Why black swans exist

Klaus Spremann

Swiss Institute of Banking and Finance, University of St. Gallen, Rosenbergstr. 52, CH-9000 St. Gallen, Switzerland, klaus.spremann@unisg.ch

1 Introduction

Till 1697, Europeans were convinced that all swans have white feathers and that there are certainly no swans bearing colors like a parrot. In particular, Europeans were persuaded by their overwhelming experience that there is no black swan. In scholarly circles even "theories" circulated which offered explanations, why swans must be white. Then the first seafarers returned from Australia and reported having seen black swans, not only one, but many. The unexpected, the unbelievable happened. The same scientists who till then explained why swans must be white revised their theories hastily. They now gave scientific reasons why black swans can exist, exist in fact, and should have been expected to exist even beforehand. And they took their new explanation as if they personally had already forecasted the existence of black swans long time ago and certainly before the evidence was reported by the seafarers.

Though the natural home of this animal is Australia, today quite a few black swans are settled in the parks of many countries around the world. Their presence does remind us how easily our experience and expectation, formed on the basis of experience, may mislead our mind. Inductive reasoning may help to formulate hypotheses, but never offers a rigorous demonstration. Sir Karl Raimund Popper (1902-1994), who lived for some time in New Zealand (where also black swans are prevalent), took statements like "all swans are white" to argue that empirical knowledge is always *preliminary*. Inductive statements derived from observations should never be called "true" even if the number of supportive observations is large and "convincing". We only surmount the border of preliminary knowledge by falsifying a statement.[1]

The black swan is not only important in the history of biology and the philosophy of science. Today we use the notion of a *black swan* to refer to an important and in most cases undesirable event which, though it is thinkable and could happen in principle, is wholly unexpected. Think of a deep crisis: a natural

[1] Popper argued 1953 that, no matter how many observations agree with a hypothesis, it may still be wrong. On the other hand, a single contrary observation can prove a statement to be false. Therefore, in order to demonstrate a hypothesis, the researcher must reject the counter hypothesis. See Popper: The Problem of Induction (1953, 1974).

disaster like the Flood, monster waves like the Tsunami in 2004, the collapse of a nuclear power station like Chernobyl 1986, the terrorist attack of September 2001, the lack of liquidity crisis in credit risk markets 2007. Apparently, the coming of a black swan means terrible bad news, or as the Economist[2] put it, "swans can give you a nasty peck". For some good sense they are called *Trauerschwan* in German. These examples underline there might be spare evidence regarding the event. At times, there are reports that the crises actually occurred in history. But we can easily push this evidence away: Is the Flood, mentioned in the bible, not something you may believe or not? Are monster waves not only cock-and-bull stories? Even if the evidence cannot be denied, like in the case of the Tsunami 2004, we may declare that the disaster must be considered to be singular and very rare.

Accordingly one might consider the black swan to be a *low-probability high-impact* risk. Such a risk could be handled by using modern techniques of risk management. Typically, these techniques proceed in three steps: (1) The probability, though being extremely small, will be assessed more precisely. The same holds for the magnitude of the possible damage. Thus, the risk is identified and quantitatively described. (2) Now the risk will be priced. It can be fractionized and allocated to different parties. Contracts will be written with the help of markets and insurance companies. (3) A few institutions, maybe the government, decides on the optimal level of protective investments which could be made beforehand.

Things are not that simple in the case of a black swan. To consider the black swan as a low-probability high-impact risk overlooks some important aspects of its proper nature. The biggest problem associated with a black swan is obviously, that the available information regarding the correct probability of the thinkable event is poor. We may have some a-priori estimates of the probability, but the sources of this a-priori knowledge are vague and very few. For one reason or another we fail in improving this poor a-priori knowledge. In the case of a black swan, we remain with our original vague initial beliefs. These probability beliefs are saying that occurrences are very, very rare.

In this article, we focus on this *failure* of mankind to update original probability beliefs (we just say: "will certainly not happen in reality"). Mankind does not work out the probability of entrance of the crisis correctly. The failure to identify precisely the probability of a black swan may have several reasons. One could be that we are not fully rational. But under this premise the question arises why we and our society with prime financial institutions should not be able to overcome psychological traps facing such important situations. The second reason for our failure to inquire and analyze the risk properly was presented by Nassim Taleb (2007) in his best-selling book. Taleb argues that we use traditional models ("Gaussian model") which, because of their simplicity, are wrong.

