Stories from the Frontier

Guido Cozzi* and Silvia Galli†

Abstract

Should basic research be publicly or privately funded? This paper studies the impact of the shift in the U.S. patent system towards the patentability and commercialization of the basic R&D undertaken by universities during the early 1980s. We interpret this change as rendering universities responsive to "market" forces. Prior to 1980, universities undertook research using an exogenous stock of researchers motivated by "curiosity." After 1980, universities patent their research and behave as private firms. This move, in a context of two-stage inventions (basic and applied research) has an a priori ambiguous effect on innovation and welfare. We build a Schumpeterian model and match it to the data to assess this important turning point from an innovation as well from a welfare-enhancing perspective.

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Keywords: Innovation Policy, Institutions, Public Basic Research, Quality Ladder, Untargetable Basic Research.

1 Introduction

What factors specifically affect innovation in the most advanced economies, at the frontier of the world technology? Basic research is a top candidate. In an international framework of advanced economies, data suggest the existence of a positive relation between basic research intensity and the multifactor-productivity growth. This can be seen by looking at figures 1.A and 1.B representing the relationship between countries' basic research intensity relative to the US and the respective MFP growth (OECD MSTI data (OECD, 2014a and OECD Productivity Database (OECD, 2014b)).

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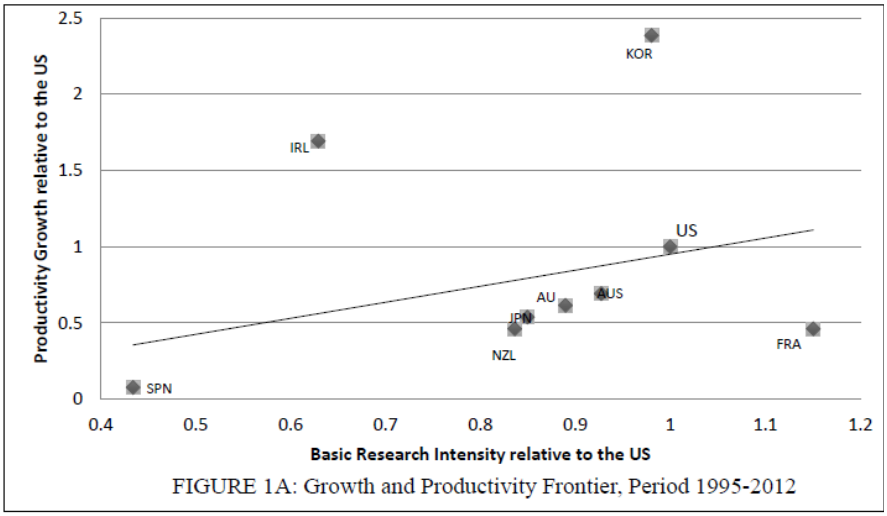


FIGURE 1A: Growth and Productivity Frontier, Period 1995-2012

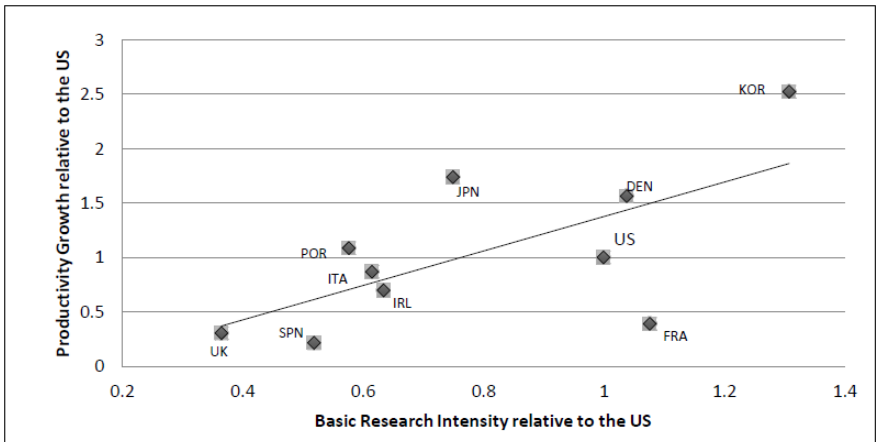


FIGURE 1B: Growth and Productivity Frontier, Year 2010

It is often argued[1] that the impact of basic research on growth is intended to become more and more relevant as the country converges to the world technological frontier: for example, US and Japan alone account for about half of the world basic research. Moreover, several advanced countries historical experience highlights the circumstance that basic research is often publicly financed by governments. Figures 2.A and 2.B show the relationship between the government funded gross domestic expenditure on research and development (GERD) intensity and MFP growth relative to the US (figure 2.A); and the public R&D expenditure and relative MFP growth (figure 2.B).[2] The first figure - GERD intensity - is a measure of R&D activities which are funded by the government, but non necessarily performed within public research institution. The second, instead, captures all R&D activities performed within higher education institutions and public R&D laboratories. In fact, for a relevant fraction basic research is performed by universities and public research institutions (see also Cozzi and Galli (2009):

[1] See for example Gersbach, Schneider and Schneller (2008 and 2013).

[2] The data are from the OECD Science, Technology, and Industry Outlook (OECD, 2012) and the OECD Productivity Database (OECD, 2014b).

public basic research is essential for developing new scientific breakthroughs and creating the basis for developing subsequent technological advancements, as recently highlighted by the U.S. Congress's Joint Economic Committee (see JEC 2010 and the insightful discussion provided by Akcigit, Hanley and Serrano-Velarde, 2014). Despite these considerations, very little attention has been devoted so far to a systematic study of the channels through which public basic research stimulate growth, innovation, and welfare.

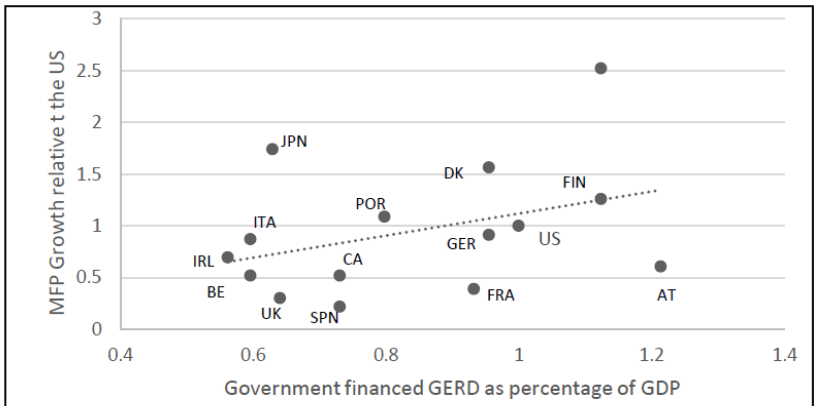


FIGURE 2A: Public R&D Expenditure and Technology Frontier, Year 2010

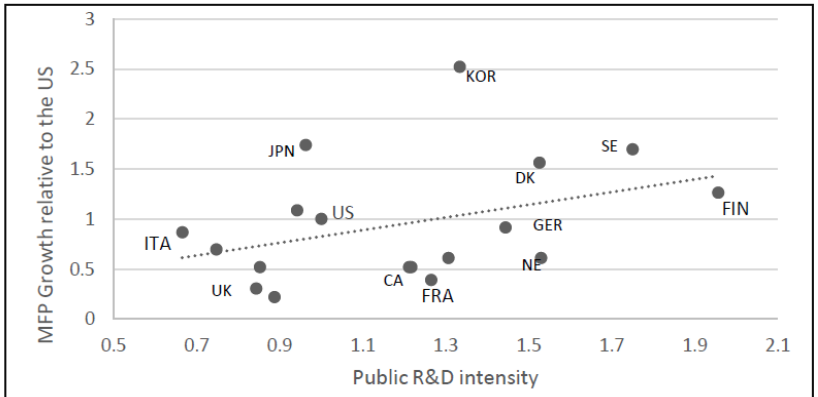


FIGURE 2B: Public R&D Expenditure and Technology Frontier, 2010 or latest year available

To try to fill this gap and shed some light on this important issue, here we incorporate publicly provided basic research into a Schumpeterian multi-sector quality ladder model. In particular, we assume that the government employs a share of researcher into basic research, and we make further assumptions which specifically characterize the behavior of the public researchers and distinguish them from the private sector researchers. We attempt to replicate the changes occurred in the US intellectual property design by modelling the institutional framework provided by different institutional scenarios. Hence, we discuss the consequences of publicly provided basic research in terms of innovative capacity and its desirability in terms of welfare. To that extent, we examine the evolution of intellectual property institutions in the U.S., with special reference to their ability to protect basic research and to promote the techno-

logical advancement of the frontier[3]. By taking the R&D sequentiality into the Schumpeterian paradigm, this paper investigates the relationship between the cumulative uncertainty involved in a two-stage (*basic-applied*) innovation process and the inefficiencies of the public research system, which is an issue often left unmodelled so far.

