



# Venture Capital Backed Growth<sup>\*</sup>

CHRISTIAN KEUSCHNIGG

christian.keuschnigg@unisg.ch

*University of St. Gallen (IFF), CEPR and CESifo, Varnbuelstrasse 19, CH-9000 St. Gallen, Switzerland*

The paper proposes a simple equilibrium model of venture capital, start-up entrepreneurship and innovation. Venture capitalists not only finance but also advise start-up entrepreneurs and thereby add value to new firms. The paper shows how a productive and active venture capital industry boosts innovation based growth. It also demonstrates the potential of tax policy to promote innovation and growth by strengthening incentives for more intensive venture capital support.

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## 1. Introduction

Creation of innovative young firms is an important source of innovation and growth. It is widely claimed among economists that the existence of a sophisticated venture capital industry is a major factor behind America's ability to encourage and sustain technological innovation and growth. The argument is that venture capital (VC) makes young firms grow faster, create more value and generate more employment than other start-ups. Empirical research in the United States has shown indeed that VC can importantly enhance the ability of new firms to create wealth and jobs.<sup>1</sup> According to Kortum and Lerner (2000), VC backed firms are more innovative and produce more and more valuable patents. Hellmann and Puri (2000, 2002) show that they aim at more radical innovations, are significantly faster in introducing their products to the market and pursue more aggressive market strategies than other start-up firms. They are found to have a higher rate of executive turnover and to be faster in hiring a specialized marketing director. It seems that the existence of an active VC sector can show up in superior macroeconomic performance. Recently, Wasmer and Weil (2000) econometrically investigated panel data of 20 OECD countries over the years 1986–1995. They found that an increase in the GNP share of VC by 0.075 would reduce the short-run unemployment rate by 0.25 percentage points while the long-run effect would amount to a reduction of 0.9 to 2.5 percentage points.<sup>2</sup>

The empirical study by Bottazzi and Da Rin (2002) suggests that European VC has

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<sup>1</sup> Gompers and Lerner (1999, 2001) summarize recent research on venture capital.

<sup>2</sup> The value of 0.075 corresponds to one standard deviation of the GNP share of VC. In Europe, this would amount to an increase in the GNP share of VC from 0.38 percent to 0.46 percent.

grown vigorously in terms of volume invested but, in contrast to the US evidence, apparently had not much influence on growth and employment of portfolio firms. This raises the question what the real contribution of VC exactly is and how incentives in VC investing are determined. Venture Capitalists (VCs) carefully screen firms, structure contracts to strengthen incentives, and monitor firms (Kaplan and Strömberg 2001). They add value to new firms, promote their professionalization, and induce them to behave more aggressively. It is often stated that the United States with an active VC sector is faster in adopting new technologies. Kortum and Lerner (2000) estimate indeed that a dollar of VC seems about three times more potent in stimulating patenting than a dollar of traditional corporate R&D. They also concluded that by 1998, venture funding accounted for about 14 percent of US innovative activity. Yet, this literature has left unspecified the precise transmission channels through which VC investing affects the economy's equilibrium rate of innovation.

The main contribution of this paper is to model the real effects of VC as a determinant of innovation driven growth. We show how an active VC sector with experienced and sophisticated investors can enhance the success of start-up entrepreneurs and can thereby raise the equilibrium rate of innovation. The model explores how the joint inputs of both entrepreneurs and VCs interact to determine the prospects of start-up firms. While entrepreneurs contribute the key technological idea, they tend to be commercially inexperienced. VCs support the firm with managerial advice that draws on their industry knowledge and commercial expertise. We study analytically how certain structural parameters determine the quality of VC finance and how this affects entrepreneurship and innovation in general equilibrium. For example, Gompers and Lerner (1999, p. 4) pointed to the fact that the skills needed for successful VC investing are difficult and time-consuming to acquire and are likely to be an important constraint in the development of an active VC industry. We thus study how an increase in investor sophistication raises the quality of VC financing and how this translates into a higher rate of innovation in general equilibrium. We also explore the consequences of other structural parameters and of policy initiatives aiming at VC backed growth.

In developing a model of VC with double moral hazard, we explain not only the incentives of entrepreneurs but also those of VCs to add value. The analysis importantly draws on Holmstrom (1982), Aghion and Tirole (1994), Repullo and Suarez (1999), Casamatta (2002), Inderst and Mueller (2002) and Schmidt (2003), to mention only a few. This literature in finance, however, is mostly confined to an analysis of contracts and the associated incentives for agents' efforts in partial equilibrium. According to Gompers and Lerner (2001), an analysis of the real effects of VC is largely missing.<sup>3</sup> The main contribution of this paper is to progress towards this end by integrating financial contracting and advising into a general equilibrium model that endogenously explains (i) the incentives of entrepreneurs to start a firm (occupational choice with an endogenously determined outside wage), (ii) the willingness of VCs to add value to start-up firms, (iii) the equilibrium venture returns, or market value of new firms, as a key determinant of

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3 This has been taken up by Kannianen and Keuschnigg (2003), Keuschnigg (2003, 2004) and Keuschnigg and Nielsen (2003a,b, 2004) who have proposed static models of industry equilibrium.

entry, and (iv) the equilibrium rate of innovation. A comparative static analysis shows how some deep structural parameters of the VC sector as well as tax policy addressed to the VC industry affect start-up entrepreneurship and innovation based growth.

The following section introduces the analytical framework. It extends the two period overlapping generations model pioneered by Diamond (1965) and its application to growth theory by Grossman and Yanagawa (1993) by including occupational choice of potential entrepreneurs and double moral hazard in the relation between entrepreneurs and VC firms. Start-up entrepreneurs if successful produce new capital goods which boost the factor productivity in final goods production. Based on comparative static analysis, Section 3 shows how key characteristics of the VC sector determine the quality and quantity of start-up investment by young entrepreneurial firms and the equilibrium innovation rate. We also consider how tax policy can promote growth by strengthening incentives for increased VC support to start-up firms. The final section concludes.

## **2. The Model**

### ***2.1. Life-Cycle of Firms***

An overlapping generations model of entrepreneurship and innovation is proposed. Production of a final good uses specialized capital goods which are introduced by innovative start-up firms. Entrepreneurs have ideas but no own capital. In addition, they are commercially inexperienced. VCs have managerial knowhow and access to capital. They collect savings and finance a portfolio of start-up firms. They not only provide finance but also add value in giving managerial advice. The success of the company in developing a marketable capital good critically depends on the effort of the entrepreneur but is further enhanced by the managerial support of the VC. The efforts of both the entrepreneur and the VC are not contractible, giving rise to double moral hazard. If the firm successfully matures to production stage, a specialized capital good is produced and supplied to final goods producers. The present value of monopolistic profits must cover the expected start-up cost of the firm.

The process of business creation starting with the entrepreneur's initial idea to the firm's successful market introduction and production of the good involves a natural sequence of events. First, an agent decides for entrepreneurship or else for a worker's career. When starting a firm, she must overcome the lack of capital and her commercial inexperience. She thus contacts a VC to obtain finance and advice. Second, the entrepreneur sells a concentrated equity stake in the firm for a price that covers the physical start-up cost and may also include a possible profit upfront. The VC accepts this offer or otherwise turns to other investment opportunities. Third, after the deal is finalized, both the entrepreneur and VC expend effort to develop the firm. Joint effort is conditional on the incentives provided by the terms of contract. Fourth, risk is resolved and firms which have successfully matured to production stage, introduce a specialized capital goods variety on the market. At this point in time, the firm is sold at a competitive price and revenues are distributed according to the agreed equity stakes. Finally, given their income, all agents choose consumption and savings.

The paper embeds this type of innovation finance in a growth model with overlapping generations. Following the principle of backward induction in solving the model, the subsequent sections first consider consumption and savings and then turn to the production side which determines venture returns. With these preliminaries, we then analyze the creation and financing of start-up firms that design new specialized capital goods. The final subsection closes the model and explains how the innovation rate is determined.

