeconomic thinking has regressed, creating an open-

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In short: Agents face "model uncertainty" ($\approx$ fundamental uncertainty).

How do they resolve this uncertainty? What could be a clever way to resolve it?

Our Approach

Propose a model of decision-making under FU that

- Stays true to the standard intertemporal optimisation and utility maximisation as much as possible (Rational Expectations as a limit case of FU)
- Is cognitively and psychologically plausible

Both the valuation of future outcomes and the expectation formation about the DGP need to be agnostic (flexible) regarding the economic system and the DGP in question.

The Economic Setting

In this paper:

- An infinitely lived representative agent confronted with stochastic stream of labour income at the aggregate level of the economy
- Agent does not know the data-generating process (DGP) of labour income (and is aware of it)
- Focus on intertemporal consumption (and therefore asset) decision
- Simulation of the RE benchmark and FU (patterns of consumption, income, income expectations and assets), comparison to empirical data

Let us begin with Rational Expectations (RE)

Agents choose consumption as to maximise their lifetime utility:

$$\begin{aligned} \max_{(c_t, c_{t+1}, c_{t+2}, \dots) \in \bar{\mathbb{B}}_t} \mathbb{E}_t \sum_{h=0}^{\infty} \beta^h u(c_{t+h}) \;\; s.t. \\ (a_t + y_t - c_t)(1+r) = a_{t+1} \\ \dots \end{aligned} \tag{10}$$

Dynamic programming and Bellman optimality principle leads to following recursive relation in the optimum, with V as a value function:

$$V_t(a_t, y_t) = \max_{c_t} \{u(c_t) + \beta \mathbb{E}_t V_{t+1}(a_{t+1}, y_{t+1})\} \tag{11}$$

Solution is a policy function $c_t = g(a_t, y_t)$ defined at each point of the event tree.

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Solving this problem is truly daunting (not only for the agents in the model!).

This problem is unsolvable under fundamental uncertainty. What is $E_t V_{t+1}$?

Fundamental Uncertainty (FU)

Key Question

How to adjust (11) so that decisions can be made under fundamental uncertainty?

Deriving A Problem Solvable Under Fundamental Uncertainty (FU)

$$V_t(a_t, y_t) = \max_{c_t} \left\{ u(c_t) + \beta \mathbb{E}_t \left[V_{t+1}(a_{t+1}, y_{t+1}) \right] \right\}$$

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$$V_t(a_t, y_t) = \max_{c_t} \{u(c_t) + \beta \mathbb{E}_t [V_{t+1}(a_{t+1}, y_{t+1})]\}$$

$$\Downarrow$$

$$\widehat{V_t(x_t)} = \max_{c_t} \left\{u(c_t) + \widehat{\beta} \cdot \widehat{\mathbb{E}_t} \left[\widehat{V_{t+1}(x_{t+1})}\right]\right\}$$

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$$V^{FU}(a_t + y_t - \bar{b}_t) = \max_{c_t} \left\{u(c_t) + \beta^{FU} \cdot \widehat{\mathbb{E}_t} \left[V^{FU}(a_{t+1} + y_{t+1} - \bar{b}_{t+1})\right]\right\}$$

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Approximating the Value Function

Keeping it as simple as possible (simplicity often implies cognitive plausibility):

- The value function needs to be independent of the true economic fundamentals ("model independent").
- What should be the argument of the value function?: Future "resources"
- Certainty equivalence (Otherwise would need to have multiple value functions...)
- The logarithmic function a simple alternative with practical properties (decreasing marginal utility)

V^{FU} is basically a valuation index for future available resources, based on agents experience. ("How good is it to have 100'000 CHF next year?")

The Solution of FU Agents

$$V^{FU}(a_t + y_t - \bar{b}_t) = \max_{c_t} \left\{ \log(c_t) + \beta^{FU} \log\left(a_{t+1} + y^*_{t+1|t} - \bar{b^*}_{t+1|t} \right) \right\} \tag{12}$$

The optimal consumption under FU becomes

$$c_t = \frac{1}{1+\beta^{FU}} \left[a_t + y_t + \left(y^*_{t+1|t} - \bar{b}^*_{t+1|t} \right) / (1+r) \right] \quad \textit{if borrowing-constraint not binding} \tag{13}$$

Expectations? Propagate with a factor of $\left(\left(1+\beta^{FU}\right)(1+r) \right)^{-1}$ into c_t

Surprising? finding: The value function significantly more relevant for agent's utility and matching the consumption and asset outcomes to patterns in data than income expectations.

Approximating $\mathbb{E}_t$

- We are focusing on point forecasts. Hence, the agents do not make forecasts over potential single events, only about their mean. This is another argument for certainty equivalence.
- Forecasts for next period income, $y^*_{t+1|t}$ could be **heuristic**,**measured** or **learnt**
- Examples of heuristic forecasting rule ("this time is different"): simple extrapolation ($y^*_{t+1|t} = y_t$), constant forecasts ($y^*_{t+1|t} = 1$)
- Example of measured expectations (e.g. University of Michigan Index of Consumer Expectations, re-scaled)
- Proposed learnt forecasting rule: historical analogies