Optimal Reinsurance Program under Default Risk

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Institute of Insurance Economics
University of St.Gallen

Normative Insurance Demand Literature (1)

Mossin, J. (1968). *Aspects of Rational Insurance Purchasing*. JPE, 76, 553-568.

→ Binary loss states

→ Full coverage is optimal iff premium is actuarially fair

→ With positive cost loading, under-insurance is optimal

Arrow, K.J. (1963). *Uncertainty and the Welfare Economics of Medical Care*. AER, 55, 941-973.

→ Continuous loss distribution

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→ With positive cost loading, full coverage above a positive deductible is optimal

Normative Insurance Demand Literature (2)

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- → Full coverage is optimal iff premium is actuarially fair
- → With positive cost loading, under-insurance is optimal

Introducing Default Risk

Doherty, N.A., Schlesinger, H. (1990). *Rational Insurance Purchasing: Consideration of Contract Nonperformance*. QJE, 105, 243-253.

- → Over- or under-insurance may be optimal, even though cost-loading is zero

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- → Continuous loss distribution
- → Full coverage is optimal iff premium is actuarially fair
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Introducing Default Risk

Mahul, O., Wright, B.D. (2004). *Implications of Incomplete Performance for Optimal Insurance*. Economica, 71, 661-670.

- → Optimal Deductible is always strictly positive
- → Above deductible, full or more-than-full coverage is optimal

Normative Insurance Demand Literature (3)

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Mitigating Default Risk

Reichel, L., Schmeiser, H., Schreiber, F. (2017). *Sometimes More, Sometimes Less: Prudence and the Diversification of Risky Insurance Coverage*. Working Paper.

→ Is it optimal to increase or decrease the coverage, if default risk can be diversified?

Mitigating Default Risk

Today's presentation:

→ **Optimal combination of (re)insurance policy and hedging instrument (e.g., CDS, Collateralization, Letter of Credit)**

→ Optimal policy, if default risk can be diversified (with several (re)insurers)

Illustration of a Reinsurance Program

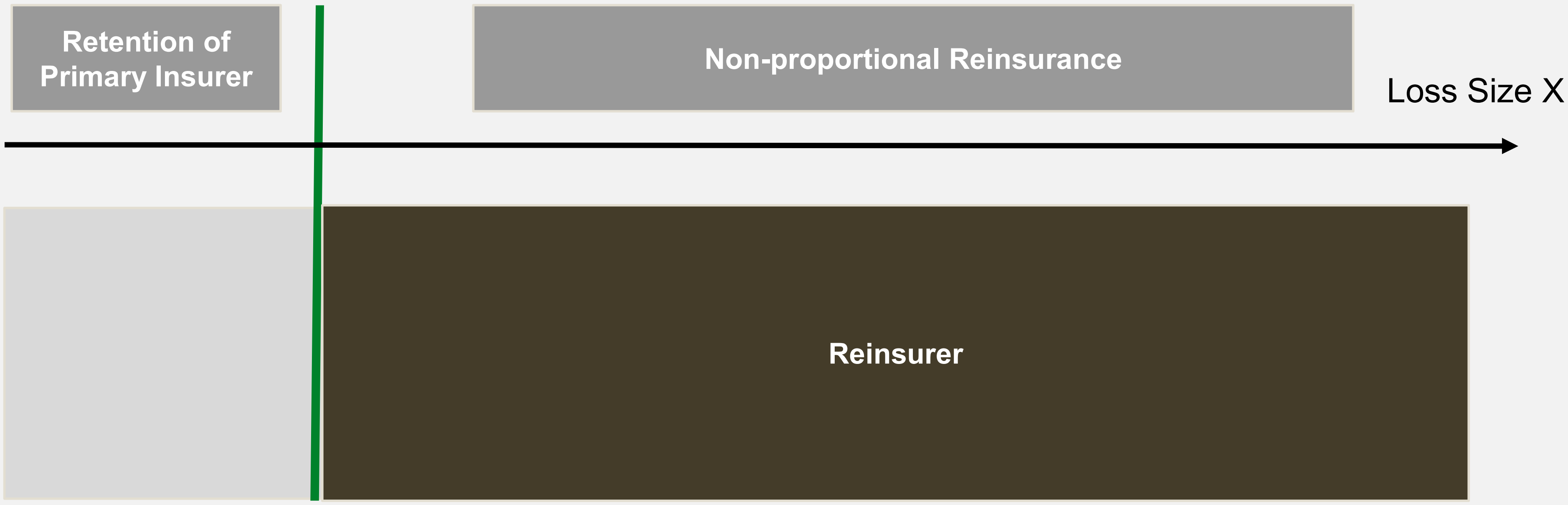
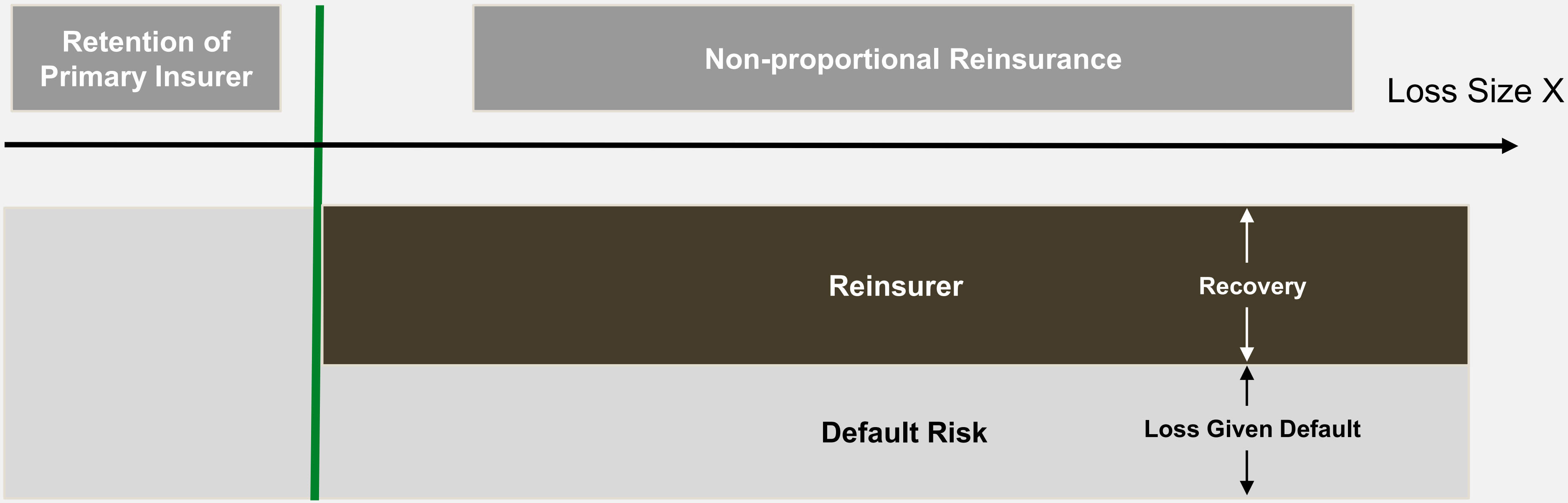