## 2.2. Consumption

It is assumed that agents are risk neutral, live for two periods and consume a final good which is the numeraire. Although they are identical ex ante, occupational risk introduces intragenerational heterogeneity ex post. Once risk is resolved, first period income  $y_t^i$  of an agent  $i \in [0,1]$  is known. Being a worker yields a safe income while an entrepreneur's income is risky as it will depend on the success of the company. Given first period income, the savings decision determines life-cycle consumption of agent  $i$  according to  $C_{1,t}^i = y_t^i - S_t^i$  and  $C_{2,t+1}^i = R_{t+1} S_t^i$ . The risk free interest factor  $R_{t+1}$  is one plus the market rate of interest. After eliminating savings to derive the intertemporal budget constraint, the life-cycle consumption problem is

$$U_t^{*i} = \max \{ u_0 \cdot (C_{1,t}^i - h_t^i)^{1-m} (C_{2,t+1}^i)^m + \lambda_t^i [y_t^i - C_{1,t}^i - C_{2,t+1}^i / R_{t+1}] \}, \quad (1)$$

where  $u_0 \equiv m^{-m} (1-m)^{-(1-m)}$  and  $h_t^i = \beta e_t^i$  is the utility cost of the effort  $e_t^i$  that the agent must put up in the first period to earn income.<sup>4</sup> Effort reduces the utility of first period consumption as is usual in intertemporal models of labor supply. Effort cost of workers is normalized to zero. The effort choice of entrepreneurs is explained later. We first solve for indirect utility, conditional on a given level of effort. The first order conditions imply  $C_{2,t+1}^i / R_{t+1} = m / (1-m) (C_{1,t}^i - h_t^i)$  which is substituted into the intertemporal budget constraint to yield first period consumption,  $C_{1,t}^i - h_t^i = (1-m)(y_t^i - h_t^i)$ . Indirect utility is, thus,

$$U_t^{*i} = U(R_{t+1}, y_t^i - h_t^i) = (R_{t+1})^m \cdot (y_t^i - h_t^i). \quad (2)$$

The linear homogeneous specification of preferences implies risk neutrality which helps to focus on incentive problems. Furthermore, savings is a constant fraction  $m$  of effort adjusted income,

$$S_t^i = y_t^i - C_{1,t}^i = m \cdot (y_t^i - h_t^i). \quad (3)$$

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4 As a convention, all parameters are defined positive,  $\beta, m > 0$ .

### 2.3. Production

The final good is the numeraire. Following Romer (1987), production is assumed to use labor and quantities  $x^j$  of the  $j \in [1, N_t]$  varieties of capital, to give a final output<sup>5</sup>

$$Y_t = Y_0 \cdot L_t^\alpha \int_0^{N_t} (x_t^j)^{1-\alpha} dj, \quad 0 < \alpha < 1, \quad Y_0 = (1 - \alpha)^{-(1-\alpha)}. \quad (4)$$

The normalization of factor productivity  $Y_0$  is for simplicity only. From cost minimization, the own price elasticity of demand for variety  $j$  is  $-(dx^j/x^j)/(dq^j/q^j) = 1/\alpha$ , where  $q^j$  is its price. Each intermediate capital good is protected by a patent and supplied by a single firm which has started up in some previous period and has successfully matured to production stage. This firm faces a perceived price elasticity of demand equal to  $1/\alpha$ . With a large number of competing varieties, cross price elasticities may be neglected as is usual in models of monopolistic competition. One unit of the specialized capital good is produced from  $\kappa$  units of the final numeraire good. An intermediate goods producer thus derives profits of

$$\pi_t^j = (q_t^j - \kappa) \cdot x_t^j, \quad \kappa = 1 - \alpha. \quad (5)$$

The choice of the input output coefficient serves to simplify the solution. Maximizing profits subject to a downward falling demand curve with own price elasticity of  $1/\alpha$ , a producer sets an output price and obtains profits equal to

$$q_t^j = \frac{1}{1 - \alpha} \cdot \kappa = 1, \quad \pi_t^j = \alpha \cdot x_t^j. \quad (6)$$

Price  $q = 1$  is a mark-up over unit cost  $\kappa$ , resulting in a profit margin of  $\alpha$  per unit. Since demand elasticities and technologies are uniform, intermediate goods production is symmetric, making output of the final good equal to  $Y = Y_0 N L^\alpha x^{1-\alpha}$ . Use  $N q x = (1 - \alpha) Y$ , (6), and  $Y_0$  in (4) to get closed form solutions

$$x_t = (1 - \alpha) L_t, \quad Y_t = N_t L_t, \quad W_t = \alpha N_t, \quad (7)$$

where  $WL = \alpha Y$  was used to obtain the wage. Equilibrium output and the wage rate relate to labor and the number of specialized capital goods. Note that  $N_t$  corresponds to the fraction of entrepreneurial firms that have been started in the pre-period and have survived the start-up phase.

It is assumed that capital goods depreciate and become obsolete with probability  $1 - \delta$  at any point in time. With probability of  $\delta$ , a given product line remains active and survives to the next period. The expected length of the remaining production period with monopoly profits protected by a patent is  $1/(1 - \delta)$ . As special cases, one may consider an infinitely lived product line enjoying permanent patent protection ( $\delta \rightarrow 1$ ), and complete obsolescence after one period at the other extreme ( $\delta = 0$ ). The value of a product line,

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5 Final goods production uses a standard technology combining labor and a preexisting range of intermediate goods. Only the design of new capital goods is a risky activity requiring inventive entrepreneurs.

or patent, is given by a no-arbitrage condition which states that the expected return must match the market rate of interest,<sup>6</sup>

$$R_{t+1}V_t = \pi_{t+1} + \delta V_{t+1}. \quad (8)$$

The value of the firm is the expected present value of future monopoly profits. Once a new firm survives the start-up period, it can be sold to other investors, if a market for young firms exists, or it can be introduced in the stock market by an initial public offering. In any case, a start-up firm with  $n$  patents is valued at a competitive price  $nV_t$ .

#### 2.4. Venture Capital

Firms are started by technology entrepreneurs who are managerially inexperienced and have no own wealth. Their main asset is an innovative business idea consisting of a number  $n$  of blueprints for specialized capital goods. We assume that the firm succeeds to develop  $n$  new patents with probability  $0 < p_t < 1$  but fails with probability  $1 - p_t$ . If the firm fails, it never reaches the production stage and all start-up investment is lost. Young innovative firms fail not only because of technological risks in further product development but also because entrepreneurs are often commercially inexperienced. Their survival chances can be significantly enhanced if they can enlist the managerial advice of an experienced VC. Taking account of this, we specify the success probability as

$$p_t = e_t \cdot p(a_t), \quad p(a_t) = \varepsilon \cdot \frac{a_t^{1-\theta}}{1-\theta}, \quad 0 < \theta < 1, \quad e_t \in \{0, 1\}, \quad (9)$$

where  $a_t$  denotes the VC's advice and  $e_t$  the entrepreneur's effort.

According to this specification, the entrepreneur's effort is critical. The firm can never succeed without the entrepreneur's effort and due diligence,  $e = 0$ . It succeeds with probability  $p > 0$  only if her effort is high,  $e = 1$ .<sup>7</sup> In this case, however, the VC may add value and boost the firm's survival chance according to  $p(a)$ . Managerial advice is assumed to be subject to decreasing returns to scale,  $p' > 0 > p''$ . The functional form in (9) is for convenience where the parameter  $\varepsilon$  indicates the VC's commercial knowhow and experience. A more experienced and knowledgeable VC will achieve, with the same number of consulting hours, a higher success rate than a less experienced one.<sup>8</sup> Neither the effort of the entrepreneur nor the advice of the VC are contractible.