In our view, this constitutes an important element of novelty for the literature on innovation in general equilibrium, which so far mostly concentrated on private R&D scenarios, where typically profit-maximizing firms engage in R&D activity in order to secure the rents associated with the introduction new markets or new technologies (Aghion and Howitt, 1992 and 1996). There are a few exceptions which explicitly consider the role of the government as a research provider in the macroeconomic growth literature. Most notably, Aghion, Dewatripont, and Stein (2008) and Spinesi (2013) analyzed the effects of technological transfer institutions (intellectual property rights) between the academia and private research firms from different perspectives. A recent scientific contribution by Akcigit et al. (2014) identified an important role for public basic research in promoting economic growth in France.[4]

Within this still thin literature, our model is the first which tries to endogenizes the public sector inefficiency in basic research. In fact, public basic research is not as targeted as private basic research, which is guided by the signalling device of future patent values. On the contrary public research is more career-motivated and less respondent to *market stimuli*[5]. This circumstance determines that the amount of inefficiency created depends on the fraction of industries where basic R&D can be effectively carried out, which is endogenous. Therefore, one cannot unambiguously rank the two institutional scenarios: patentable or unpatentable basic research? In some cases it would be best to keep basic research publicly driven, while in others it would be best to facilitate privatizing institutions with basic research patents. In this paper, depending on the parameters to be calibrated, the most innovation-fostering and the socially optimal institution will be determined.

Over the last 35 years, the U.S. patent system switched from the doctrine limiting the patentability of early-stage scientific findings - lacking in current commercial value - to the conception that also fundamental basic scientific discoveries - with no current tradeable application - fall in the general applicability of the patent system. This fundamental turning point marked the year 1980, when two important events characterized this new idea of the patentability requirements:

1. the United States Supreme Court's decision on the Diamonds vs. Chakrabarty case established that microorganisms produced by genetic engineering could be patented;

[3]Curiously, the US National Institute of Healt (NIH) recentely introduced the concept of "appropriate patenting", according to which "patenting is one of the tools available to the NIH for transferring publicly funded technology to the market" (see OECD, 2011).

[4]In an early contribution, Pelloni (1997) builds an endogenous growth model with public research only, where the government faces a trade-off between financing public research or public education.

[5]For a complementary discussion on the role of relevant spillovers from the stock of academic basic knowledge on industry, see also Spinesi (2012) and Akcigit et al. (2014).

2. the passing of the Patent and Trademark Act Amendments (P.L. 96-517, known as the Bayh-Dole Act) facilitated universities an public laboratories in patenting their innovations.

Such jurisprudential and juridical reforms opened the way to a flow of private funds into the academia, as well as facilitated professors in patenting their own research without incurring in legal obstacles linked to the public financing their research activities.

Recent studies focussed on the U.S. university licensing activity. In particular, Jensen and Thursby (2001) studied the licensing practices of 62 US universities and found that "Over 75 percent of the inventions licensed were no more than a proof of concept (48 percent with no prototype available) or lab scale prototype (29 percent) at the time of license!"

This process, which determined a cultural shift in the U.S. basic research culture, was reluctantly followed by Europe, where only in 1998 the European Directive on Biotechnologies aiming at extending patentability to most basic research patenting was adopted (see European Parliament and Council, 1998). Many observers commented on how such a Directive has always been implemented in contradictory ways, leading to a sort of an undefined situation for Europe. For this reason, we believe that an analysis of the U.S. turning point may give good insight to start a scientific debate rich of relevant policy implications also for Europe.

This paper is organized as follows. Section 2 explains the modifications in Schumpeterian theory needed to analyze the two-stage innovation process stylizing the basic innovation mechanism in the "targetable" basic research version of the model. In order to facilitate readability, this section intentionally focusses only on the most original aspects of the model, leaving the most standard parts to the Appendix 1. Section 3 applies this new framework to a stylized pre-1980 US scenario: basic research findings are conceived in public institutions and put into the public domain, triggering patent races by freely entering perfectly competitive private R&D firms aiming at inventing a better quality product. Section 4.1 models a stylized post-1980 US scenario with "targetable" basic research. Section 4.2 sets up the stylized post-1980 US scenario under the alternative assumption of "untargetable" basic research. Here basic R&D achievements are patented and, afterwards, developed into tradable applications within a completely privatized economy. Free entry patent races only occur in the basic research, whereas as soon as a research tool is discovered it will be developed by its patent holder. Section 5 matches the different scenarios developed by the model to the US data prevailing at the time of the jurisprudence and legislative change. We estimate the relevant technological parameter and we undertake numerical simulations in order to assess under which underlying assumption (targetable or untargetable basic research) the reform has enhanced innovation. Section 6 concludes.

2 The Model

2.1 Overview

Consider an economy with a continuum of differentiated final good sectors with corresponding differentiated research and development (R&D) sectors, along the lines of Grossman and Helpman (1991). In each final good sector vertical innovation takes place, hence price-competition among firms determines - under the usual constant returns to scale assumption - the market monopolist, the owner of the patent on the highest quality product in its industry.

2.2 The Mechanics of R&D

Product improvements occur in each final good industry, and, within each industry, firms are distinguished by the quality of the final good they can produce. When the state-of-the-art quality product in an industry $\omega \in [0,1]$ is $j_t(\omega)$, research efforts are necessary in order to achieve the $j_t(\omega) + \frac{1}{2}$th inventive step, and then other researchers engage in a patent race to implement it in the $j_t(\omega) + 1$st quality product[6]. So, in each industry, the R&D activity is a two-stage innovation process by which, first a new idea is invented through basic research activity and then it is used by applied researchers to find the way to introduce a higher quality product. Our definition of basic research output essentially coincides with a *research-tool*: "the full range of tools that scientists use in the laboratory" including "cell lines, monoclonal antibodies, reagents, animal models, growth factors, combinatorial chemistry libraries, drugs and drug targets, clones and cloning tools (...) methods, laboratory equipment and machines, databases and computer software", according to the definition[7] provided by the US National Institute of Health (see NIH (1998) and OECD (2011)). Nearly all research tools became patentable in the US, thanks to the juridical innovations that took place in the last 30 years (see Cozzi and Galli (2014)).

The whole set of industries $\{\omega \in [0,1]\}$ gets partitioned into two subsets of industries: at each date t, there are industries $\omega \in A_0$ with (temporarily) no research tool and, therefore, with one quality leader (the final product patent holder), no applied research and a mass of basic researchers; and the industries $\omega \in A_1 = [0,1] \setminus A_0$, with one research tool and, therefore, one quality leader and a mass of applied researchers directly challenging the incumbent monopolist.

We will here assume two different basic research technologies: "Targetable" and "Untargetable".

[6] Of course, upstream ideas could be as difficult to get as are Nobel prizes: see, for example, the Cohen-Boyer patents on the basic method and plasmids for gene cloning (granted in 1990).

[7] Note how this definition relies on the implicit assumption that basic research bears no utility increase for the consumers.

2.2.1 Targetable Basic Research

Let us define a *perfectly targeted research economy* when basic research focusses exclusively on industries $\omega \in A_0$, whose output can therefore be used by profit-motivated R&D firms engaging in applied R&D activity aimed at a final product innovation only in A_1 industries. When eventually a quality improvement occurs within an A_1 industry, the innovator becomes the new quality leader and the industry switches from A_1 to A_0. Similarly, when a discovery arises in an industry $\omega \in A_0$ this industry switches to A_1.This process can be better understood by considering the industry dynamics illustrated by the two-lakes representation of the economy in FIGURE 3: notice that in our multi-sector two-stage perpetual innovation process, basic R&D alternates with applied R&D in all sectors of the economy. The two sets A_0 and A_1 change over time, even if the economy will eventually tend to a steady state.