Illustration of a Reinsurance Program with Default Risk



Literature on Optimal Reinsurance with Default Risk

Cummins, J.D, Mahul, O. (2003) *Optimal Insurance with Divergent Beliefs About Insurer Total Default Risk*. Journal of Risk and Uncertainty, 27, 121-138

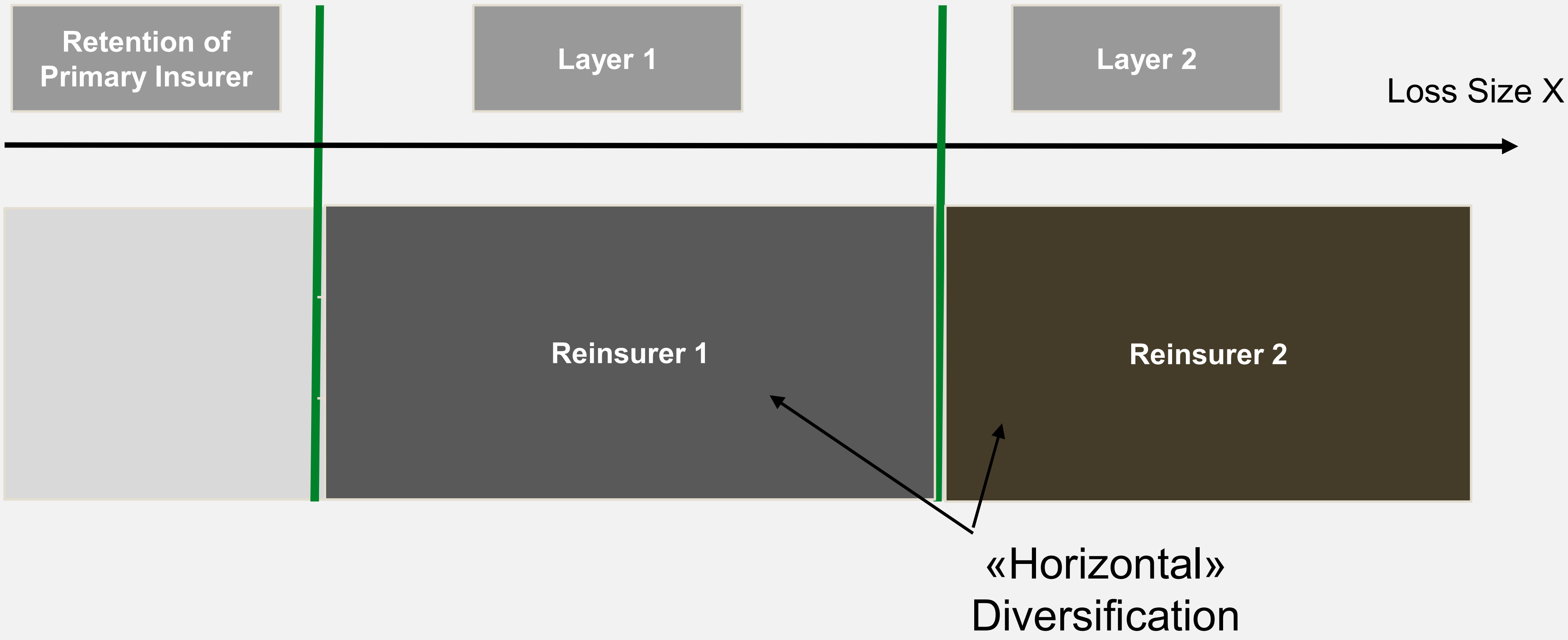
Mahul, O., Wright, B.D. (2004). *Implications of Incomplete Performance for Optimal Insurance*. Economica, 71, 661-670.