The entrepreneur must hire  $k$  workers at a wage cost  $kW$  to start the firm. Having no own wealth, she sells a share  $1 - s$  to the VC and asks for a price  $Q = (b + k)W$  that covers at

6 The VC cycle occurs within the first period of life, starting with occupational choice at the beginning of period, and a start-up investment during the period. The within period interest rate is zero.

7 Alternatively, and at the cost of some further complexity, one could treat the entrepreneur's effort continuous as well. This would not change any basic insights. The main difference would be in the response of the entrepreneur's equity share and her effort decision while the other results are unaffected in qualitative terms, see Keuschnigg and Nielsen (2004) in a static context.

8 Parameters must be chosen such that the equilibrium level of advice implies  $0 < p_t < 1$ . In the following, we suppress the time index, except when needed for clarity.

least the start-up cost plus a variable upfront payment  $bW$ . In total, her expected income amounts to  $y^E = p(a)snV + bW$  and must compensate at least for the foregone job opportunities in industry at wage  $W$  and for the effort cost  $h = \beta W$  expended on developing the firm to maturity.<sup>9</sup> In structuring the deal, she maximizes her own surplus  $\Omega W = y^E - \beta W - W$ , subject to three constraints. First, the deal must be sufficiently profitable for the VC (Financier) to be acceptable and must satisfy her participation constraint  $PC^F$ . Second, the entrepreneur must anticipate how the terms of the proposed contract determine incentives for the joint inputs in the start-up phase. The contract must thus fulfil her own and the VC's incentive constraints  $IC^E$  and  $IC^F$ . The surplus is conveniently expressed relative to the wage rate:

$$\begin{aligned} \Omega W &= \max_{s,B} y^E - \beta W - W \geq 0, \quad y^E = p(a)snV + bW, \\ (a) \quad PC^F : y^F &= p(a)(1-s)nV - \gamma aW - (b+k)W \geq 0, \\ (b) \quad IC^E : p(a)snV - \beta W &\geq 0, \\ (c) \quad IC^F : ep'(a)(1-s)nV &= \gamma W. \end{aligned} \tag{10}$$

Once the terms of the contract are specified and the firm is started, the entrepreneur expends managerial effort with an opportunity cost  $\beta W$  expressed in terms of wages. The VC supports the firm with advice. To this end, she allocates  $\gamma a$  consulting hours at a wage cost  $\gamma aW$  to the firm. When there are  $E$  entrepreneurial firms in total, the VCs must hire  $\gamma aE$  consultants to support their portfolio firms.<sup>10</sup> The crucial assumption is that the VC fund cannot commit to assign its managers' or consultants' time to any specific one of their portfolio companies. A promised level of support could not be verified and enforced by the entrepreneur. Since the same holds for the entrepreneur's effort, a double moral hazard problem arises. Both inputs are decided only *after* the contract is signed, as indicated by the incentive constraints. The inputs must be elicited by means of financial incentives, that is, by relating compensation to performance and allowing both agents to participate in the upside potential of the firm. When offering a contract, the entrepreneur must thus anticipate how the proposed profit sharing determines incentives in the subsequent start-up phase.<sup>11</sup>

The solution follows backward induction. When the firm succeeds in developing  $n$  patents and matures to production stage, it can be sold at a price  $nV$  in an initial public

9 The participation constraint requires, in fact, that utility from entrepreneurship is higher than utility from a worker's career. Noting (2), the constraint is  $R^m \cdot (y^E - \beta W) \geq R^m \cdot W$ , and reduces to (10) after eliminating common terms. Furthermore, to support sustained growth, effort cost must grow at the same rate as other wages. See, for example, Section 6 of Michelacci and Suarez (2002).

10 With  $F$  VCs and  $E$  entrepreneurial firms, each VC fund finances and advises  $E/F$  portfolio companies on average. In assuming linear cost, the number of portfolio companies is indeterminate at the individual level but is implied in equilibrium by a free entry condition. With convex cost, Kannianen and Keuschnigg (2003) derive an optimal number of portfolio companies per VC. Alternatively, the search approach of Keuschnigg (2003) explains portfolio size as a result of VCs searching for investments prior to contracting.

11 In focussing on the importance of VC value added for long-run growth, and to keep the model simple, we have excluded the outside option of bank financing where projects could be funded without

offering, or a trade sale. The entrepreneur retains a share  $s$ , yielding a further expected gain equal to  $psnV$  while the VC is entitled to a share  $1 - s$  of revenues. When agents decide about effort, the terms of contract are fixed and initial investments are sunk. At this stage, the VC can influence only the part  $ep(a)(1 - s)nV - \gamma aW$  of expected profit. Taking the effort of the entrepreneur as given, she chooses a level of advice as in (10c). The entrepreneur, in turn, chooses effort among discrete alternatives  $e \in \{0, 1\}$  whichever yields a higher return  $e \cdot [p(a)snV - \beta]$ , giving rise to the incentive constraint (10b). The entrepreneur's incentive to put up high effort increases when the VC advises more intensively and, thereby, raises the survival fortunes of the firm. The VC, in turn, advises only if the entrepreneur's effort is high, but wastes no time when the entrepreneur is shirking. The incentive constraints are thus mutually interdependent, implying that efforts are strategic complements.

The solution of the contract reflects the idea that agents want to maximize the joint surplus before distributing it.<sup>12</sup> When an entrepreneur confronts competitive VCs, she will allow them no more than to break even,  $y^F = 0$ , so that the VC's participation constraint is binding. Whatever the value of  $s$ , she will extract the maximum possibly upfront payment  $bW$ . Substituting this from (10a) into the objective function shows that the entrepreneur is able to appropriate the entire joint surplus of the project,

$$\Omega W = p(a)nV - \gamma aW - \beta W - kW - W. \quad (11)$$

When the entrepreneur proposes a deal, she anticipates how the terms of contract affect the VCs and her own incentives in the effort stage. From equation (11), it is immediately obvious that her return would be a maximum if the VC would expend a first best level of advice,  $p'(a)nV = \gamma W$ . Comparing to (10c), the VC should thus be full residual claimant. But  $s = 0$  is not possible since this would undermine the entrepreneur's own incentives at a later stage and thus violate the incentive constraint. To retain her own incentives for high managerial effort which is critical for the survival prospects of the firm, she must keep a minimum profit share  $s$  satisfying  $IC^E$  with equality. Hence, the VC cannot be full residual claimant as would be required for the first best, and will accordingly advise less intensively.<sup>13</sup> Any higher share  $s$  would only subtract from the VC's share, reduce advice even more and would further diminish joint surplus.

Conditions (10b,c) simultaneously determine the equity share and the level of managerial support such that  $e = 1$ . With  $s$  and  $a$  already fixed, the entrepreneur will sell the share  $1 - s$  at the highest possible price  $Q = (b + k)W$  by extracting a maximum upfront payment  $bW$  until the VC's participation constraint becomes binding. With  $IC^E$  binding as well, the entrepreneur's objective in (10) reduces to  $bW - W \geq 0$ . Given that

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managerial support. See Ueda (2002) on the entrepreneur's choice between bank and VC financing, and Keuschnigg and Nielsen (2003b) for a model allowing for coexistence of bank and VC financing in market equilibrium.