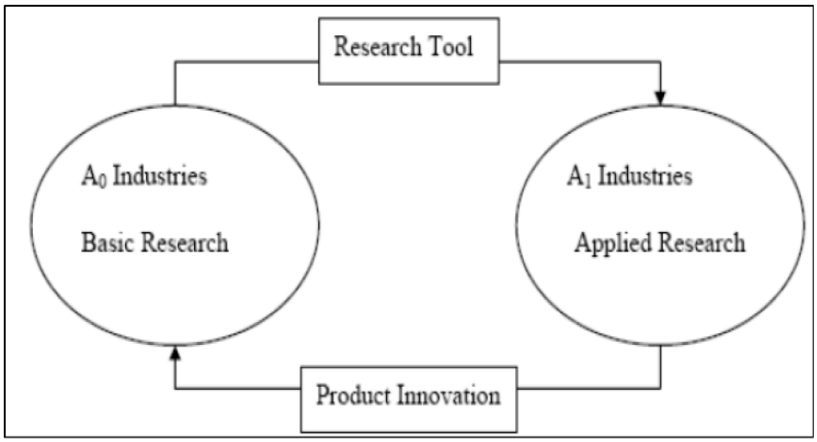


FIGURE 3: Targeted research economy by flows of industries

Suppose that at any instant one can measure the two sets A_0 and A_1. Let m_0 denote the measure of A_0; and m_1 respectively denote the measure of A_1. By construction, $m_1 = 1 - m_0$. In the steady-state equilibrium the two measures shall be constant, as the two-flows in and out of the lakes (the arrows denoted *research tool* and *product innovation* in FIGURE 3) will offset each other. However, the endogenous nature of the steady-state distribution of sectors allows the model to analyze the effects of different institutional scenarios on the technology dynamics and the aggregate innovative performance.

Let index $i = B, A$ denote basic or applied research respectively. $n_i(\omega, t)$ indicates the mass of skilled workers employed in the two sages of the innovation process in sector $\omega \in [0, 1]$ at time t. We specify the per-unit time Poisson probability intensity of an innovative step (basic or applied) to occur in a generic sector ω as:

$$\theta_B(\omega, t) \equiv \lambda_0 n_B(\omega, t)^{1-a}, \omega \in A_0, \quad (1)$$

$$\theta_A(\omega, t) \equiv \lambda_1 n_A(\omega, t)^{1-a}, \omega \in A_1 \quad (2)$$

where $\lambda_k > 0$, $k = 0, 1$, are R&D productivity parameters and constant $0 < a < 1$ is an intra-sectorial congestion parameter, capturing[8] the risk of R&D duplication, knowledge theft, and other diseconomies of fragmentation, external to the single firm in competitive industries. Each Poisson process - with arrival rates described by (1)-(2) - is independent across researchers and across industries. Hence the probability per unit time of inventing a research tool in a sector $\omega \in A_0$ at date t is $\theta_B(\omega, t)$, and the probability of completing a final blueprint in a sector $\omega \in A_1$ is $\theta_A(\omega, t)$.

Moreover, in all our scenarios, symmetric equilibria exist, allowing simpler notation: $n_B(\omega, t) \equiv n_B(t)$ and $n_A(\omega, t) \equiv n_A(t)$.

2.2.2 Untargetable Basic Research

So far we have assumed the ability of intellectual property rights, here represented by patents, to channel basic research efforts towards more profitable venues, thus implicitly assuming that market signals can be useful to direct research. In terms of our model, we assumed that granting basic researchers intellectual property is a viable option to increase the efficiency of the technological transfer aggregate mechanism.

However, a broad consensus in the literature on basic research share the a more pessimistic view about the possibility for the patent system to effectively operate in this dimension. In particular, it is often observed how basic research cannot be targetable because its outcomes are hard to predict, and in many instances, the motivations behind its creation are pure intellectual curiosity and desire to achieve academic promotions. Given the importance of this issue, to gain a deeper understanding of the potential problems of a privatization of basic research has caused, one needs to explicitly study this case. For this purpose, this section modifies the privatized economy model to include untargetable basic research.

To define untargetable basic R&D technology, we re-specify the per-unit time Poisson probability intensity of innovation postulated in eq.s (1)-(2) as:

$$\theta_B^u(\omega, t) \equiv \lambda_0 n_B(t)^{1-a}, \omega \in A_0, \tag{3}$$

$$\theta_A(\omega, t) \equiv \lambda_1 n_A(\omega, t)^{1-a}, \omega \in A_1 \tag{4}$$

Notice that, unlike the previous "targetable" case, here the probability per unit time of inventing a research tool in a sector $\omega \in A_0$ at date t, that is $\theta_B^u(\omega, t)$, only depends on aggregate basic R&D $n_B(t)$: in fact, a basic researcher is not able ex-ante to predict the exact application or the potential impact of his/her discovery.

In a symmetric equilibrium applied R&D will satisfy $n_A(\omega, t) \equiv n_A(t)$.

2.2.3 Manufacturing

Adopting the unskilled wage as the numeraire, we will endogenously determine the skill premium, as summarized by the skilled labour (relative) wage w_s.

[8] As in Jones and Williams' (1998 and 2000) specification of the R&D technology.

In all our equilibria, the skilled labour employed in manufacturing sector $\omega \in [0,1]$ at time t, labeled $x(\omega,t)$, will be constant across sectors and equal to $x(\omega,t) = x(t)$. In fact, in the Appendix 1 we prove that the manufacturing employment of the skilled labour obeys the following decreasing function of the relative skilled wage w_s:

$$x(\omega,t) = \frac{1}{w_s(t)}\left(\frac{\alpha}{1-\alpha}\right)M \equiv x(t), \tag{5}$$

where $0 < \alpha < 1$ is the skilled labour elasticity of output. Appendix 2 also show that profit flows are constant and equal to $\pi = (\gamma - 1)\frac{1}{1-\alpha}M$, where $\gamma > 1$ is the size of each product quality jump.

Since the total mass of sectors in the economy is normalized to 1, $x(t)$ also denotes the aggregate employment of skilled in manufacturing. Hence, defining $Y(t)$ the aggregate final good production, $x(t)w_s(t) = \alpha Y(t)$ and $M = Mw_u(t) = (1-\alpha)Y(t)$.

In light of the previous discussion, and dropping time indexes for simplicity[9], we can express the skilled labor market equilibrium as:

$$L = \frac{1}{w_s}\left(\frac{\alpha}{1-\alpha}\right)M + m_0 n_B + m_1 n_A. \tag{6}$$

Eq. (6) states that, at each date, the aggregate supply of skilled labor, L, finds employment in manufacturing and in basic and applied R&D.

3 The Public Basic Research Economy

In order to depict a pre-1980 US normative environment, in this section we assume unpatentable basic scientific results. In fact, pre-1980 the prevailing practice in public basic research was granting open access to its scientific findings. Besides, public researchers were paid regardless of the development opportunities arising from their discoveries: their activity was "curiosity-driven" and totally indifferent to sectorial profitability, thus their efforts were potentially wrongly targeted from a social point of view. Therefore, while this research outcome is inevitable if basic research is untargetable[10], it instead points to an inefficiency of the public sector with the "targetable" basic research technology.

We depict these two cases with a unique model, by assuming that under a targetable basic research case public researchers are allocated across different industries according to a uniform distribution[11]. Please note that this assumption could be microfunded within an incomplete contract setting between the university (principal) and the public basic researchers (agents).

[9] Of course time dependence is implicit, as employment variables, wage, and the mass of sectors in which a half idea is present, respectively absent, keep changing over time, except in the steady state.

[10] It would be present even if basic research were privazed.

[11] This assumption captures the idea of ivory-tower-oriented basic researchers, mainly concerned with academic advancement in an eviroment shaped by values such "universalism, disinterestedness, originality, skepticism, and communalism" (Davis, Larson and Lotz (2009)).

The university bureaucrats do not exert effective authority on the different research activities, which are carried out by the researchers employed in R&D. Hence, university bureaucrats have only very limited control. In particular, the bureaucrats do not have the authority to stipulate complete contracts, since they are unable able to effectively specify in what sector basic research should be carried out by the academic researcher at each instant in time and to enforce the contract terms (see Aghion and Tirole, 1997).