Bernard, C., Ludkovski, M. (2012). *Impact of Counterparty Risk on the Reinsurance Market*, North American Actuarial Journal, 16, 87-111.

Asmit, A.V., Badescu, A.M., Cheung, K.C. (2013). *Optimal Reinsurance in the Presence of Counterparty Default Risk*, IME, 53, 690-697.

Cai, J., Lemieux, C., Liu, F. (2014). *Optimal Reinsurance with Regulatory Initial Capital and Default Risk*, IME, 57, 13-24.

Diversification of Default Risk



Literature on Multi-Layering (without consideration of default risk)

Boyer, M., Nyce, C.M. (2013) *An Industrial Organization Theory of Risk Sharing*. North American Actuarial Journal, 17, 283-296.

Malamud, S., Rui, H., Whinston, A. (2016) *Optimal Reinsurance with Multiple Tranches*. Journal of mathematical Economics, 65, 71-82.

Boonen, T.J., Tan, K.S., Zhuang, S.C. (2016) *The Role of a Representative Reinsurer in Optimal Reinsurance*. IME, 70, 196-204.

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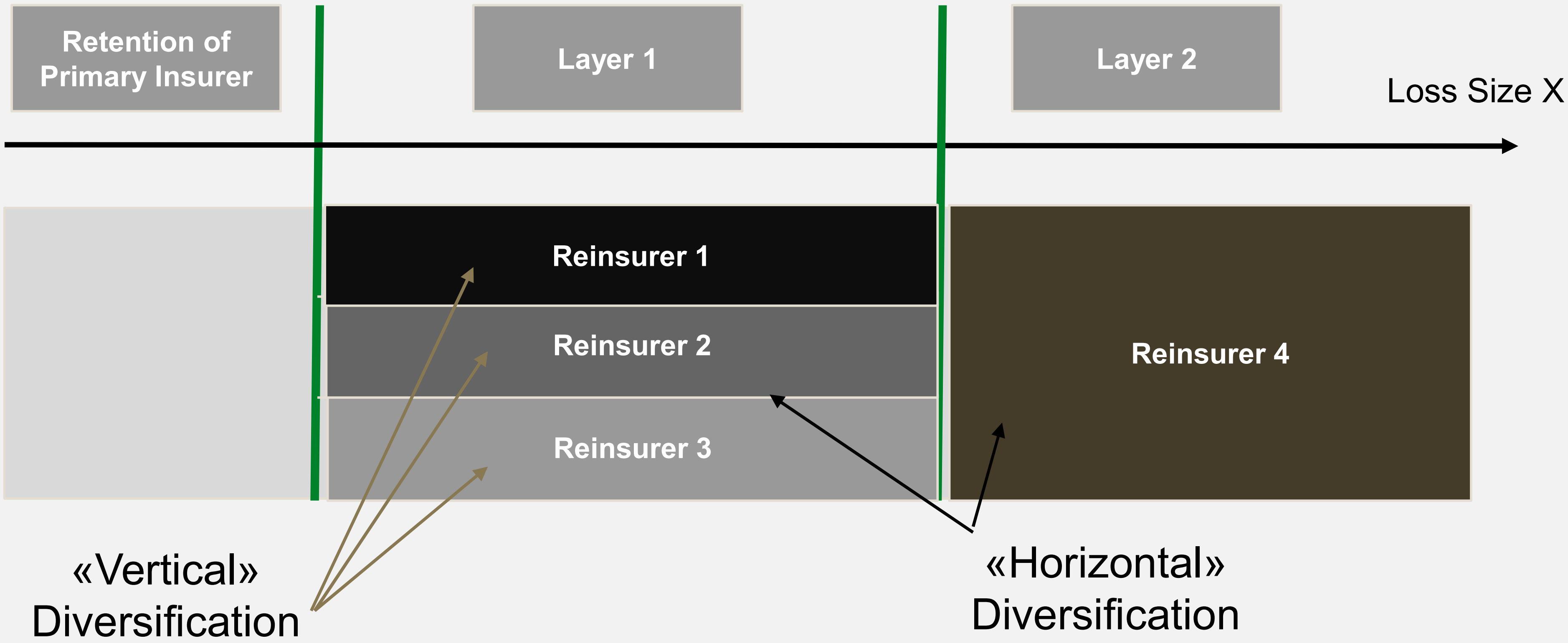