12 This corresponds to the concept of a Pareto optimal contract as in Inderst and Mueller (2002).

13 This is parallel to Tirole's argument that the entrepreneur's incentive constraint reduces her "pledgeable income" and may thereby prevent funding by outside financiers of some projects with positive net present value (Tirole, 2001, p. 5).



free entry eliminates any remaining rents, the profit maximizing solution to the entrepreneur's problem in a free entry equilibrium is, thus,

$$\begin{aligned} (a) \quad PC^E : b &= 1, \\ (b) \quad IC^E : p(a)v \cdot s &= \beta, \quad v \equiv nV/W, \\ (c) \quad IC^F : p'(a)v \cdot (1 - s) &= \gamma. \end{aligned} \tag{12}$$

It is convenient to define relative venture returns  $v$  which is the IPO value  $nV$  of a firm with  $n$  patents divided by the wage rate. Note also the recursive structure of the problem. Conditions (12b,c) determine  $a(v, \cdot)$  and  $s(v, \cdot)$  in terms of venture returns and other parameters. The equity share fixes the minimum required bonus at effort stage to reward the entrepreneur for her critical input. The upfront payment is then chosen to extract any remaining rent from the VC. Finally, free entry of entrepreneurs will squeeze the upfront payment to eliminate entrepreneurs' rents as well.

## 2.5. Equilibrium

The paper studies symmetric equilibria. Free entry of firms eliminates rents. Continued entry into entrepreneurship diverts labor from production so that output and profits per product line and accordingly patent values fall, see equations (6)–(8). Entry squeezes relative venture returns  $v$  until a zero profit condition holds in equilibrium. Setting  $\Omega = 0$  in equation (11) and dividing by the wage rate yields

$$p(a(v)) \cdot v = \Phi(v), \quad \Phi(v) \equiv 1 + k + \beta + \gamma a(v). \tag{13}$$

In free entry equilibrium, relative venture returns  $v$  are exclusively determined by cost to market  $v = \Phi/p$ . The total cost of starting a firm in terms of wages is  $\Phi$ , including managerial effort and consulting costs. Given the risk involved in start-up entrepreneurship, one needs to start  $1/p$  firms on average to bring one firm to production stage. To achieve break even, the value of a mature firm must at least match cost to market.

Agents are either entrepreneurs or workers, with a total mass of unity. With each entrepreneur starting only one firm, the number of projects that must be financed each period, is equal to the number of entrepreneurs. With  $E$  entrepreneurs, this leaves  $1 - E$  workers. The workforce is decomposed into  $L$  production workers,  $kE$  workers in start-up firms, and  $\gamma aE$  advisors in VC firms. The resource constraint is, thus,

$$L + (1 + k + \gamma a)E = 1. \tag{14}$$

Workers and consultants each earn a wage income,  $y^L = W$ , while entrepreneurs receive an expected capital income of  $y^E = psnV + bW$  when the start-up firm is sold. Using  $b = 1$  and (10b), aggregate income is

$$y = \int_0^1 y^i di = y^L \cdot (1 - E) + y^E \cdot E = W + \beta W \cdot E. \tag{15}$$

According to equation (3), savings is a fraction of income adjusted for effort cost.

Income of an entrepreneur less effort cost is  $y^E - \beta W = W$  by free entry, yielding aggregate savings of

$$S = m \cdot \int_0^1 (y^i - h^i) di = m \cdot W. \quad (16)$$

The value of a mature firm at the end of period  $t$ , equal to the present value of profits earned in period  $t + 1$  or later, is  $V_t$ . The number of product lines at the beginning of next period is  $N_{t+1}$ , so that  $N_{t+1}V_t$  is the aggregate value of mature firms at the end of period  $t$ . Savings of young agents must buy all outstanding assets (patents),

$$S_t = N_{t+1}V_t. \quad (17)$$

A last equilibrium condition states how the number of product lines evolves over time. Given  $N_t$  units at the beginning of the period, a fraction  $1 - \delta$  of them becomes obsolete, leaving  $\delta N_t$  at the end of the period. By the law of large numbers, only a fraction  $p$  of the start-up projects succeeds in the aggregate while a fraction  $1 - p$  fails. Given  $E$  start-ups,  $n$  patents per firm and a success rate  $p$ , the number of varieties next period is augmented by  $pnE$  new firms maturing to production stage:

$$N_{t+1} = \delta N_t + p(a_t)n_tE_t. \quad (18)$$

The model is now complete. Appendix A derives the market clearing of final goods which follows by Walras' Law.

The solution is much simplified by the recursive structure of the model. The incentive constraints in equation (12b,c) simultaneously yield the equilibrium profit share and level of VC advice in terms of relative venture returns,  $s(v)$  and  $a(v)$ . Taking account of this, the free entry condition (13) autonomously determines competitive returns  $v$  in equilibrium and, in turn,  $s(v)$ ,  $a(v)$  and the success rate  $p$ . The nature of the dynamic equilibrium, however, depends on the extent of spillovers in private research.

### 2.5.1. Transitional Growth

If knowledge spillovers are absent, entrepreneurs have a fixed productivity  $n$  in developing new products. They generate  $n$  patents per researcher if they are successful, and none if they fail. The equilibrium number of product designs follows from the savings investment equations (16)–(17),  $mW_t = N_{t+1}V_t$ . Multiplying by  $n/W_t$  and noting relative venture returns  $v = nV/W$  yields, after a slight rearrangement,

$$N_{t+1} = m \cdot n / v_t. \quad (19)$$

There is no transitional dynamics since equilibrium returns are time autonomous by the free entry condition (13). The steady state is attained within one period. Quite intuitively, product variety increases with a higher savings rate  $m$ , a larger research productivity  $n$ , and falls with higher relative venture returns or patent value  $v$ . High patent value reflects large outputs and profits per product line, see equations (6)–(7). Hence, a large part of the labor resource is absorbed in production, leaving only a few entrepreneurs to invent new

designs. Given (19), the equilibrium level of entrepreneurship is residually determined by (18). Wages are proportional to product variety as in (7). The equilibrium return  $v_t$  implies, together with  $N_t$ , patent value  $V_t$ . Since  $\pi_{t+1}$  depends on next periods allocation, the interest rate must adjust to satisfy (8).

### 2.5.2. Endogenous Growth

Knowledge spillovers boost research productivity and the number of inventions per firm. Growth becomes self-sustained if inventiveness increases linearly with the aggregate stock of knowledge (see, for example, Grossmann and Helpman, 1991; Aghion and Howitt, 1998; Michelacci and Suarez, 2002, in the context of a model with business creation),

$$n_t = \eta \cdot N_t. \quad (20)$$

The solution is much the same as before, except that the definition of  $v$  in (12) must now take account of (20). Relative venture returns remain stationary, however, as long as firm values ( $\eta N_t V_t$ ) and wages  $W_t = \alpha N_t$  both grow with the same factor  $G$ . Relative venture returns  $v_t = V_t \eta / \alpha$  are strictly proportional to the value  $V_t$  of a single patent which remains constant in the long-run with a constant allocation of labor. To obtain the growth rate, multiply the savings investment condition (16) and (17),  $N_{t+1} V_t = m W_t$ , by  $n_t / W_t$  and use (20) to get

$$G_{t+1} = m \cdot \eta / v_t, \quad G_{t+1} \equiv N_{t+1} / N_t. \quad (21)$$

The growth rate is thus fully determined by equilibrium venture returns  $v$ . The same intuition as in (19) applies in the endogenous growth case as well. Again, knowledge accumulation in (18) residually determines equilibrium entrepreneurship. Using (20) and dividing by  $N_t$  yields

$$G_{t+1} = \delta + p(a(v_t)) \eta \cdot E_t. \quad (22)$$

The stationary growth rate is attained within one period. Once the number of entrepreneurs is determined, and knowing the labor requirement for consulting per project, one can compute by (14) the labor allocated to manufacturing and thereby final and intermediate goods production according to the closed form solutions given in (7).