We also make the assumption that the government exogenously sets the fraction, $\bar{L}_G \in [0, L]$, of the skilled workers to be allocated to basic research laboratories, funded by lump-sum taxes[12]. Given that the mass of sectors normalized to 1, $\bar{L}_G$ is also equal to the per-sector amount of R&D. Therefore the probability that in any sector $\omega \in A_0$ a basic research result appears is $\theta_B \equiv \bar{L}_G^{1-a}\lambda_0$, whereas the probability that an existing research tool generates a new marketable product is $\theta_A = n_A^{1-a}\lambda_1$.

Let v_L^0 denote the value of a monopolistic firm producing the top quality product in a sector $\omega \in A_0$, and consistently let v_L^1 be the value of a monopolistic firm producing the top quality product in a sector $\omega \in A_1$. These two types of quality leaders earn the same profit flow, π, but the first type has a longer expected life, before being replaced by the new quality leader, i.e. by the patent holder of the next version of the product it is currently producing. In sectors that are currently of type A_0 no applied R&D firms enters because there is no research tool to develop: they shall wait until public researchers invent one, causing that sector to switch into A_1. Instead, in an A_1 sector, applied R&D firms hire skilled workers in order to complete the freely available basic research result. Since there is free entry into applied research, the R&D firm's expected profits are dissipated. From a welfare perspective, entry into applied R&D could be excessive, thereby generating distortions.

Defining r as the relevant real interest rate, the following equations hold:

$$w_s = \lambda_1 n_A^{-a} v_L^0 \tag{7a}$$

$$r v_L^0 = \pi - \bar{L}_G^{1-a}\lambda_0 \left(v_L^0 - v_L^1\right) + \frac{dv_L^0}{dt} \tag{7b}$$

$$r v_L^1 = \pi - n_A^{1-a}\lambda_1 v_L^1 + \frac{dv_L^1}{dt}. \tag{7c}$$

Eq. (7a) is the free entry condition in applied research in each sector $\omega \in A_1$, equalizing the unit cost of R&D (the skilled wage) to the probability $\lambda_1 n_A^{-a}$ of inventing the next version of the final product times the value of its patent, v_L^0. Eq. (7b) is the financial arbitrage equation stating that v_L^0 is determined by equating the risk-free interest income attainable by realizing the stock market value of an industry leader in A_0, $r v_L^0$, to the flow of profit π minus the expected capital loss from being challenged by subsequent basic research activity generating in a new research-tool, $\bar{L}_G^{1-a}\lambda_0 \left(v_L^0 - v_L^1\right)$, plus the gradual appreciation in the case of such event

[12] This guarantees that governmental R&D expenditure does not imply additional distortions on private decisions.

not occurring, $\frac{dv_L^0}{dt}$. In a steady state $\frac{dv_L^0}{dt} = 0$.

Eq. (7c) equates the risk free income per unit time deriving from the liquidation of the stock market value of a leader in an A_1 industry, rv_L^1, with the relative flow of profit π minus the expected capital loss, $n_A^{1-a}\lambda_1 v_L^1$, due to the downstream applied researcher firms' R&D, plus the gradual appreciation if replacement does not occur, $\frac{dv_L^1}{dt}$. In a steady state $\frac{dv_L^1}{dt} = 0$.

All jump processes are independent across industries. Hence, by the law of large numbers, the dynamics of the mass of industries is described by:

$$\frac{dm_0}{dt} = (1 - m_0)\, n_A^{1-a}\lambda_1 - m_0 \bar{L}_G^{1-a}\lambda_0. \tag{8}$$

The skilled labor market clearing condition imposes:

$$x + \bar{L}_G + (1 - m_0)n_A = L. \tag{9}$$

Recall the equilibrium value of x derived by equation (5): $x = \frac{1}{w_s}\left(\frac{\alpha}{1-\alpha}\right) M$; by combining this expression with the skilled labor market clearing equilibrium, we get:

$$n_A = \frac{L - \frac{1}{w_s}\left(\frac{\alpha}{1-\alpha}\right) M - \bar{L}_G}{(1 - m_0)}. \tag{10}$$

Hence the dynamics of this economy is completely characterized by system (7a)-(7c), (8), and (10).

3.1 Steady-State Equilibrium

In a steady state equilibrium all variables are constant except the average quality of consumer goods[13], and therefore the instantaneous utility index, which grows at a constant rate[14] $\ln(\gamma) g_{PUBBL}$ proportional to the aggregate innovation rate $g_{PUBBL} = m_0 \bar{L}_G^{1-a}\lambda_0 = (1 - m_0)\,\lambda_1\,(n_A)^{1-a}$. Based on the previous characterization, we can state:

Definition 1. *A steady state equilibrium of the Public Basic Research economy is a vector* $[m_0, n_A, v_L^0\ v_L^1, w_s, x, g_{PUBBL}] \in R_+^7$, *satisfying* $m(A_0) \in [0, 1]$ *and the following equations:*

[13] Since we are following Grossman and Helpman's (1991) framework, it is the geometric average $D(t) = \exp\left[\int_0^1 \ln\left[\gamma^{j_t^*(\omega)} d_{j_t^*(\omega)t}(\omega)\right] d\omega\right]$ that matters. Appendix 1 clarifies these aspects in detail.

[14] This is a usual property of quality ladder models (see e.g. Grossman and Helpman, 1991). Find more on this in the welfare calculations in Appendix 1.

$$
\begin{aligned}
w_s &= \lambda_1 n_A^{-a} v_L^0 && \text{(11a)}\\
r v_L^0 &= (\gamma-1)\tfrac{1}{1-\alpha} M - \bar{L}_G^{1-a}\lambda_0\left(v_L^0 - v_L^1\right) && \text{(11b)}\\
r v_L^1 &= (\gamma-1)\tfrac{1}{1-\alpha} M - n_A^{1-a}\lambda_1 v_L^1 && \text{(11c)}\\
x &= \frac{1}{w_s}\left(\frac{\alpha}{1-\alpha}\right) M && \text{(11d)}\\
(1-m_0)\, n_A^{1-a}\lambda_1 &= m_0 \bar{L}_G^{1-a}\lambda_0 && \text{(11e)}\\
x + \bar{L}_G + (1-m_0) n_A &= L && \text{(11f)}\\
g_{PUBBL} &= \lambda_1 (1-m_0)\, n_A^{1-a}. && \text{(11g)}
\end{aligned}
$$

Given the high non-linearity of system (11a)-(11g), we performed numerical simulations in Matlab[15]. In all simulations a unique economically meaningful steady state equilibrium exists. Moreover, analyzing the eigenvalues of the Jacobian matrix of the fully dynamic (out of steady state) system shows that the steady-state equilibrium is saddle-point stable. Therefore the equilibrium is determinate.

Moreover, one can prove the uniqueness of the steady-state. In fact, the following lemma holds:

Lemma 1. *In the Public Basic Research economy there can exist no more than one steady state equilibrium.*

Proof. See Appendix 2.

4 The Privatized Basic Research Economy

4.1 Targetable Basic Research

In this section, stylizing a post-1980 US scenario, we assume that once a research tool is invented in an A_0 sector, it gets protected by a patent with infinite legal life. The presence of perfectly enforced intellectual property rights on the research tools permits the existence of a market for basic research findings. We will here assume that the basic is perfectly efficient[16]. Let us remark that this scenario does not preclude the existence of public universities, as long as their attitudes and internal incentive system is profit-seeking as well[17].

Let v_A, denote the value of a research-tool patent owned by an applied R&D firm. Such a firm will optimally choose to hire an amount n_A of skilled research labour to maximize the difference between its expected gains from completing its own first stage - probability of

[15] The Matlab and Dynare files used to simulate the model are available from the authors upon request.

[16] This means that basic researchers target their activity only in the A_0 sectors.

[17] Belenzon and Schankerman's (2009) empirical analysis shows that the private or public university ownership does not change their licensing performance, provided they adopt the same incentive pay. Also see Lach and Schankerman (2004).

inventing, $(n_A)^{1-a}\lambda_1$, times the net gain from inventing the final product, $(v_L^0 - v_A)$ - and the implied labour cost $w_s n_A$. The optimal applied R&D employment in an A_1 sector is

$$n_A^* = \left[\frac{(1-a)\lambda_1(v_L^0 - v_A)}{w_s}\right]^{\frac{1}{a}}. \tag{12}$$

Unlike the previous section, now the sole research-tool patent holder can undertake applied R&D in its industry[18], whereas free entry is relegated to the basic research stage, where researchers vie for inventing the research-tool that will render the winner the only owner of a research tool patent worth v_A. Hence their freely entering and exiting mass will dissipate any excess earning, by equalizing wage to the probability flow $\lambda_0 n_B^{-a}$ times the value of a research tool patent, v_A. Therefore excessive entry into basic research can determine welfare losses.