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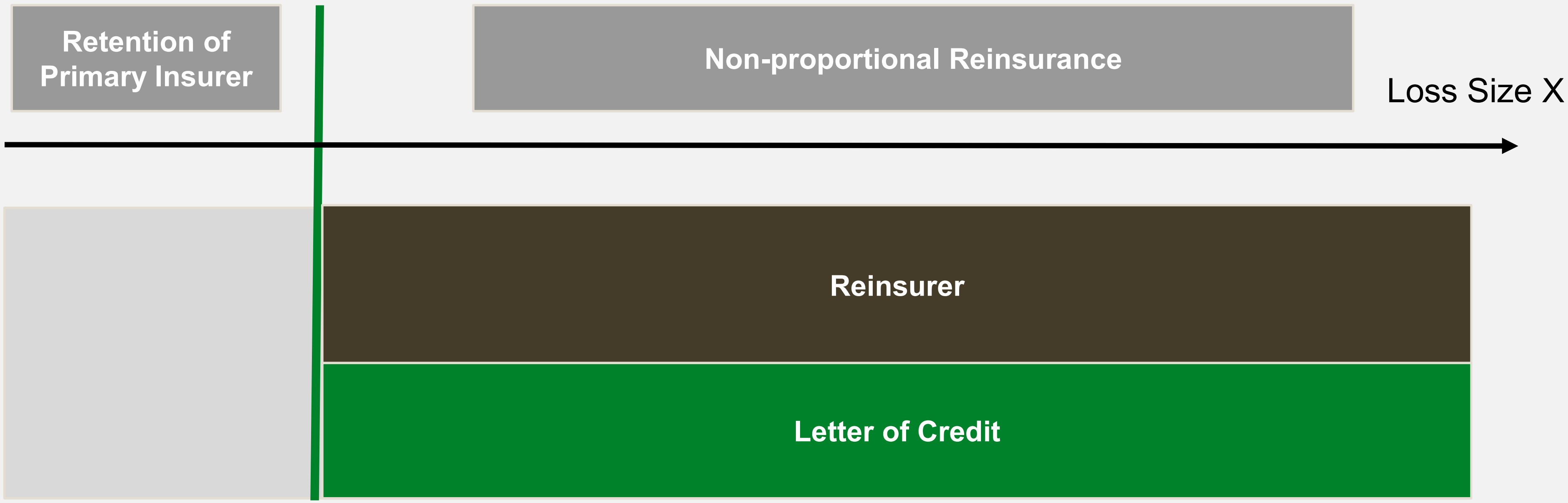
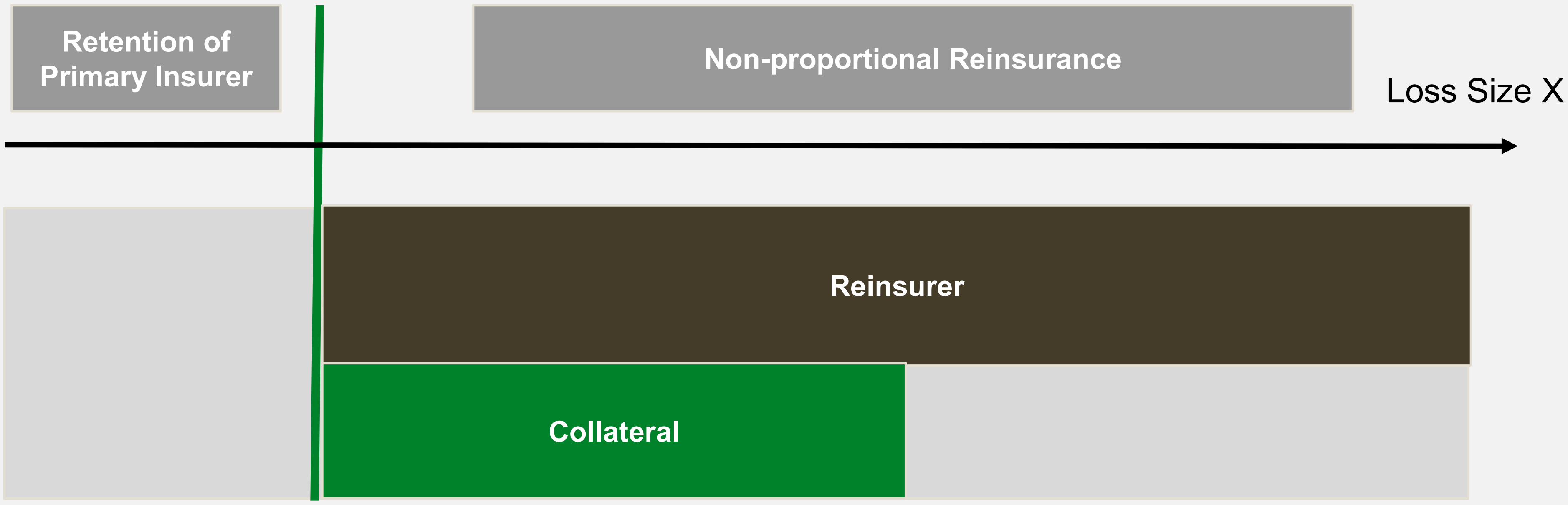
Hedging of Default Risk: Letter of Credit