[2] The Economist: "The crisis to watch for in 2008", 22 Dec 2007, p. 112.

Thus he advocates the use of superior models ("Mandelbrot variations") which could estimate the correct probability of a black swan more accurately. But also here we must admit that the Mandelbrot variations and other superior models are known for some time, though not very popular in the broad academic community. Again the question is why the financial market, research academies, and other institutions do not turn to these superior models in order to come up with better estimates of the probability of a black swan which would open the way to risk management.

Our argumentation reveals a third reason why we and our institutions avoid the effort to identify the probability of a black swan. We argue that, under certain circumstances, it is not rational to devote time and resources to learn more about the risk. It thus remains opaque and risk management techniques fail.[3] The reason is the fact that the *information value* associated with getting more details regarding the correct probability of the risk under discussion is zero or very low. Since informational activities always come at a cost, the small information value does not warrant these costs. It is thus optimal and economical efficient to stay with the vague a-priori probability rather than devoting resources for a deeper inquiry. The cause for the smallness of the information value – which is lower than any cost for learning more about the risk – is the decision-theoretic definition. It says that the information value is calculated on the basis of a-priori probabilities. If our a-priori probability of a black swan is very low, the information value turns out to be very low as well. In view of the mentioned costs to get more information, a rational decision maker will not waste time and resources to learn more about the black swan and the correct probability of the disaster. Thus, the risk which we call black swan will never be identified precisely. It remains in the fog of our vague a-priori beliefs. Thus it cannot be addressed by the modern techniques of risk management.

Our arguments are organized as follows. In section 2 we argue why a black swan is not just a low-probability high-impact risk. Then, basic elements of decision theory (Sections 3, 4 and 5) are recapitulated. Sections 6 and 7 show how a black swan can be treated in this framework. As our main result we explore the "trap of vagueness": If our a-priori belief of the probability of a crisis or disaster is low, it is rational not to investigate into learning more about the situation. Thus, it will remain vague, and a black swan exists. Section 8 is a conclusion.

[3] Asking this question, the reader will be reminded to the work of Klaus Schredelseker, which is devoted to situations in which it is of no value to know more – different from the common belief that the value of information is always non-negative.

2 Decision-theoretic analysis

Let us now develop the decision-theoretic framework to analyze the black swan. We start with the basic question whether measures of protection and hedging against the possible crises should be undertaken or not. Consider this tableau of *actions* and *states* (Table 1).

Table 1. Actions, states and payoffs in the basic decision problem

	States and their probabilities	
	s_1 "normal"	s_2 "crisis"
	p_1	p_2
Action a_1 "no hedge"	$z_{1,1} = f(a_1, s_1)$	$z_{1,2} = f(a_1, s_2)$
Action a_2 "hedge"	$z_{2,1} = f(a_2, s_1)$	$z_{2,2} = f(a_2, s_2)$

- There are two actions, a_1 and a_2. The action a_1 means not taking any measures to protect against the crisis. The action a_2 stands for hedging and building shelters in advance, which is costly.
- The outcomes of these actions depend on whether a crisis occurs or not. So we have two states, s_1 and s_2, respectively. Thereby s_1 means "no emergency" while s_2 is the state in which the disaster happens, the black swan appears and bites.

The monetary outcome of action a_i if state s_k occurs is denoted by $z_{i,k} = f(a_i, s_k)$.

- If no money is spent on shelter or hedging (action a_1), and if no crisis occurs (state s_2), the corresponding outcome $z_{1,1} = f(a_1, s_1)$ is the highest of all. But if, under this action not to hedge, the crisis occurs (state s_2), the outcome is the worst of all. Thus, $z_{1,2} = f(a_1, s_2)$ is very small and $z_{1,2} \ll z_{1,1}$.
- If, under the decision to protect in advance (action a_2), there is no crisis (state s_1), the outcome $z_{2,1} = f(a_2, s_1)$ is lower compared to $z_{1,1} = f(a_1, s_1)$, since the protection requires resources: $z_{2,1} < z_{1,1}$. If again protective measures are taken but the crisis occurs, the outcome $z_{2,2} = f(a_2, s_2)$ will be much larger compared to $z_{1,2} = f(a_1, s_2)$ because of the protection or insurance provided by the undertaken measures, $z_{1,2} \ll z_{2,2}$. Arguably, $z_{2,2}$ will be smaller than $z_{2,1}$ since the agent will seek no over-protection.