## 3. Venture Capital Backed Growth

How do innovation and growth depend on the parameters that determine the quality of VC finance? We are particularly interested in the comparative static effects of changes in the VC's managerial productivity  $\varepsilon$  and her effort cost  $\gamma$ , the entrepreneur's effort cost  $\beta$ , and the size of the start-up investment  $k$ . A higher value of  $\varepsilon$  indicates increased knowhow, experience and industry knowledge of VC firms, making consulting activity more effective. Any given time input by the VC then adds more value to the firm and achieves a higher success probability. We have argued that the entrepreneur contributes the

technological innovation and business idea while the VC adds commercial knowhow. Larger consulting cost  $\gamma$  might thus reflect a more difficult business environment under rapidly changing market conditions which presumably makes the commercial input more critical for the success of the firm. An increased effort cost  $\beta$  of entrepreneurs, in contrast, might result from greater risks of technological development, requiring a larger effort to sustain the prospects of the firm. Finally, product development and innovation might simply become more capital intensive and require a larger start-up investment  $k$ .

We now provide a comparative static analysis of VC backed growth by log-linearizing the stationary transformation of the model at an initial steady state position.<sup>14</sup> This exercise reveals how various shocks affect the quality of VC finance, entrepreneurship, and innovation driven growth. Since we start from a position of stationary equilibrium, the initial values of stationary variables are constant and time autonomous. The hat notation, for example,  $\hat{a} = da/a$ , indicates percentage changes relative to constant initial values after a shock occurs. All shocks are assumed permanent. The analysis applies, of course, only to variables that remain constant in the long-run. We henceforth consider only the case of endogenous growth.

### 3.1. *Venture Capital Value Added*

One of the key propositions of the empirical literature is that VC adds value to young firms, making them grow faster and larger, and making them less vulnerable to business failure. By this propagation mechanism, VC can importantly enhance a country's ability to innovate and thereby promote growth. On the other hand, the productive contribution of VC is not a guaranteed matter as part of the empirical evidence for Europe seems to suggest. In the present model, the VC's advisory role addresses the commercial inexperience of young entrepreneurs and results in a larger success rate. We are interested in how important structural parameters determine the quality of VC finance, as measured in terms of VC involvement in new firms. Depending on her managerial productivity  $\varepsilon$  as a measure of industry knowledge, technological knowhow and investment experience, the VC's consulting input determines the firm's success probability as in (9),

$$\hat{p} = \hat{\varepsilon} + (1 - \theta)\hat{a}. \quad (23)$$

The constraints (12b,c) show how the incentives to help with managerial advice and the propensity for entrepreneurial effort depend on the profit sharing rule and other parameters. In fact, (12b) gives the minimum profit share  $s$  that still makes the entrepreneur put up her critical effort. When the expected return increases, either because of a higher relative firm value  $v$  or a higher success chance  $p$ , a smaller share suffices to compensate the entrepreneur for her effort, or human capital contribution. When her effort

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14 Helpman (1993) formulates an endogenous growth model such that it is linear in some endogenous stationary variables, including the growth rate. Here, the stationary transformation of the model is non-linear but we can uncover the comparative statics by means of log-linearization.

cost increases, she requires, of course, a higher equity share. This, in turn, implies a lower share for the VC which diminishes incentives for advice:

$$\begin{aligned} \text{(a)} \quad \hat{s} &= \hat{\beta} - \hat{\varepsilon} - \hat{v} - (1 - \theta) \hat{a}, \\ \text{(b)} \quad \theta \hat{a} &= \hat{\varepsilon} + \hat{v} - \frac{s}{1-s} \hat{s} - \hat{\gamma}. \end{aligned} \quad (24)$$

These constraints are mutually interdependent and simultaneously fix advice and incentive compatible profit sharing,

$$\begin{aligned} \text{(a)} \quad \hat{a} &= \frac{1}{\theta - s} \left[ \hat{v} + \hat{\varepsilon} - (1 - s) \hat{\gamma} - s \hat{\beta} \right], \\ \text{(b)} \quad \hat{s} &= -\frac{1-s}{\theta - s} \left[ \hat{v} + \hat{\varepsilon} - (1 - \theta) \hat{\gamma} - \theta \hat{\beta} \right], \end{aligned} \quad (25)$$

where  $\theta > s$  by the following stability argument.<sup>15</sup> Starting with an exogenous shock to her effort cost  $\beta$ , the entrepreneur requires in (24a) a higher share  $s$  to reward her effort. When the financier is left with a smaller share  $1 - s$ , she feels less interest in the firm and advises less, see (24b). As the success probability falls and the project becomes more risky, the entrepreneur demands an even higher equity share to reward her effort which again erodes the VC's share, and so on. This cycle converges when  $\theta > s$ , giving overall effects on advice and profit share as in (25).

### 3.2. Equilibrium Venture Returns

As we have shown in Section 2.5, long-run growth critically depends on equilibrium venture returns  $v$ . With competitive VCs, free entry of entrepreneurs eventually exhausts all rents to VC backed start-up activity. Equilibrium venture returns just compensate for all relevant costs incurred by inventive start-up entrepreneurs. To determine the effect of structural parameters on venture returns, we log-linearize the entrepreneur's surplus in (13). Substituting (12b), we have  $\Omega = (1 - s)pv - \gamma a - (1 + k)$ .<sup>16</sup> Starting from a zero profit position, we define  $\hat{\Omega} \equiv d\Omega$ . Given the condition (12c), the effect of a change in advice drops out, leaving  $\hat{\Omega} = (1 - s)p\hat{v}(\hat{v} + \hat{\varepsilon}) - sp\hat{v}\hat{s} - k\hat{k} - a\gamma\hat{\gamma} = 0$ . Replacing  $a\gamma$  by (A.7) and subsequently  $1 + k$  by (A.5), we get

$$\hat{v} = \frac{s}{1-s} \hat{s} + \theta \xi \hat{k} + (1 - \theta) \hat{\gamma} - \hat{\varepsilon}, \quad \xi \equiv \frac{k}{1+k}, \quad (26)$$

where we have defined by  $\xi$  the share of start-up cost  $kW$  in the overall price  $(1 + k)W$  that the VC pays for the stake  $1 - s$  in the firm.

15 The Appendix states in (A.8) a condition for  $\theta > s$  which is always fulfilled if the informational friction indicated by the entrepreneur's unobserved effort cost  $\beta$  is not too high.

16 Alternatively, if (10b) holds with equality,  $\Omega = b - 1$  in (10). Given that the entrepreneur is able to extract the entire rent of a VC by asking for a higher upfront payment  $b$ , (10b) holds with equality as well, giving  $(1 - s)p\hat{v} - \gamma a - (1 + k) = b - 1 = \hat{\Omega}$ .

An increase in effort cost  $\gamma$  or a higher start-up cost  $k$  require higher venture returns to retain profitability in VC investing. An increase in the entrepreneur's minimum required equity share, leaving a smaller share to the VC and thereby discouraging VC support, reduces joint surplus and similarly requires a higher return  $V$  to restore a break even. A higher success rate on account of an increased productivity of advice, reflecting more experience and knowhow on the part of the VC, allows for lower returns. Substituting the entrepreneur's incentive compatible profit share from (25b) shows how the return in free entry equilibrium adjusts to shocks,

$$\hat{v} = (1 - \theta)\hat{\gamma} + s\hat{\beta} + (\theta - s)\xi\hat{k} - \hat{\varepsilon}. \quad (27)$$

Increases in effort cost of entrepreneurs or VCs and a higher initial start-up cost  $k$  all require higher equilibrium venture returns to retain profitability of VC investments while increased effectiveness of VC advice allows for lower returns. These returns feed back on the level of advice and the entrepreneur's equity share in zero profit equilibrium. Substituting into (25) gives<sup>17</sup>

$$\hat{a} = \xi\hat{k} - \hat{\gamma}, \quad \hat{s} = (1 - s)(\hat{\beta} - \xi\hat{k}). \quad (28)$$

In equilibrium, larger start-up cost requires higher returns and thereby boosts advice. As a result, the success probability increases which allows for a lower incentive compatible profit share of the entrepreneur. Larger consulting cost of the VC reduces equilibrium advice despite of higher venture returns. Larger entrepreneurial effort cost requires a larger share of the entrepreneur to elicit her crucial input.