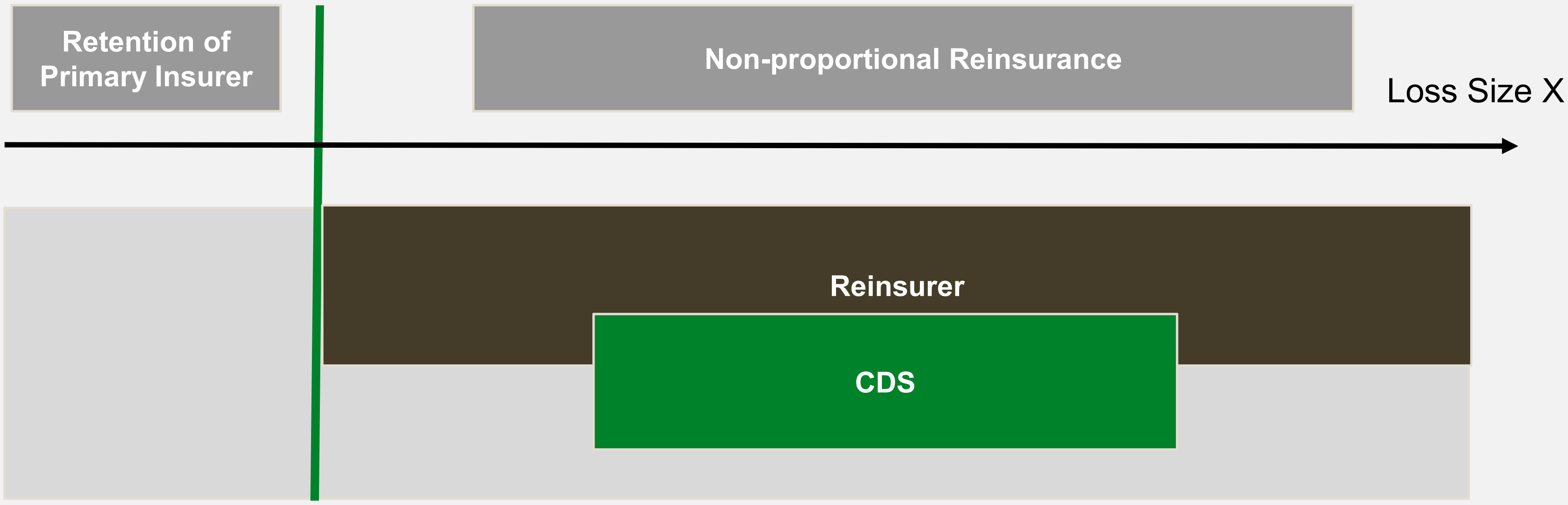
Letter of Credit: A third party issues a guarantee to resume the primary insurer's claims deficit, if the reinsurer goes bankrupt.

Hedging of Default Risk: Collateral



Collateral: Securities of the reinsurer are withheld in a trust and become available for the primary insurer, if the reinsurer goes bankrupt.

Hedging of Default Risk: Credit Default Swap



Credit Default Swap: Primary insurer purchases a CDS with the reinsurer as reference entity; fixed notional (less recovery) is paid, if the reinsurer goes bankrupt.

Research Question

From the perspective of the primary insurer: What is the optimal payoff structure of such a hedging instrument and how is it optimally combined with the reinsurance coverage?

The Model: One Reinsurer, Default Risk and Hedging

Basic Setting (cf. Arrow, 1963)

a_0: non-stochastic initial wealth, e.g., the primary insurer's assets

X: loss size, modeled as a non-negative random variable with density $f(x)$ on the support $[0, \bar{x}]$

$r(X)$: reinsurance reimbursement as contracted given loss size X, constraints: $r(0) = 0$ and $r(X) \geq 0$

Default Risk of the Reinsurer (cf. Mahul/Wright, 2004)

$\mathcal{r}(X, D) = (1 - D)r(X)$: actual reimbursement with default risk D

D is an indicator variable with $\mathbb{P}(D = \tau) = q$ and $\mathbb{P}(D = 0) = 1 - q$

τ is the loss-given-default (non-stochastic, $1 - \tau$ is the recovery rate); X and D are stoch. independent

π_r is the premium: $\pi_r = (1 + \lambda_r)\mathbb{E}[\mathcal{r}(X, D)] = (1 + \lambda_r)(1 - q\tau) \int_0^{\bar{x}} r(x)dx$, with cost loading $\lambda_r \geq 0$

Extension: Hedging Instrument

Provides payoff $\hbar(X, D)$, with $\hbar(X, 0) = 0$ and $\hbar(X, \tau) = h(X) \geq 0$ (no default risk!)