Putting these relations together, the decision to hedge or not to hedge can be characterized by this relationship:

$$z_{1,2} \ll z_{2,2} \leq z_{2,1} < z_{1,1}. \tag{1}$$

Here is a simple numerical illustration which reflects this structure: If no protective measure is taken and there is no crisis, the outcome is $z_{1,1} = f(a_1, s_1) = 1$. But if under this decision to go on as usual the crisis occurs, the outcome is a disastrous $z_{1,2} = f(a_1, s_2) = -99$. If the decision maker decides to hedge and no crisis occurs, the outcome is $z_{2,1} = f(a_2, s_1) = 0$, a bit lower compared to $z_{1,1} = f(a_1, s_1) = 1$ because of the costs of the protection. If, again under hedging, the crisis occurs, the outcome could also have this level: $z_{2,2} = f(a_2, s_2) = 0$. It could also be a bit lower if only part of the risk is hedged, and it could be a bit higher in the case of over-insurance. But for simplicity, the example suggests $z_{2,2} = z_{2,1} = 0$.

We will use this numerical example below for some illustrative calculations. But before, we characterize the optimal decision in the general tableau with two actions and two states where the outcomes reflect (1).

Clearly a *decision under uncertainty* must be taken, since the choice between a_1 (no hedge) and a_2 (hedge) must be made before it is known which of the two states s_1 (no crisis) and s_2 (crisis) occurs. Decision theory mentions a number of criteria according to which a person might decide under uncertainty. Asking which criterion is the most adequate, finds a straightforward answer if *probabilities* can be assigned to the states of nature. Accordingly we proceed under the assumption that state s_2 (crisis) occurs with a certain probability p. Consequently, state s_1 (no crisis) occurs with probability $1-p$. Here p is just a number which the decision maker is willing to accept in his calculations as if it were a probability. So we consider p as a *subjective probability* of the decision maker. It is the outcome of a-priori beliefs which are formed on the basis of the general knowledge of the decision maker.

Considering the probability p as given, the state becomes a random variable which will be denoted by $\tilde{s}$. This random variable assumes the realization s_1 (no crisis) with probability $1-p$ and s_2 (crisis) with probability p.

As soon as the person is willing to take p like a probability into account, she is going to make a *decision under risk*. The person chooses among lotteries: If she decides for action $a \in \{a_1, a_2\}$, she receives the lottery $f(a, \tilde{s})$, i.e., the outcome $f(a, s_1)$ with probability $1-p$ and $f(a, s_2)$ with probability p.

Decision theory offers much more advice to make "reasonable" decisions under risk compared to the general case of uncertainty. One criterion is expected utility; Kahneman/Tversky and others have suggested modifications with more explanatory power. Here we will not enter into this large literature. In order not to burden the notation, we suppose the decision maker to be *risk neutral*: For each action, she is only looking at the expected outcome:

$$z(a) = \mathbb{E}[f(a, \tilde{s})] = (1-p) \cdot f(a, s_1) + p \cdot f(a, s_2). \tag{2}$$

Hence, the person is choosing the action with the highest expected outcome.[4] She is maximizing (2) for $a \in \{a_1, a_2\}$. This way, she can generate the maximal expected outcome denoted by z^*,

$$z^* = \max_{a \in \{a_1, a_2\}} \mathbb{E}[f(a, \tilde{s})]. \tag{3}$$

In decision theory, sometimes the probabilities of the states must be considered as imprecise entities or variables. So we also can consider p, the probability of a crisis, as a variable or as a parameter. We are now addressing the question which decision will be made as a function of this parameter p. Obviously, if p is large, it is optimal to hedge. But if p is smaller and smaller, at some point it will become optimal not to hedge. The crucial probability of a crisis, where the decision maker is indifferent between hedging and not hedging, is denoted by p^*. We are now calculating this crucial probability.