### 3.3. Innovation and Start-up Entrepreneurship

The number  $N$  of specialized capital goods is a measure of factor productivity and determines final output that can be produced by a given amount of labor, and wages as in (7). The introduction of new goods and the ensuing gains from specialization boost factor productivity and growth. If spillovers in industrial research are sufficiently large, that is, linear in the aggregate knowledge stock as measured by the number of goods, growth is self-sustained. Clearly, with a constant labor allocation, output and profits of a given product line remain constant as in (6) and (7), implying a constant patent value  $V_t$  in (8) as well. The equilibrium growth rate then follows from the savings investment identity (16) and (17),  $mW_t = N_{t+1}V_t$ . Wages and savings grow along with knowledge  $N$ , so that successive young generations can buy a growing number of patents. The growth rate is stated in (21) and depends on venture returns which reflect the costs of start-up entrepreneurship in free entry equilibrium. We are interested in how the structural parameters of the VC industry affect the growth rate. Combining (21) and (27), we have

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<sup>17</sup> As a check,  $\hat{a}$  and  $\hat{s}$  are directly obtained by log-linearizing (A.5) and (A.7) which already reflect the zero profit condition. Substituting (28) into (24a) yields (27). The more roundabout derivation in the text helps to interpret the economic adjustment mechanism.

$$\hat{G} = -\hat{v} = \hat{\varepsilon} - (1 - \theta)\hat{\gamma} - s\hat{\beta} - (\theta - s)\xi\hat{k}. \quad (29)$$

Innovation reflects entrepreneurship  $E$  which is driven by the propensity of agents to start a firm rather than opt for a safe industry career. It also reflects the contribution by VCs whose managerial advice promotes the professionalization and commercial expertise of entrepreneurs and raises their survival chance  $p$ . In competitive equilibrium, free entry eliminates rents so that the entrepreneurs' participation constraint holds with equality. Entrepreneurs are therefore indifferent between starting a firm and employment in industry. Entrepreneurship is thus determined exclusively by the demand for new goods to sustain innovation based growth. It follows residually from the accumulation relationship (22). The log-linearized version is  $\hat{G} = \sigma \cdot (\hat{p} + \hat{E})$  where  $\sigma$  is the share of new goods in the overall number of varieties next period. Using (23), (28) and (29) yields

$$\sigma\hat{E} = (1 - \sigma)[\hat{\varepsilon} - (1 - \theta)\hat{\gamma}] - s\hat{\beta} - [\theta - s + (1 - \theta)\sigma]\xi\hat{k}, \quad \sigma \equiv p\eta E/G < 1. \quad (30)$$

To summarize, Section 3.1 shows the results from partial equilibrium which determines a solution for profit sharing, advice and a success rate for a given venture return. Section 3.2 endogenizes venture returns, relative to the wage rate, in the free entry equilibrium which feed back on the incentives for joint effort and thereby changes equilibrium VC support and profit sharing. Venture returns reflect all relevant costs incurred in VC backed start-up entrepreneurship, that is, the foregone wage of the entrepreneur, an initial start-up cost, plus the effort and consulting costs of the entrepreneur and VC. Restricting the analysis to the case of linear knowledge spillovers, Section 3.3 explains the equilibrium growth rate as an outcome of the savings investment identity. The following subsection discusses several comparative static scenarios that demonstrate how some important structural characteristics of the VC industry can affect innovation based growth.

### 3.4. Comparative Statics

#### 3.4.1. Consulting Productivity

Suppose that knowhow and experience in the VC industry and, therefore, managerial productivity  $\varepsilon$  improve permanently. When VCs are more sophisticated, the same time input into the consulting activity receives a higher marginal expected return which strengthens incentives for more active VC involvement. When VCs allocate more consulting time to their portfolio companies, and this time input is more productive, the success probability increases on both accounts. With a higher expected return on the project, a smaller profit share suffices to secure the entrepreneur's critical effort, leaving for the same price a higher share to the VC. More investor sophistication boosts profits. As more firms get started, venture returns will eventually decline until rents are exhausted. Lower returns diminish incentives for advice which tends to offset the initial encouragement on account of higher consulting productivity. In zero profit equilibrium, consulting hours thus remain the same (see (28)) but they are more productive, giving  $\hat{p} = \hat{\varepsilon}$  in (23).

With lower venture returns (relative to the wage rate), the same amount of savings can

buy more patents or intermediate goods firms next period. Consequently, higher consulting productivity boosts innovation and growth. The growth rate increases by  $\hat{G} = \hat{\varepsilon}$ , see (29), which raises the demand for new entrepreneurial firms. On the other hand, since start-up firms are more successful on account of closer VC involvement, a larger fraction  $p$  of them survives to the mature production stage. Hence, the same mass of new firms could be generated by a smaller number of entrepreneurs. However, the net effect on entrepreneurship in (30) is positive,  $\sigma\hat{E} = (1 - \sigma)\hat{\varepsilon}$ .

### 3.4.2. Managerial Effort Cost

Rapidly changing market conditions, red tape and other regulations might increase the VC's cost of providing finance and advice. With more costly managerial effort, VCs will obviously cut back on their consulting activity which raises start-up risk. With expected returns  $pv$  lower, entrepreneurs demand a higher profit share to compensate for their own critical effort, see (25). As VCs become less inclined to add value to their portfolio companies, profitability in start-up entrepreneurship must suffer, forcing potential entrepreneurs to give up their business plans and pursue employment instead. When the number of mature firms declines but more labor is allocated to production, output and profits of capital goods firms expand and makes these firms more valuable. In competitive free entry equilibrium, venture returns  $v$  must increase to an extent that allows start-up firms to break even again,  $\hat{v} = (1 - \theta)\hat{\gamma}$  in (27). Despite this encouraging advice, the *net* effect on VC support remains negative as in (28).

Due to the increased consulting cost, venture returns are higher. The same amount of savings can thus buy only a smaller number of mature firms so that the growth rate in (29), or (21), declines. Slower growth squeezes the demand for start-up entrepreneurship. Inspecting (30) and (28), we find that increased consulting cost not only shrinks the number of start-up firms but also makes them less successful. Since advice per project is lower, a smaller fraction of the same number of start-ups is successful, with obvious consequences for innovation based growth.

### 3.4.3. Entrepreneurial Effort Cost

When starting a firm is perceived more costly in terms of required entrepreneurial effort, entrepreneurs must insist on a larger equity share to secure their own critical effort during the start-up period. Unfortunately, the entrepreneur's share is at the expense of the VC's share which undermines the VC's incentive to get involved in the firm. When projects receive less VC support, they become less profitable. As the rate of business creation declines, venture returns will eventually increase which restores again the incentives for advice. In free entry equilibrium, the entrepreneur retains a higher profit share, the VC supplies the same level of advice, and venture returns are higher, see (27) and (28).

As mature firms become more valuable, aggregate savings acquire only a smaller number of them. The innovation rate accordingly declines by  $\hat{G} = -s\hat{\beta}$  which reduces the demand for new firms and retards start-up entrepreneurship,  $\sigma\hat{E} = -s\hat{\beta}$ , see (29) and (30).



### 3.4.4. *Size of Start-up Investment*

When business formation requires a larger scale  $k$  per project, VCs need to inject a larger amount of equity to get the firm started. Managerial advice and profit sharing are not directly affected because these investment outlays are unrelated to the agents' effort. Larger start-up cost, however, erodes profits and thereby dictates exit which subsequently raises venture returns to restore profitability. In zero profit equilibrium, mature firms become more valuable. Expecting larger returns, VCs start to advise more intensively. This tendency is reinforced by the fact that entrepreneurs require a lower profit share to secure their critical input and thus willingly cede a larger profit share to the VC, see (24) and (25). Entrepreneurs receive more VC value added in return as is shown in (28). When venture returns are up, fewer of these firms can be acquired out of savings so that growth slows down by  $\hat{G} = -(\theta - s)\xi\hat{k}$  as in (29). Demand for entrepreneurship shrinks accordingly as the last term in (30) indicates. The rate of business creation is smaller and eventually results in slower growth.