π_h is the hedging fee: $\pi_h = (1 + \lambda_h)\mathbb{E}[\hbar(X, D)] = (1 + \lambda_h)q \int_0^{\bar{x}} h(x)dx$, with cost loading $\lambda_h \geq 0$

Expected-Utility-Optimization

- The primary insurer's equity after reinsurance and hedging is:

$$E(X, D; r, h) = a_0 - \pi_r - \pi_h - X + \mathscr{r}(X, D) + \mathscr{h}(X, D)$$

- It is assumed that the primary insurer (or its risk-managers) behaves like a risk-averse decision-maker with concave utility function u (cf. Froot/Stein, 1998).

- It is aimed at finding the two-dimensional function $(r^*(x), h^*(x))$ that maximizes the expected utility $\mathbb{E}\big[u\big(E(X, D; r, h)\big)\big]$:

$$\max_{r,h,\pi_r,\pi_h} \mathbb{E}\big[u\big(E(X, D; r, h)\big)\big] \text{ subject to the constraints}$$

$$r(x) \geq 0, \text{ for } x \in [0, \bar{x}],$$

$$h(x) \geq 0, \text{ for } x \in [0, \bar{x}],$$

$$\pi_r = (1 + \lambda_r)(1 - q\tau) \int_0^{\bar{x}} r(x)dx,$$

$$\pi_h = (1 + \lambda_h)q \int_0^{\bar{x}} h(x)dx.$$

Result

Let Δ_λ be the spread between the cost loading for reinsurance and the cost loading for hedging:

$$\boxed{\Delta_\lambda := \lambda_h - \lambda_r}$$

Proposition

(1) If $\Delta_\lambda = 0$, then there is a constant $d \geq 0$, so that the optimal reinsurance-hedging combination is given by

$$(r^*(x), h^*(x)) = ([x-d]^+, \tau[x-d]^+).$$

(2) If $\Delta_\lambda > 0$, then there are constants $d_2 > d_1 > 0$ and a function $r_1(x)$, which is non-decreasing on $[0, \bar{x}]$ with $r_1(0) = 0, r_1'(x) \geq 1$ on $[d_1, d_2]$ and $r_1'(x) = 0$ on $[d_2, \bar{x}]$, so that

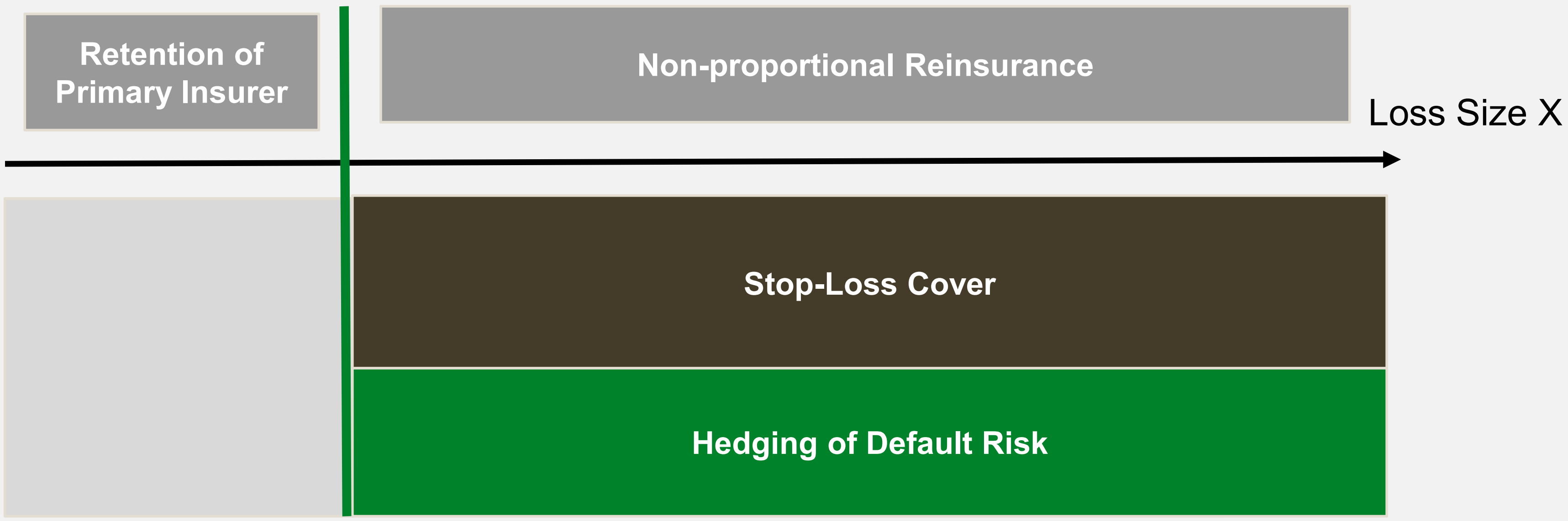
$$(r^*(x), h^*(x)) = (r_1(x) + [x-d_2]^+, \tau[x-d_2]^+).$$