Costless arbitrage between risk free loans and firms' equities implies:

$$w_s = \lambda_0 n_B^{-a} v_A \tag{13a}$$

$$rv_A = (n_A^*)^{1-a}\lambda_1(v_L^0 - v_A) - w_s n_A^* + \frac{dv_A}{dt} \tag{13b}$$

$$rv_L^0 = \pi - (n_B)^{1-a}\lambda_0\left(v_L^0 - v_L^1\right) + \frac{dv_L^0}{dt} \tag{13c}$$

$$rv_L^1 = \pi - (n_A^*)^{1-a}\lambda_1 v_L^1 + \frac{dv_L^1}{dt} \tag{13d}$$

The first equation, (13a), characterizes the free entry condition in basic research. The second equation equalizes the risk free income deriving liquidating the expected present value of the research tool patent in an A_1 industry, rv_A, and the expected increase in value from becoming a top quality leader, $(n_A^*)^{1-a}\lambda_1(v_L^0 - v_A)$, minus the relative R&D cost, $w_s n_A^*$, plus the gradual appreciation in the case of R&D success not arriving, $\frac{dv_A}{dt}$.

The interpretation of the third and forth equation is like that of equations (7b) and (7c) in the previous section.

Plugging $w_s = \lambda_0 n_B^{-a} v_A$ into the expression of the skilled labour wage ratio (in Appendix 1), we obtain[19]:

$$x = \frac{1}{w_s}\left(\frac{\alpha}{1-\alpha}\right)M = \min\left(\frac{n_B^a}{\lambda_0 v_A}, 1\right)\left(\frac{\alpha}{1-\alpha}\right)M. \tag{14}$$

The skilled labor market clearing condition states:

$$x + m_0 n_B + (1 - m_0)\, n_A^* = L \tag{15}$$

[18] Here, perfect IPRs successfully restrict entry into applied R&D to only those (patent holder or *ex ante* licensees) legally entitled to do so. For an alternative scenario, with weaker IPR protection, in which free entry into downstream research vanifies any attempt to impose to *ex ante* licensing, see Cozzi and Galli (2014).

[19] We have implicitly assumed that $w_s \geq 1$, because skilled workers always have the option to work as unskilled workers. Therefore skilled employment in manufacturing is inversely related to the market value of patented research tools.

Hence, since wages are pinned down by the optimal firm size and by the zero profit conditions in the perfectly competitive basic research labor markets, the unique equilibrium per-sector mass of entrant basic R&D firms consistent with skilled labor market clearing (15) is determined by solving equation (15) for n_B:

$$n_B = \frac{1}{m_0}\left(L - x - (1-m_0)n_A^*\right). \tag{16}$$

To complete our analysis, let us look more closely at the inter-industry dynamics depicted by Figure 3. In the set of basic research industries a given number of perfectly competitive (freely entered) basic researchers, n_B^*, have a flow probability of becoming applied researchers, while in the set of the applied R&D industries each of the n_A^* per-industry applied researchers has a flow probability to succeed. By the law of large numbers, the industrial dynamics of this economy is described by the following first order ordinary differential equation:

$$\frac{dm_0}{dt} = (1-m_0)\,\lambda_1\,(n_A^*)^{1-a} - m_0\,(n_B)^{1-a}\,\lambda_0. \tag{17}$$

System (13b)-(13d) and eq. (17) - jointly with cross equation restrictions (14) and (16) - form a system of four first order ordinary differential equations, whose solution describes the dynamics of this economy for any admissible initial value of the unknown functions of time v_L^0, v_L^1,v_A, and System (13b)-(13d) and eq. (17) - jointly with cross equation restrictions (14) and (16) - form a system of four first order ordinary differential equations, whose solution describes the dynamics of this economy for any admissible initial value of the unknown functions of time v_L^0, v_L^1,v_A, and $m(A_0)$. In a steady state, $\frac{dv_L^1}{dt} = \frac{dv_L^0}{dt} = \frac{dv_A}{dt} = \frac{dm(A_0)}{dt} = 0$.

Let us remark that, unlike in the unpatentable research-tools case, here there is - potentially excessive - endogenous entry into basic research. Moreover, in this privatized scenario, congestion in applied research is internalized by the basic patent holder.

4.1.1 Steady State Equilibria

In the steady state equilibrium all variables are constant except the average quality of consumer goods, and therefore the instantaneous utility index, which grows at a constant rate $\ln(\gamma)g_{PRIV}$ proportional to the aggregate innovation rate $g_{PRIV} = m_0\,(n_B)^{1-a}\,\lambda_0 = (1-m_0)\,\lambda_1\,(n_A^*)^{1-a}$. Based on the previous characterization, we can state:

Definition 2. *A steady state equilibrium of the Privatized Targetable Basic Research economy is a vector* $[m_0, n_B, n_A^*, v_A, v_L^0\ v_L^1, w_s, x, g_{PRIV}] \in R_+^9$ *satisfying* $m_0 \in [0,1]$ *and the following equations:*

$$
\begin{aligned}
w_s &= \lambda_0 n_B^{-a} v_A && \text{(18a)}\\
rv_A &= (n_A^*)^{1-a}\,\lambda_1(v_L^0 - v_A) - w_s n_A^* && \text{(18b)}\\
n_A^* &= \left[\frac{(1-a)\lambda_1(v_L^0 - v_A)}{w_s}\right]^{\frac{1}{a}} && \text{(18c)}\\
rv_L^0 &= \pi - (n_B)^{1-a}\,\lambda_0\left(v_L^0 - v_L^1\right) && \text{(18d)}\\
rv_L^1 &= \pi - (n_A^*)^{1-a}\,\lambda_1 v_L^1 && \text{(18e)}\\
(1-m_0)\,\lambda_1\,(n_A^*)^{1-a} &= m_0\,(n_B)^{1-a}\,\lambda_0 && \text{(18f)}\\
L &= x + m_0 n_B + (1-m_0)\,n_A^* && \text{(18g)}\\
x &= \frac{1}{w_s}\left(\frac{\alpha}{1-\alpha}\right) M && \text{(18h)}\\
g_{PRIV} &= (1-m_0)\,\lambda_1\,(n_A^*)^{1-a}. && \text{(18i)}
\end{aligned}
$$

In all numerical simulations of the fully dynamic system, the steady state turned out to be saddle-point stable.

Also for the current scenario, the uniqueness of the steady-state holds:

Lemma 2. *In the Privatized Basic Research economy there can exist no more than one steady state equilibrium.*

Proof. See Appendix 2.

To replicate a post-1980 U.S. scenario with untargetable basic research, we need to modify the framework provided by Definition 2, in particular by changing the labour market equilibrium to:

$$L = x + n_B + (1-m_0)\,n_A^* \tag{19}$$

Notice that, compared to eq. (18f), in (19) n_B does not multiply m_0 anymore, because what previously was the per-sector basic research is now the aggregate basic research employment.