### 3.5. *Taxation of the VC Industry*

The preceding analysis has demonstrated that the structural characteristics of the VC industry can importantly influence innovation based growth. However, these deep parameters are probably beyond the influence of public policy. An important policy question obviously is whether VC backed start-up investment can be influenced more directly by means of taxes or subsidies. Indeed, most countries offer many programs to encourage start-up entrepreneurship, and part of it is explicitly addressed towards the VC industry. Most of these programs subsidize the cost of capital by means of subsidized interest, credit guarantees to facilitate access to bank finance at lower interest, or direct subsidies to start-up investments.<sup>18</sup> These subsidies have in common that they are not performance related and therefore cannot strengthen incentives. Within our formal framework, these subsidies can be understood as a subsidy  $zk$  to start-up cost of a VC backed firm. Another concern by representatives of the industry is the capital gains tax. Since these firms do not generate revenues during the start-up phase, the return to entrepreneurs and investors mainly consists of capital gains. For this reason, policy makers often propose to cut the capital gains tax to encourage innovative young firms.<sup>19</sup>

In the present model, there would be indeed a need for a policy that strengthens VC advice and promotes growth. First, when a VC advises more intensively, the benefits from the firm's higher success probability are shared with the entrepreneur. Depending on the incentive compatible profit share, the VC appropriates only part of the returns but must bear the full private effort cost of managerial advice. As we have argued in the paragraph

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<sup>18</sup> See, for example, Lerner (1999) on the SBIR program in the United States.

<sup>19</sup> Keuschnigg (2004) and Keuschnigg and Nielsen (2003a, 2004) provide a more detailed tax analysis in static models of start-up entrepreneurship with VC finance. See Poterba (1989) and Gompers and Lerner (1998) for empirical analysis of capital gains taxation on VC financing and entrepreneurship.

following equation (11), VC involvement in new firms tends to be too low compared to a first best allocation. Second, in a model of horizontal product differentiation, the innovation rate is too low since entrepreneurs cannot appropriate the spillovers that their own inventions create for subsequent generations of entrepreneurs by adding to the public knowledge stock (see Grossman and Helpman, 1991, Chapter 3).

This section shows that it is possible to restructure the taxation of VC backed entrepreneurship in a way that avoids handing out any net subsidy to the industry but nevertheless is able to sharpen incentives and boost growth. We consider the following self-financed policy initiative: instead of subsidizing start-up investment, we tax it, and use the proceeds to finance a targeted cut in the capital gains tax. Since we want to keep the policy analysis simple, we start from an initial untaxed equilibrium (to avoid complicated tax base effects) and therefore literally cannot analyze a tax cut. We can, however, mimic a tax cut by introducing a small revenue subsidy  $\tau pv$ . We thus impose the following budget constraint:

$$\tau pv = zk, \quad (31)$$

where  $z$  is now a tax that adds to the private start-up cost, and  $\tau$  is a revenue subsidy (mimicking a capital gains tax cut).

With the presence of taxes and subsidies, the solution to the problem in (10) reflects the same economic considerations. The entrepreneur is able to extract the entire surplus of the project which is, however, reduced to zero by free entry,

$$\Omega = (1 + \tau) p(a)v - \gamma a - \beta - (1 + z) k - 1 = 0. \quad (11')$$

Comparing (11') and (11) shows that the private project surplus remains unchanged when the government follows the policy given by (31). This reflects the fact that the policy is self-financing and avoids any net subsidy to VC financed start-up firms. By the same arguments as in Section 2.4, the VC contract is now given by the conditions

$$\begin{aligned} \text{(a)} \quad PC^E : b &= 1, \\ \text{(b)} \quad IC^E : p(a)v \cdot (1 + \tau)s &= \beta, \\ \text{(c)} \quad IC^F : p'(a)v \cdot (1 + \tau)(1 - s) &= \gamma. \end{aligned} \quad (12')$$

The formal comparative static analysis of tax policy is relegated to Appendix C. Quite intuitively, a subsidy on venture returns strengthens incentives for effort. Its direct effect is therefore to boost VC advice. Further, a smaller minimum profit share suffices to elicit the entrepreneur's critical effort level, see (A.9). The induced increase in consulting intensity is very desirable since VC advice is too low compared to a first best allocation. As we argued in the paragraph following (11), the VC should be full residual claimant on the firm's returns to be willing to expend a first best level of consulting support. But the entrepreneur must also receive a minimum profit share to assure her own critical input. The need to share revenues implies that the VC bears the full marginal cost of consulting but cannot claim the full benefits of advice. Joint surplus is not maximized at the privately determined level of VC support which is given by the incentive constraint  $(1 - s)p'v = \gamma$ . Joint surplus would be maximized with  $p'v = \gamma$ . With a revenue subsidy chosen such that

$(1 + \tau)(1 - s) = 1$ , the VC's choice according to her incentive constraint (12') would coincide with the first best level of advice.

Despite of the fact that the policy (31) is self-financing and does not provide any net subsidy, it raises joint surplus by stimulating value increasing VC advice which is too low in private equilibrium. A higher joint surplus attracts new start-up firms until rents are exhausted in the free entry equilibrium. Following the analysis in (10–12), we find that venture returns fall after the policy is put in place.<sup>20</sup> Although lower venture returns clearly discourage private effort, the direct effect of the revenue subsidy remains dominant and thereby assures more VC support in equilibrium, see (A.13). As the value of mature firms declines, the young generation of savers can acquire a larger number of firms next period, leading to an increase in the growth rate, see (21), or its change in (A.14). Increased growth boosts demand for entrepreneurship. On the other hand, start-ups are more successful so that fewer entrepreneurs are needed to introduce a given number of new goods to the market. If the share  $\sigma$  of new goods relative to the total product range is small, and entrepreneurial effort cost  $\beta$  is sufficiently large (giving a large profit share  $s$ ), the demand effect is likely to dominate so that the policy expands entrepreneurship as shown in (A.15).

The analysis of this subsection shows that tax policy can strengthen incentives in VC financing, contribute to more success in VC backed start-up investment, and boost innovation based growth. This is achieved without any net cost to the general tax payer since the policy is constructed to be self-financing. Our model of VC backed growth also calls for an output subsidy on mature firms to offset mark-up pricing of intermediate capital goods, see (6). Since the role of an output subsidy is well understood, we do not repeat it in this paper. There is a new twist, however, as the analysis of Keuschnigg (2003) has shown in a static model with differentiated capital goods. An output subsidy not only reduces the demand price to bring it closer to marginal cost, but it also raises venture returns by making mature firms more valuable. It thereby not only raises the reward for start-up investment but also strengthens incentives for VC advice. In our framework, the output subsidy would thus hold a double dividend in targeting markup pricing and strengthening VC support.

#### 4. Conclusions

The policy interest in establishing an active VC industry mainly derives from the expectation that young innovative firms are an important source of innovation and growth and that VC can importantly enhance the performance of start-up firms. This expectation is confirmed by the empirical literature which established that VC backed companies pursue more radical innovations and more aggressive marketing strategies and that VC speeds up the professionalization and growth of these firms. In line with these arguments, Kortum and Lerner (2000) found indeed that VC funding accounts for a disproportionately large share of US innovative activity. Yet, the literature has left the precise transmission

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20 Note that  $\hat{\tau} > 0$  introduces a small revenue subsidy and  $\hat{z} > 0$  a small tax on start-up cost.

channels largely unspecified. Almost without exception, the theoretical finance literature on VC is partial equilibrium in nature, dealing mostly with the incentive problems between entrepreneurs and outside financiers and how they are dealt with by financial contracting. The link to economy wide innovation was not formalized. Despite of the considerable interest of policy makers in VC as a means to promote innovation based growth, the literature has not yet analyzed the role of public policy to determine VC backed innovation in equilibrium.