The crucial probability has the property that the expected outcome under hedging is the same as the expected outcome under not hedging, $z(a_1) = z(a_2)$. Using (2), this condition says $(1-p) \cdot f(a_1, s_1) + p \cdot f(a_1, s_2) = (1-p) \cdot f(a_2, s_1) + p \cdot f(a_2, s_2)$. Solving for p gives the crucial probability:

$$p^* = \frac{f(a_1, s_1) - f(a_2, s_1)}{f(a_1, s_1) - f(a_1, s_2) - f(a_2, s_1) + f(a_2, s_2)}. \tag{4}$$

Thus, the crucial probability is larger if protection is expensive, i.e., if the nominator $z_{1,1} - z_{2,1}$ is large. Basically the crucial probability depends on the possible outcomes under the possible actions and states.

- By definition, the optimal decision will be not to hedge if the subjective probability assigned by the person to the disaster is smaller than the crucial probability, i.e., if $p < p^*$.
- The optimal decision will be to protect and to hedge, if the subjective probability is larger than the crucial probability, i.e., if $p^* < p$.

Consider our numerical example: For the specific outcomes $z_{1,1} = f(a_1, s_1) = 1$, $z_{1,2} = f(a_1, s_2) = -99$, $z_{2,1} = f(a_2, s_1) = 0$ and $z_{2,2} = z_{2,1} = 0$, the crucial probability (4) is $p^* = (1-0)/(1-(-99)-0+0) = 1/100$. Hence, the decision maker will hedge if and only if she thinks that the probability of the disaster will exceed one percent.

3 The information value

In all decisions under risk, knowing the probabilities of the states is a critical input. These probabilities are subjective, i.e., numbers which the decision maker

[4] Some of the results, in particular regarding the information value, can be extended to the case of constant risk aversion. See Bamberg/Spremann 1981.

is willing to use in his decision calculus as if they were objective probabilities. We also stated that very often subjective probabilities are formed on the basis of the general knowledge and experience of the decision maker. In some situations, the probabilities are estimated on the historical evidence. Sometimes, more data can be generated by sampling techniques. But in still other situations the decision maker has only vague information regarding the probabilities (but is nevertheless willing to use her a-priori belief to make decisions under risk).

However vague they may be, a-priori probabilities very often are no final destiny. With effort and resources, very often additional pieces of information can be received and processed. Thus, original a-priori probabilities can be transformed into a-posteriori probabilities. The latter refers to probabilities as the outcome of costly informational activity.

Knowing more is not *per se* valuable (as Klaus Schredelseker has pointed out in a number of publications). The value of knowing more depends on the situation. Getting more information appears to be beneficial only if, by eventually revising the original decision which would be optimal for a-priori probabilities, an improved result warrants the costs of acquiring and processing additional information. In decision theory, the value of knowing more is determined as the so-called value of information.

To recall the formula for the *value of information* we start by focusing on *perfect* information. By having perfect information, the decision maker learns which state will occur even before having to choose the action. Once the person knows that the state $s \in \{s_1, s_2\}$ is the one that will occur later, she will choose the action $a \in \{a_1, a_2\}$ such that she achieves the highest possible outcome in this state:

$$z^*(s) = \max_{a \in \{a_1, a_2\}} \{f(a, s)\}. \tag{5}$$

We should imagine that learning more about the probability in this way: Before deciding for an action, the person can purchase a closed envelope (from an information provider). She knows that inside the envelope there is a letter which announces correctly the state of nature $s \in \{s_1, s_2\}$ that later will occur and become known to everybody. The information value answers this question: How much will the decision maker at most pay for receiving the closed envelope?

The answer is easy.

- If, after opening the envelope it turns out that state s_1 will occur, the person will choose the best action and receive the outcome $z^*(s_1)$.
- If, instead the opened envelope tells that s_2 will occur, the person will choose the action which is best for s_2. Her outcome will be $z^*(s_2)$.