Moreover, the basic research free entry condition (18a) now becomes

$$w_s = m_0 \lambda_0 n_B^{-a} v_A. \tag{20}$$

The r.h.s. of eq. (20), unlike in (18a), multiplies m_0 because a basic researcher has an equal probability of discovering a research-tool in any sector $\omega \in [0,1]$, thereby risking to generate a duplicate of an already existing research-tool in a sector $\omega \in A_1$. Since the probability to end up in a sector $\omega \in A_0$ where research-tools are efficient is indeed m_0, it has to be included in the computation of the expected benefit of basic research. All other equations of Definition 2 remain unchanged. We can therefore define the steady-state equilibrium of this variant of the privatized basic research scenario as:

Definition 3. *A steady state equilibrium of the Privatized Untargetable Basic Research economy is a vector* $[m_0, n_B, n_A^*, v_A, v_L^0\ v_L^1, w_s, x, g_{PRIVu}] \in R_+^9$ *satisfying* $m_0 \in [0,1]$ *and the following equations:*

$$
\begin{aligned}
w_s &= m_0 \lambda_0 n_B^{-a} v_A && \text{(21a)}\\
r v_A &= (n_A^*)^{1-a} \lambda_1 (v_L^0 - v_A) - w_s n_A^* && \text{(21b)}\\
n_A^* &= \left[\frac{(1-a)\lambda_1 (v_L^0 - v_A)}{w_s} \right]^{\frac{1}{a}} && \text{(21c)}\\
r v_L^0 &= \pi - (n_B)^{1-a} \lambda_0 \left(v_L^0 - v_L^1 \right) && \text{(21d)}\\
r v_L^1 &= \pi - (n_A^*)^{1-a} \lambda_1 v_L^1 && \text{(21e)}\\
(1-m_0)\, \lambda_1 \, (n_A^*)^{1-a} &= m_0 \, (n_B)^{1-a} \, \lambda_0 && \text{(21f)}\\
L &= x + n_B + (1-m_0)\, n_A^* && \text{(21g)}\\
x &= \frac{1}{w_s} \left(\frac{\alpha}{1-\alpha} \right) M && \text{(21h)}\\
g_{PRIVu} &= (1-m_0)\, \lambda_1 \, (n_A^*)^{1-a} . && \text{(21i)}
\end{aligned}
$$

Also for the current scenario, the uniqueness of the steady-state can be proved along the lines of the proof of Lemma 2. Moreover, in all numerical simulations of the fully dynamical system, the steady-state turned out to be saddle-point stable.

5 Quantitative Analysis

5.1 Observed Regularities

In general, simulating our models[20] suggests that an economy in which public basic research is conducted in a non-profit oriented manner can induce less or more innovations and/or welfare than an economy in which basic R&D is privately carried out. For example, we have run simulations under the constrain of an equal amount of basic research employment: we first ran the privatized scenario, then plugged the steady state equilibrium level of basic research as L_G in the public economy. We have consistently obtained that the targetable privatized basic research economy outgrows the public basic research economy when the applied R&D productivity parameter, λ_1, becomes very low: in such cases the equilibrium innovative performance of the privatize economy with patentable research tools becomes better than the equilibrium performance of the economy with a public R&D sector. In fact, if λ_1 is very small or λ_0 is high, the flow out of A_1 will be scarce, whereas the flow out of A_0 will be intense. Therefore in the steady state $m(A_0)$ will be small, thereby exalting the wasteful nature of the public R&D activity uniformly diluted over $[0,1] - A_0$: in this case the social cost of a public R&D blind to the social needs signalled by the invisible hand would overwhelm the social costs of the restricted entry into the applied R&D sector induced by the patentability of research tools.

When instead we have run the same simulations in the case of untargetable basic research, the public scenario would always generate more innovations. In fact, despite the same amount

[20] The codes we have used are available upon request.

of basic research undertaken, applied R&D is lower in the privatized scenario. However, as a consequence, there is more consumption, with ambiguous effects on welfare.

While the discussion so far highlights the innovation perspective, the aggregate consumer utility - welfare - is also affected negatively by the potentially excessive entry associated with patent races. Since in either regime there is free entry into one of the two types of research activities, this may lead to excessive entry into basic research in the private regime, and excessive entry into development in the public regime[21]. While the lack of commercial focus in basic research can make publicly funded research worse, excessive entry into basic research in the private regime can potentially counter this handicap. Hence, it is not possible a priori to rank the two regimes.

In the next sections we will estimate the unknown parameters and use others taken from the literature, in order to evaluate the alternative patenting regimes. We will undertake our calibrations under the simplifying assumption that the US economy was in an unpatentable research tools steady state equilibrium before 1980. This will deliver the parameter values with which to simulate the alternative scenarios at the last year[22] of the public basic R&D regime (1979).

5.2 Calibration

In this section we calibrate our model to a steady state using U.S. data from 1973 to 1979, obtaining the values of these parameters as well as the endogenous variables in the unpatentable research tools case, which we believe prevailed during that period. Our exercise will obtain an estimation of the difficulty of R&D, summarized inversely by the basic and applied productivity parameters, λ_0 and λ_1. Consistently with our theoretical model, we use only skilled and unskilled labour as inputs and numbers of qualified innovations as R&D output, as represented by patents.

5.2.1 Description of the Procedure and the Data

1. Exact estimation of the values of the unobservable parameters λ_0, λ_1, γ, α, and a based on U.S. 1973-1979 data on the following moments: number of yearly patents/skilled labour employment ratio, equal to 0.000309692 (DATA); and skilled labour in manufacturing as a fraction of the labour force[23]; applied R&D labour as a fraction of the labour force[24], equal

[21] However, in our stylized framework, research tool patentability should reduce applied research, as compared to the unpatentable basic research scenario. This is corroborated by the important evidence provided by Galasso and Schankerman's (2013) careful identification strategy (based on judges propensity of invalidating patents), compellingly showing that following patent invalidation an idea gets more often cited in successive research.

[22] Qualitative results would not change if we had chosen another year, or included an average of four years before 1979.

[23] In our model economy, this underlies the macroeconomic trade-off in the allocation of skilled labour between manufacturing and R&D, as emphasized in the Schumpeterian literature (Aghion and Howitt, 1992, etc.).

[24] Which pins down the allocation of R&D labour between basic and applied R&D, at the essence of our contribution. Normalization by labour force is a hallmark of the previously mentioned dilution effect.

to 0.00428941 (DATA); number of patents/basic research labour[25], equal to 0.197070187 (DATA); the skill premium[26], equal to 1.228 (DATA). The results are shown in Table 1.

2. Use of the estimated parameter values $\hat{\lambda}_0$ and $\hat{\lambda}_1$, $\hat{\gamma}$, $\hat{\alpha}$, and $\hat{a}$, along with other parameters shown in Table 1 in the system of equations of the steady state equilibrium of the Privatized Targetable Basic Research Economy.

3. Use of the estimated parameter values $\hat{\alpha}$, $\hat{\lambda}_0$ and $\hat{\lambda}_1$, along with other parameters shown in Table 1 in the system of equations of the steady state equilibrium of the Privatized Untargetable Basic Research Economy.

4. Comparison of the steady state innovation rates and welfare levels of the two policy scenarios of steps 2 and 3 with the Public Basic Research Economy that has generated the data.

L is the percentage of people who were 25 year old or more and who had completed at least 4 years of college, collected by the U.S. Census (2010a), Current Population Survey, Historical Tables[27].

$\bar{L}_G$ is doctorate holders employed in science and engineering[28]. The relevant series of the expenditure on basic research in our estimations is the total basic R&D expenditure net of the industry performed basic R&D[29].

w_s is the skilled premium estimated by Krusell, Ohanian, Rios-Rull and Violante (2000).

The g_{PUBBL} data (according to our model, the measure of the actual U.S. innovation rate before 1980) are the number of utility patents granted to U.S. residents per million inhabitants[30].

As for the real rate of return on consumer assets, we adopt the usual $r = \rho = 0.05$, consistently with Mehra and Prescott's (1985) estimates for the pre-1980 period.

The following Table 1 reports the parameters we have used and their sources.

[25] R&D productivity measure of the successful interaction between basic and applied research.
[26] Which responds to the allocation of incentives between basic and applied research (Cozzi and Galli, 2014).
[27] Available at: www.census.gov/population/socdemo/education/tabA-2.xls
[28] Source: National Science Foundation (2005).
[29] Both series are taken from the NSF Science & Engeneering Indicators (2005).
[30] Source: USPTO (2010).

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Table 1: Input - Structure and Sources

Parameter	Description	Value	Source
L	Skilled Labour (intensity 1979)	0.164	U.S. Census, Current Population Survey
$\bar{L}_G$	S&E Doctorate Holders	0.00157	National Science Foundation
M	Unskilled Labour (intensity 1979)	0.836	U.S. Census, Current Population Survey
r	Subjective Discount Rate	0.05	Mehra and Prescott (1985)
λ_0	Basic Research Productivity	0.00149	Estimation
λ_1	Applied Research Productivity	0.10637	Estimation
γ	Mark-up	1.8291	Estimation
α	Skilled Share in Manufacturing	0.189	Estimation
a	R&D congestion	0.7565	Estimation

Our estimated value of γ is consistent with that estimated by Roeger (1995) and Martins et al. (1996).