The main contribution of this paper is to include VC finance in a dynamic general equilibrium model of innovation and to investigate the effects of public policy. Innovation results from the joint efforts of both entrepreneurs and informed financiers. While the entrepreneur contributes the key technological idea in starting up a new firm, the VC supports the firm with managerial advice which draws on her extensive business experience and industry knowledge. The model endogenously explains (i) the incentives of entrepreneurs to start a firm, (ii) the willingness of VCs to become actively involved and to add value to start-up firms, (iii) the equilibrium venture returns, or market value of new firms, and (iv) innovation based growth. It was shown how structural parameters of the VC industry, such as experience and market knowledge of VCs, determine the growth rate. Tax policy holds interesting options to strengthen VC support and promote growth. With horizontal product innovation and double moral hazard in VC financing, both the innovation rate and the degree of VC involvement in new firms tend to be too low in private equilibrium compared to a first best allocation. Policy should thus aim at both margins. It seems though that many of the existing programs including interest subsidies, loan guarantees or direct investment subsidies do not address the main problem. Although they promote entry, they do not succeed to sharpen incentives for effort since they are largely unrelated to performance. A targeted tax relief on capital gains, instead, applies only in case of success and thereby directly strengthens the reward to effort. The analysis in this paper suggests a revenue neutral restructuring of public policy by shifting public funds away from subsidies to the cost of capital and using the proceeds to cut capital gains taxes on VC backed firms. This policy approach would indeed stimulate VC support and promote innovation based growth.

## A. Appendix

### A.1. Final Goods Market

Young agents must buy all assets as in (17). Using (18) and replacing  $pnV$  by (13) yields

$$S_t = N_{t+1}V_t = \delta N_t V_t + (1 + k + \beta + \gamma a_t) W_t E_t. \quad (\text{A.1})$$

Aggregate budgets for first and second period consumption are  $C_{1,t} = y_t - S_t$  and  $C_{2,t} = R_t S_{t-1}$ . Replacing  $y_t$  by (15) and using the previous equation yields

$$\begin{aligned} C_{1,t} &= y_t - S_t = W_t - \delta N_t V_t - (1 + k + \gamma a_t) W_t E_t, \\ C_{2,t} &= R_t S_{t-1} = (\pi_t + \delta V_t) N_t. \end{aligned} \quad (\text{A.2})$$

The budget  $C_{2,t}$  follows by substituting  $S_{t-1} = N_t V_{t-1}$  and replacing  $R_t V_{t-1}$  by (8). Adding up (A.2) then yields  $C_t \equiv C_{1,t} + C_{2,t} = W_t - (1 + k + \gamma a_t) W_t E_t + \pi_t N_t$ . The resource constraint (14) implies  $C_t = W_t L_t + \pi_t N_t$ . Substitute the definition of  $\pi$  in (5) and note  $Y_t = W_t L_t + q_t x_t N_t$  from the production side:

$$Y_t = C_t + \kappa x_t N_t. \quad (\text{A.3})$$

### A.2. Zero Profit Restrictions

From (9), we have  $ap' = (1 - \theta)p$ . Therefore, the  $IC^F$ -constraint (12c) implies

$$\gamma a = (1 - \theta)(1 - s)pv. \quad (\text{A.4})$$

Using this to replace  $\gamma a$  on the r.h.s. of the zero profit condition (13), and using (12b) to replace  $\beta$  as well, we obtain after some rearrangement

$$(1 - s)\theta pv = 1 + k, \quad \frac{1 - s}{s} = \frac{1 + k}{\theta\beta}. \quad (\text{A.5})$$

With this result, we can explicitly compute the equity shares,

$$s = \frac{\theta\beta}{\theta\beta + 1 + k}, \quad 1 - s = \frac{1 + k}{\theta\beta + 1 + k}. \quad (\text{A.6})$$

Substituting  $pv$  from (A.5) into (A.4) gives an explicit solution for equilibrium advice,

$$a = \frac{1 - \theta}{\theta\gamma}(1 + k). \quad (\text{A.7})$$

Finally, use (A.6) to compute

$$\theta - s = \theta \cdot \frac{1 + k - (1 - \theta)\beta}{1 + k + \theta\beta} > 0 \quad \Leftrightarrow \quad \frac{1 + k}{1 - \theta} > \beta. \quad (\text{A.8})$$

### A.3. Taxation of the VC Industry

Comparative static analysis of tax policy in Section 3.5 follows the same lines as before. We consider small taxes, yielding small deviations from an untaxed equilibrium. The relative change in tax rates is defined as  $\hat{\tau} \equiv d\tau/(1 + \tau) = d\tau$  and  $\hat{z} = dz/(1 + z) = dz$ . By (23), VC advice boosts the success probability by  $\hat{p} = (1 - \theta)\hat{a}$ . The log-linearization of the IC-constraints in (12') and their simultaneous solution thus yields

$$\hat{a} = \frac{1}{\theta - s}(\hat{v} + \hat{\tau}), \quad \hat{s} = -\frac{1 - s}{\theta - s}(\hat{v} + \hat{\tau}). \quad (\text{A.9})$$

In free entry equilibrium, venture returns adjust to allow the entrepreneur and VC to break even on all relevant start-up costs as in (11'). It is convenient to write the surplus

as  $\Omega = (1 + \tau)(1 - s)pv - \gamma a - [1 + (1 + z)k]$ . Log-linearization yields  $\hat{\Omega} = (1 - s)pv(\hat{v} + \hat{\tau}) - spv\hat{s} - k\hat{z} = 0$ . Isolating  $\hat{v}$  and using (A.5) which still holds since we start from an untaxed initial zero profit equilibrium, yields

$$\hat{v} = \frac{s}{1-s}\hat{s} + \theta\zeta\hat{z} - \hat{\tau}, \quad \zeta \equiv \frac{k}{1+k}. \quad (\text{A.10})$$

The government budget constraint in (31) shows the tax on start-up cost which is required to finance the subsidy on venture returns. Starting from an untaxed position and using (A.5), the log-linearized version is

$$\hat{z} = \frac{pv}{k}\hat{\tau} = \frac{1}{(1-s)\theta\zeta}\hat{\tau}. \quad (\text{A.11})$$

Substitute  $\hat{s}$  from (A.9) and use the government budget restriction (A.11) to obtain equilibrium venture returns,

$$\hat{v} = (\theta - s)\zeta\hat{z} - \hat{\tau} = -\frac{(1-\theta)s}{\theta(1-s)}\hat{\tau}. \quad (\text{A.12})$$

The adjustment of venture returns feeds back on advice and profit sharing in (A.9),

$$\hat{a} = \zeta\hat{z} = \frac{1}{(1-s)\theta}\hat{\tau}, \quad \hat{s} = -(1-s)\zeta\hat{z} = -\frac{1}{\theta}\hat{\tau}. \quad (\text{A.13})$$

The conditions (21) and (22) determining the equilibrium growth rate and entrepreneurship are unchanged. Upon log-linearization of (21) and substituting (A.12), the self-financed revenue subsidy affects growth by

$$\hat{G} = -\hat{v} = \frac{(1-\theta)s}{\theta(1-s)}\hat{\tau} > 0. \quad (\text{A.14})$$

Entrepreneurship is determined residually by (22) which is  $\hat{G} = \sigma \cdot (\hat{p} + \hat{E})$  in log-linearized form. Using (23) and (A.13–14) gives

$$\sigma\hat{E} = (s - \sigma)\frac{1-\theta}{(1-s)\theta}\hat{\tau}, \quad \sigma \equiv \frac{p\eta E}{G} < 1. \quad (\text{A.15})$$

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