In other words, by purchasing the closed envelope the person either will arrive at the result $z^*(s_1)$ or at $z^*(s_2)$. Now the person knows the two uncertain consequences associated with buying the information (envelope). What will she expect? Now, a very important point for our argumentation is that the decision maker will expect messages that correspond to her a-priori belief. According to her actual information, the person expects to read in the envelope the announcement of state s_2 (crisis) with probability p and she thinks that the letter will announce s_1 with probability $1-p$. So the expected outcome, after having bought the information, is

$$z^{**} = (1-p) \cdot z^*(s_1) + p \cdot z^*(s_2). \tag{6}$$

To say it once more: The decision maker assesses the value of what can be reached having the envelope or information by taking the perspective of her *a-priori probabilities*.

Now, the value of perfect information is the result of a simple comparison. The person compares the outcome (6) which can be expected if the envelope will be purchased with the outcome (5) when no additional information is available. The difference $V = z^{**} - z^*$ is the value of perfect information:

$$V = \mathbb{E}\left[\max_{a \in \{a_1, a_2\}} \{f(a,s)\}\right] - \max_{a \in \{a_1, a_2\}} \{\mathbb{E}[f(a, \tilde{s})]\}. \tag{7}$$

Students remember this formula (7) always by memorizing "V = E-Max minus Max-E". A simple analysis can show that this value of information is always non-negative, $V \geq 0$. It is the highest price the decision maker would pay for receiving the envelope.

4 Optimal action for a black swan

Having recalled the information value in a decision-theoretic framework, we now calculate this value for a black swan. Action a_1 means not taking any protective measures while action a_2 means to hedge and to build shelters of all sort. State s_1 means that no crisis occurs while s_2 is the state in which the disaster happens. Because now the tableau describes a black swan, the entries must be in the relationship (1). In addition, the a-priori probability p which the decision maker assigns to the state s_1 (crisis) is very, very small. For a numerical illustration we come back to $z_{1,1} = 1$, $z_{1,2} = -99$, $z_{2,1} = 0$ and $z_{2,2} = 0$.

Without having further information, the person decides for the lottery with the highest expected outcome. If she chooses a_1, her expected outcome is

$$z(a_1) \stackrel{(2)}{=} (1-p) \cdot 1 + p \cdot (-99) = 1 - 100p. \tag{8}$$

If the person chooses a_2, the (expected) outcome is

$$z(a_2) = (1-p) \cdot 0 + p \cdot 0 = 0. \tag{9}$$

Table 2. Actions, states and payoffs in our numerical example

	States and their probabilities	
	s_1 "normal"	s_2 "crisis"
	$p_1 = 1 - \varepsilon$	$p_2 = \varepsilon$
Action a_1 "no hedge"	$z_{1,1} = 1$	$z_{1,2} = -99$
Action a_2 "hedge"	$z_{2,1} = 0$	$z_{2,2} = 0$

Obviously, if the a-priori probability p assigned to the disaster is almost zero, it will turn out to be smaller than the crucial probability p^* determined in (4). This crucial probability was calculated for the numbers given here to be $p^* = 1\%$. If her subjective a-priori probability belief is smaller than one percent, the person prefers not to spend money for protection. Only if the person thinks that the probability of a crisis is larger than $p^* = 1\%$ she prefers to protect (action a_2). The overall outcome of the decision situation, optimal action taking assumed, is

$$z^* \overset{(3)}{=} \max_{\{z(a_1), z(a_2)\}} \overset{(8),(9)}{=} \begin{cases} 1 - 100p \Leftrightarrow p < 0.01 \\ 0 \qquad\qquad \Leftrightarrow p \geq 0.01 \end{cases} . \tag{10}$$

5 The trap of vagueness

Now we determine the value of perfect information. If the message in the envelope is that state s_1 (no crisis) will occur, the person will obviously choose a_1 (no protection) and receive the outcome $z^*(s_1) = 1$, see (5). If the message in the envelope is instead that state s_2 (disaster) will occur, she will choose a_2 and subsequently gets the outcome $z^*(s_2) = 0$.