Our estimate the intra-sectorial congestion parameter a is consistent with Jones and Williams' (1998) and (2000) calibrations.

The reason why we have also estimated parameter α - the high skilled labour [31] elasticity in manufacturing production - instead of relying on available statistics on labour shares, is that they fail to single out the fraction of high skilled labour in production[32], consistently with our stylized economy.

5.3 Policy Comparisons

In this section we utilize the previously estimated values of the technological parameters, along with the previous exogenous variable to compute the hypothetical steady state equilibrium of the two scenarios - patentable research tools under targetable and untargetable basic research - for the year 1979, i.e. the last year of the non-patentable research tools regime. It is important to remark that the qualitative results do not change if instead we use any combinations of the data in the last 5 years time interval (from 1975 to 1979).

We have also simulated the welfare levels[33]

$$\begin{aligned} Welf_s &= \int_0^\infty e^{-rt}\left[\log(\gamma)g_s t + \log(x_s^\alpha M^{1-\alpha})\right] dt = \\ &= \frac{\log(\gamma)g_s}{r^2} + \frac{\log(x_s^\alpha M^{1-\alpha})}{r},\ s = PUBBL,\ PRIV, \text{ and } PRIVu. \end{aligned} \tag{22}$$

associated with the different IPR and targetableness scenarios. In order to provide a cardinal measure of the utility change associated with each reform, we have also computed the equivalent

[31] In this paper's restrictive interpretation as highly skilled workers with at least college education, and able to perform R&D activities competently.

[32] For example, the ratio of non-production workers in operating establishments to total employment in 1979 was 0.248 (Berman, Bound, and Griliches, 1994), but this would include a large fraction of not highly skilled workers, as well as people actually undertaking knowledge-related activities.

[33] See Appendix 1 for the derivation of this expression.

steady-state consumption compensating variations from the public research scenario. In this model, it is achieved using the following simple formula: $\Delta c_s = (Welf_s - Welf_{PUBBL})\,\rho$, with $s = PRIV$, and $PRIVu$.

Table 2 lists the steady state innovation rates, basic research per-sector, fraction of sectors needing a research tool, and consumption welfare compensating variations - based on the 1979 data and estimated parameter values - of the public basic research regime, and the privatized targetable and untargetable, g_{PRIVu}, basic research regimes:

Table 2

Scenarios	Innovation Rate	n_B	m_0	Δc_s
Public	0.00030969	0.157%	0.996	0
Private Targetable	0.00034024	0.234%	0.994	0.383%
Private Untargetable	0.00033965	0.232%	0.997	0.382%

As the data in the table show, the privatized basic research scenarios outgrows the public basic R&D regime, regardless of the basic research being targetable or untargetable. When basic research can be targeted to specific products, the private sector can do better than the public sector in allocating it. What is surprising is that even when basic research cannot be targeted to specific products, privately guided basic R&D can outgrowth the purely public case. The representative US family, moreover, would have liked privatization: the equivalent consumption welfare gain would amount to about 0.38%, which must have facilitated the consensus needed for that historical reform of the intellectual property rights institutions.

This is further confirmed by the the fraction - m_0 - predicted by either regime, which according to our computation was quite high. Consequently, basic research was needed in almost all sectors: hence the public scenario with unpatentable research tools would not waste much research effort relative to even the patentable targetable basic research. As a result of our simulations, the main effect of patentability has been to stimulate massive additional entry into basic research, which more than compensates the entry restrictions in applied research.

Notice that our results do not mean that in the privatized scenarios public research would disappear. On the contrary, it just says that private basic research would just add the difference $n_B - L_g$, which, according to our simulations, amounts to about 50% of L_g. Moreover, the possibility of earning a new source of revenue in terms of patent royalties on research tools, would not only attract profit-seeking private basic R&D firms, but also increase the revenues of the public research institutions, universities, and colleges.

6 Final Remarks

The debate on the effects of the patentability of research tools on the incentives to innovate is still very controversial, not only in the US but also in Europe and in other important areas of

the world. This paper has analyzed from a general equilibrium perspective the US policy shift towards the extension of patentability to research tools and basic scientific ideas that took place around 1980. These normative innovations have been modifying the industrial and academic lives in the last three decades, raising doubts on their desirability. The losses from the free entry into basic research and the monopolization of applied research induced by intellectual property of the research tools have been compared with the inefficacy of public research institutions to promptly react to downstream market opportunities and the potentially excessive entry into applied R&D.

Results were not a priory unambiguous, which motivated us to use the available data and calibrate and simulate our model in order to check if the US did it right in changing their institutions around 1980. A broad consensus in economic literature, also confirmed by recent studies (see Lam 2009, OECD 2012, Howitt 2013), has been suggesting that the motivations for basic researcher goes beyond personal income and particularly include the opportunities to advance the scientist's research agenda. Related with such contributions, a large literature pointed out the technological impossibility to direct or target basic research by granting the inventor property right on the basic research output.

In this paper, we have robustly found that under the assumption of either targetable and untargetable basic research, the U.S. assigning property rights to basic research findings and creating a market for research tools was mildly innovation-enhancing. It is important to remark that none of this necessarily implied the demise of the innovative State in the United States economy after 1980, as is clear from three decades of high public involvement. On the contrary, more and more patenting has not reduced the basic research budget, rather it has given it an additional source of funds.

Therefore we can say that the 1980 US normative change was mildly positive innovation-enhancing institutional response to the underlying technological modifications, but above all a mean to guarantee the public research institutions more funding coming from the industry and less from the taxpayers. In this sense it helped the public R&D effort to better sustain itself, which has facilitated the expansion and future success of a world model of entrepreneurial state (Mazzuccato, 2013).

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Appendix 1
Model Details

This Appendix explains the details of the quality ladder model used in the main text. It may be skipped by readers familiar with this literature.

Population level is normalized to 1. The representative household preferences are represented by the following intertemporally additive utility functional[34]:

$$U = \int_0^\infty e^{-rt} \ln D(t) dt, \tag{23}$$

where $r > 0$ is the subjective rate of time preference, and $D(t)$ is an intra-household consumption index reflecting the household's taste for variety and for product quality. Per-family member instantaneous utility is given by:

$$\ln D(t) = \int_0^1 \ln \left(\sum_j \gamma^j d_{jt}(\omega) \right) d\omega, \tag{24}$$

where $d_{jt}(\omega)$ is the individual consumption of a good of quality $j = 1, 2, ...$ (that is, a product that underwent up to j quality jumps) and produced in industry ω at time t. Parameter $\gamma > 1$ measures the size of the quality upgrades. This formulation, common to Grossman and Helpman (1991) and Segerstrom (1998), assumes that each consumer prefers higher quality products of different varieties. Since we are not incorporating horizontal innovation, the set of varieties is bounded and normalised to the unit interval.

[34] We skip starting with an expectational operator in order to save notation. A more general setting of the consumer problem would not change results, as in our framework, due to perfectly diversifiable risks, law of large numbers, and perfect financial markets, the consumer's wealth evolves deterministically in equilibrium.

The representative consumer is endowed with $L > 0$ units of skilled labor and $M > 0$ units of unskilled labor summing to 1. Since population is normalized to 1, L and M will also equal, in equilibrium, the supply of skilled, respectively, unskilled labour. Unskilled labor can only be employed in the final goods production. Skilled labour is able to perform R&D activities.

Focussing on the set $J_t(\omega)$ of the existing quality levels with the lowest quality-adjusted prices, the household, at each instant, allocates maximizes the instantaneous utility (24) according to the following static constraint

$$E(t) = \int_0^1 \sum_{j \in J_t(\omega)} p_{jt}(\omega) d_{jt}(\omega)\, d\omega, \tag{25}$$

where $E(t)$ denotes a given consumption expenditure and $p_{jt}(\omega)$ is the price of a product of quality j produced in industry ω at time t. Let us define $j_t^*(\omega) \equiv \max\{j : j \in J_t(\omega)\}$. Using the instantaneous optimization results, we can re-write (24) as

$$u(t) = \int_0^1 \ln\left[\gamma^{j_t^*(\omega)} E(t)/p_{j_t^*(\omega)t}(\omega)\right] d\omega = \tag{26}$$

$$= \ln[E(t)] + \ln(\gamma) \int_0^1 j_t^*(\omega) d\omega - \int_0^1 \ln[p_{j_t^*(\omega)t}(\omega)] d\omega \tag{27}$$

The solution of this maximization problem yields the static demand function:

$$d_{jt}(\omega) = \begin{cases} E(t)/p_{jt}(\omega) \text{ for } j = j_t^*(\omega) \\ 0 \quad \text{otherwise.} \end{cases} \tag{28}$$

where we posit that if two products have the same quality-adjusted price, consumers buy the higher quality product.