(3) If $\Delta_\lambda < 0$, then there are constants $d_2 > d_1 \geq 0$, so that

$$(r^*(x), h^*(x)) = ([x-d_2]^+, min\{[x-d_1]^+, d_2 - d_1\} + \tau[x-d_2]^+).$$

Optimal Reinsurance-Hedging Combination (1)

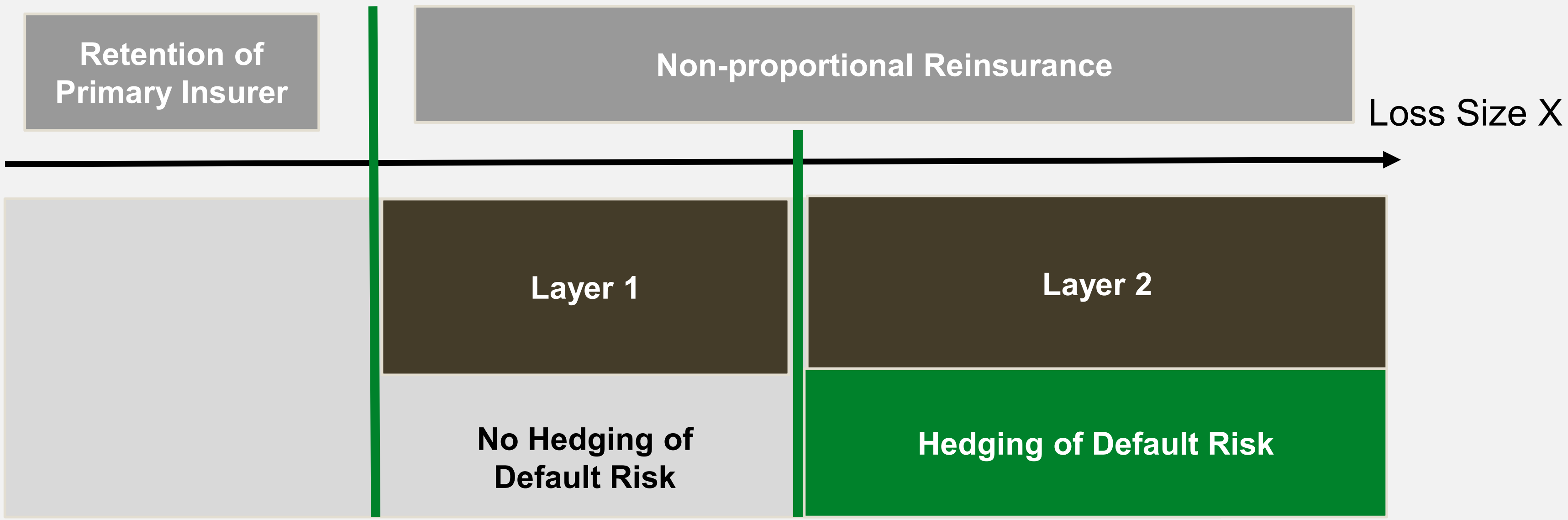
(1) Zero Spread: $\Delta_\lambda \coloneqq \lambda_h - \lambda_r = 0$ → Full hedging is optimal



- Stop-loss cover is the optimal reinsurance policy
- Full hedging of default risk from stop-loss cover is optimal
- Deductible is the same as for the default-free demand model

Optimal Reinsurance-Hedging Combination (2)

(2) Positive Spread: $\Delta_\lambda \coloneqq \lambda_h - \lambda_r > 0$ → Under-hedging is optimal



- Coverage is split in two layers: first layer remains unhedged; second layer is fully hedged
- Deductible is slightly higher than in the default-free model
- More-than-full coverage for layer 1 is optimal, if $0 < \tau < 1$; for $\tau = 1$: full coverage

Optimal Reinsurance-Hedging Combination (3)

(3) Negative Spread: $\Delta_\lambda \coloneqq \lambda_h - \lambda_r < 0$ → Over-hedging is optimal