- Thus, if the person has the possibility to acquire and process information regarding the correct probability ("buy the envelope"), her result is either $z^*(s_1) = 1$ or $z^*(s_2) = 0$.
- *According to her a-priori belief*, the person thinks that $z^*(s_1) = 1$ has a probability of $1 - p$ while $z^*(s_2) = 0$ has a probability of p.
- So, with the help of the envelope, she *expects* to get:

$$z^{**} \overset{(5)}{=} p_1 \cdot z^*(s_1) + p_2 \cdot z^*(s_2) = (1 - p) \cdot 1 + p \cdot 0 = 1 - p. \tag{11}$$

Using (11) and (10), the information value $V = z^{**} - z^*$ can be calculated:

$$V = 1 - p - \begin{Bmatrix} 1 - 100p \Leftrightarrow p < 0.01 \\ 0 \qquad\qquad \Leftrightarrow p \geq 0.01 \end{Bmatrix} = \begin{cases} 99p \quad \Leftrightarrow p < 0.01 \\ 1 - p \Leftrightarrow p \geq 0.01 \end{cases} \tag{12}$$

What does that mean?

- If the person has an a-priori belief of $p \approx 0.01$, the critical probability where she starts to think that it might be worthwhile to hedge, the information value is quite high. The person is therefore well prepared to devote time, effort, and resources to become better informed on the correct probability.
- If however the a-priori probability of the decision-maker is much lower than the critical probability (4), let us say $p \approx 0.001$ or $p \approx 0.0001$, the value of perfect information is only 0.099 or 0.0099. Without knowing the actual costs to acquire and process information in detail, one can conclude that the information value will be larger than these costs. The person will not find it worthwhile to pay for getting more information.

In the end the decision maker who, for whatever reason, has a very low a-priori probability belief is sitting in a *trap*: It is rational not to look for more information in order to get a more precise description of the decision situation. It is optimal to go on and not to think about how one could come up with better estimates of the probability of the possible disaster. We call it the *trap of vagueness*. Although there might be some pieces of information around, it is rational not to devote time or effort to take note of them. Just put them aside by saying there are "still some doubts".

Only when you have a-priori probability beliefs in the range of the critical probability (4), where action a_1 and action a_2 have a similar outcome, the information value assumes a magnitude in which it might be advisable to take the cost to gain a better description of the situation.

How could we overcome the trap of vagueness? We fall in the trap if and only if our initial probability belief says that the event is very, very rare. Then, as we have seen, it is in fact rational not to devote time or effort to learn more about the black swan.

Mankind can easily sit in the trap of vagueness. If we allow our intellect to form a rough initial probability belief for certain events which are very close to zero, it turns out to be rational, methodological and economical not to attempt to learn more about the exact probabilities. Thus we remain in the trap of vagueness.

In order to avoid this trap, we must exclude the formation of initial probability beliefs of almost zero in our thinking. This is a question of the general way of our reasoning. We always should keep commands like "never say never" in mind. Education is required. If, by whatever cultural process, in the very beginning of our first reasoning, we would give high impact events a probability of occurrence which is in the range of the critical probability (4), we will find it beneficial to devote more time, effort and other resources to know more.

6 Conclusion

The "black swan" stands for a crisis which in principle could happen with a probability which is commonly thought to be zero or almost zero.

But the black swan is not just a low-probability high-impact event. Low-probability high-impact events are usually dealt within risk management using best practices and methods. The first step in risk management is to identify and to specify the event. All available information is used in order to come up with good estimators of the probability. In addition, the possible consequences are made transparent and measured with reasonable precision. This first step in risk management creates the basis for the second step, in which the risk will be priced, allocated and in which decisions on protection and prevention are made.

In the case of a black swan, the low-probability high-impact event remains vague. Nobody is devoting resources, effort or time to identify it with precision or to come up with a better estimate of the probability of occurrence. As a result, there is no informational basis to price the risk, to allocate it through the various means including the market mechanism, and to find the best dimension of shelter. Black swans are treated as if they are out of the range of rational reasoning. Why is this so?

It all depends on our initial belief regarding the probability of occurrence. On whatever argument, we assign to a thinkable event a certain probability. Now two cases must be distinguished.

If it happens that our a-priori probability belief is in the range of the break-even probability such that rational decision making suggests to look whether taking shelter is a viable option, the value of information is considerably high. Most likely we acquire more information and transform the possible event into to well-specified low-probability high-impact risk which can be rationally managed by classical methods of risk management. If, it happens that our initial probability belief is lower than the break-even probability, the value of information is quite low. Most likely we find the effort required to inform us better to be unreasonably high. We fall into the trap of vagueness.

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