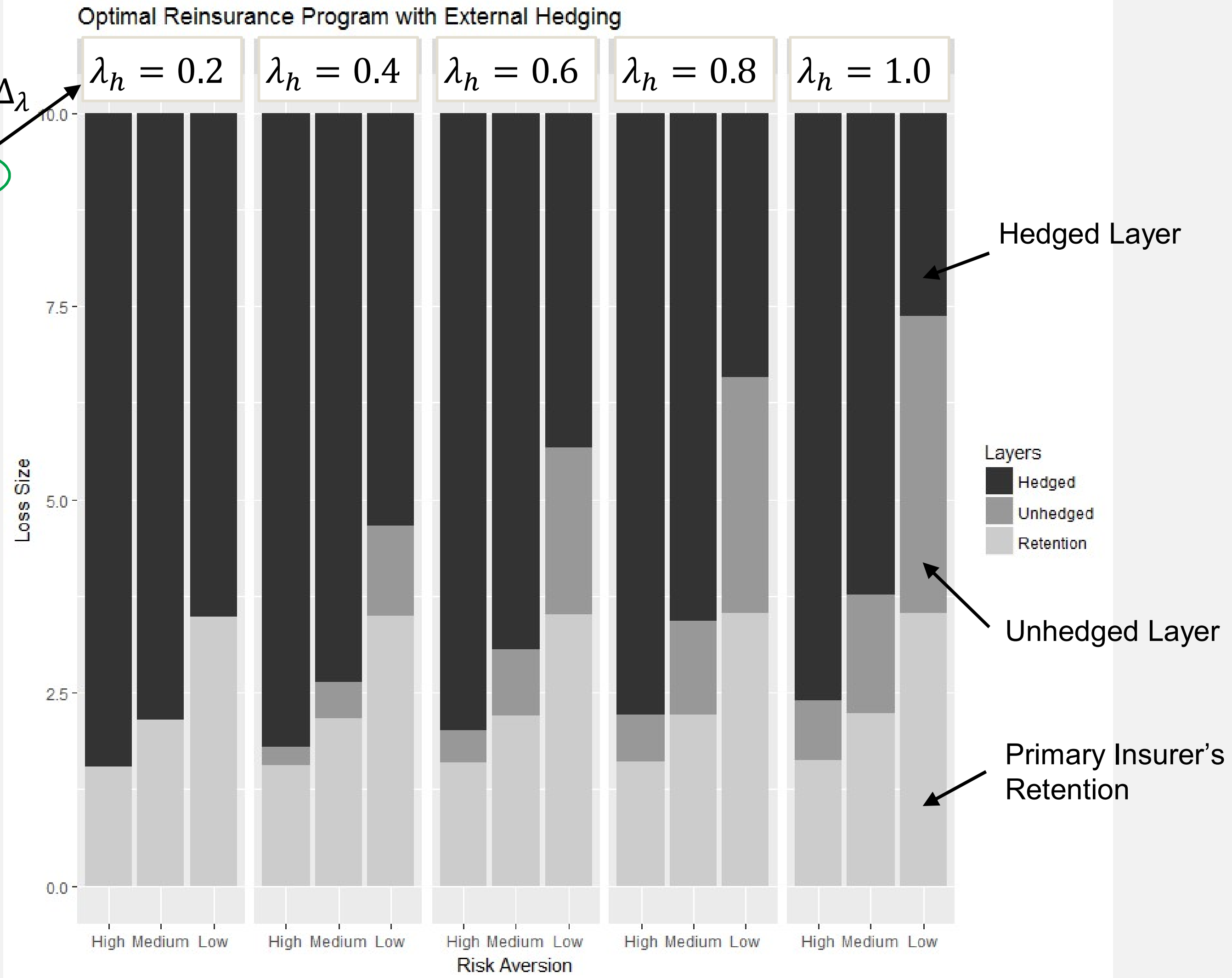
- Stop-loss cover is optimal reinsurance policy
- More-than full hedging is optimal (speculating on reinsurer's default)
- Deductible is slightly smaller than in the default-free case

Numeric Example

Parameter Setting

- Initial assets $a_0 = 15$
- Reinsurance cost loading: $\lambda_r = 0.2$
- Default risk of reinsurer: $q = 0.05$, $\tau = 1.0$ (total default)
- Loss size X has a truncated exponential distribution with $\bar{x} = 10$
- Primary insurer has exponential utility function with risk-aversion-parameter β
- High Risk Aversion: $\beta = 10/a_0$
- Medium Risk Aversion: $\beta = 5/a_0$
- Low Risk Aversion: $\beta = 2/a_0$

Δ_λ



$Risk\ Aversion: High\ -Medium\ -Low$

Concluding Remarks

Results can be seen from different angles:

<u>Cat Bonds</u>

- Cat bonds are a fully collateralized type of reinsurance
- The optimal combination of conventional reinsurance, its hedging and Cat bonds can be drawn from the given results:
- *If the costs for Cat bonds are less than the weighted sum of costs for reinsurance and its hedging instrument (weights depend on default risk), the primary insurer is advised to replace hedged reinsurance by Cat bonds (at least in the idealized model world).*
- Results are in line with Lakdawalla/Zanjani (2012) and Trottier/Lai (2017)

<u>Reinsurer Allocation Problem</u>

- Assume there is a risky reinsurer 1 (with default risk, no hedging possible) selling reinsurance with cost loading λ_1 and a risk-free reinsurer 2 (without default risk) selling reinsurance with cost loading λ_2:
- *If $\lambda_1 < \lambda_2$, risky but cheaper reinsurer should optimally cover lower layer 1, safe but more expensive reinsurer covers upper layer 2 (at least in the idealized model world).*
- From the reinsurer’s perspective: What is the optimal trade-off between enhancing solvency (lower default risk) and the associated costs?

Thank You

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Hedging Instruments