Therefore the consumer chooses the piecewise continuous per-family member expenditure trajectory, $E(\cdot)$, that maximizes:

$$U = \int_0^\infty e^{-rt} \ln[E(t)] dt. \tag{29}$$

Households possess equal shares of all the firms at time $t = 0$, hence later. Letting $A(0)$ denote the present value of human capital plus the present value of asset holdings at $t = 0$, each household's intertemporal budget constraint is:

$$\int_0^\infty e^{-I(t)} E(t) dt \leqq A(0) \tag{30}$$

where $I(t) = \int_0^t i(s) ds$ represents the equilibrium cumulative real interest rate up to time t.

Finally, the representative consumer chooses the time pattern of consumption expenditure to maximize (29) subject to the intertemporal budget constraint (30). The equilibrium expenditure trajectory satisfies the Euler equation:

$$\dot{E}(t)/E(t) = i(t) - r \tag{31}$$

- where $i(t) = I(t)$ is the instantaneous market interest rate at time t - along with the usual transversality condition and the no-Ponzi game condition.

Since preferences are homothetic, in each industry aggregate demand is proportional to the representative consumer. E denotes the aggregate consumption spending and d denotes the aggregate demand.

As for the production side, we assume constant returns to scale technologies in the (differentiated) manufacturing sectors represented by the following production functions:

$$y(\omega) = x^{\alpha}(\omega)\, m^{1-\alpha}(\omega), \text{ for all } \omega \in [0,1], \tag{32}$$

where $\alpha \in (0,1)$, $y(\omega)$ is the output flow per unit time, $x(\omega)$ and $m(\omega)$ are, respectively, the skilled and unskilled labour input flows in industry $\omega \in [0,1]$. Letting w_s and w_u denote the skilled and unskilled wage rates, in each industry the quality leader seeks to minimize its total cost flow $C = w_s x(\omega) + w_u m(\omega)$ subject to constraint (32). For $y(\omega) = 1$, the conditional unskilled (33) and skilled (34) labour demand per-unit of output are:

$$m(\omega) = \left(\frac{1-\alpha}{\alpha}\right)^{\alpha} \left(\frac{w_s}{w_u}\right)^{\alpha}, \tag{33}$$

$$x(\omega) = \left(\frac{\alpha}{1-\alpha}\right)^{1-\alpha} \left(\frac{w_u}{w_s}\right)^{1-\alpha}. \tag{34}$$

Thus cost is:

$$C(w_s, w_u, y) = c(w_s, w_u)y \tag{35}$$

where $c(w_s, w_u)$ is the per-unit cost function:

$$c(w_s, w_u) = \left[\left(\tfrac{1-\alpha}{\alpha}\right)^{-(1-\alpha)} + \left(\tfrac{\alpha}{1-\alpha}\right)^{-\alpha}\right] w_s^{\alpha} w_u^{1-\alpha}. \tag{36}$$

Since unskilled labour is uniquely employed in the final good sectors and all price variables (including wages) are assumed to instantaneously adjust to their market clearing values, unskilled labour aggregate demand $\int_0^1 m(\omega)\, d\omega$ is equal to its aggregate supply, M, at any date. Since industries are symmetric and their number is normalized to 1, in equilibrium[35] $m(\omega) = M$.

Unskilled labour as numeraire implies $w_u = 1$. From equations (33) and (34) we get the firm's skilled labour demand function:

$$x(\omega) = \frac{1}{w_s}\left(\frac{\alpha}{1-\alpha}\right) M. \tag{37}$$

[35] More generally, with mass $N > 0$ of final good industries, in equilibrium $m(\omega) = \frac{M}{N}$.

In each industry, at each instant, firms compete in prices. Given demand function (28), within each industry product innovation is non-drastic[36], hence the quality leader will fix its (limit) price by charging a mark-up γ over the unit cost:

$$p = \gamma c(w_s, 1) \Rightarrow d = \frac{E}{\gamma c(w_s, 1)}. \tag{38}$$

Hence each monopolist earns a flow of profit equal to

$$\begin{aligned} \pi &= \frac{\gamma - 1}{\gamma} E = (\gamma - 1) \frac{w_s x}{\alpha} \\ \pi &= (\gamma - 1) \frac{1}{1 - \alpha} M. \end{aligned} \tag{39}$$

From eq.s (39) follows:

$$\frac{\gamma - 1}{\gamma} E = (\gamma - 1) \frac{1}{1 - \alpha} M \Rightarrow E = \frac{\gamma}{1 - \alpha} M. \tag{40}$$

Interestingly, eq. (40) implies that in equilibrium total expenditure is always constant. Therefore, eq. (31) implies a constant real interest rate:

$$i(t) = r. \tag{41}$$

Steady State Welfare

We here derive the equation used in our simulations to assess the steady state welfare associated with each scenario. In equilibrium the instantaneous utility function (24), after reminding that $d_{j_t^*(\omega)t}(\omega) = x^\alpha M^{1-\alpha}$, becomes

$$\ln D(t) = \int_0^1 \ln \left[\gamma^{j_t^*(\omega)} d_{j_t^*(\omega)t}(\omega) \right] d\omega = \log(\gamma) \int_0^1 j_t^*(\omega) d\omega + \log(x^\alpha M^{1-\alpha}). \tag{42}$$

In equilibrium $j_t^*(\omega) = j_t(\omega)$ in all industries. Focussing on steady state equilibria, we can assume that the economy starts from the steady state value of all variables (including $m(A_0)$). Hence:

$$\ln D(t) = \log(\gamma) g_s t + \log(x_s^\alpha M^{1-\alpha}) + \log(\gamma) \int_0^1 j_0^*(\omega) d\omega, \tag{43}$$

with index $s = PUBBL$, $PRIV$, and $REexem$, depending on the institutional scenario chosen. In fact, $\int_0^1 j_t^*(\omega) d\omega = g_s t + \int_0^1 j_0^*(\omega) d\omega$. To understand this, it is important to remember that all processes are independent, all sectors are symmetric within A_0 and A_1, and there is an infinite number of them. Define $\phi(t) \equiv \int_0^1 j_t^*(\omega) d\omega$. Consider a positive and small time

[36] We are following the standard definition of drastic innovation as generating a sufficiently large quality jump to allow the new monopolist to maximize profits without risking the re-entry of the previous monopoly. Given the unit elastic demand, here the unconstrained profit maximizing price would be infinitely high: that would induce the previous incumbent to re-enter.

increment Δt, and the increment $\phi(t+\Delta t)-\phi(t)=\int_0^1\left[j^*_{t+\Delta t}(\omega)-j^*_t(\omega)\right]d\omega$. Notice that, by the properties of Poisson processes, $j^*_{t+\Delta t}(\omega)-j^*_t(\omega)=0$ or 1, except for events with probability of a zero of higher order than Δt, which we write $o(\Delta t)$. By the law of large numbers the average number of jumps is equal to its expected value. Hence:

$$\begin{aligned}\phi(t+\Delta t)-\phi(t) &= \int_{A_1(t)}\left[0*\left(1-\left(n^*_A\right)^{1-a}\lambda_1\Delta t\right)+1*\left(n^*_A\right)^{1-a}\lambda_1\Delta t\right]d\omega+0(\Delta t)= \\ &= \left(1-m(A_0)\right)\left(n^*_A\right)^{1-a}\lambda_1\Delta t+o(\Delta t).\end{aligned}$$

Dividing both sides by Δt and taking the limit $\Delta t\to 0$, and remembering that $\lim_{\Delta t\to 0}o(\Delta t)/\Delta t=0$, gives $\phi'(t)=\left(1-m(A_0)\right)\left(n^*_A\right)^{1-a}\lambda_1\equiv g_s$. Along a steady state g_s is constant, and hence $\phi(t)=g_st+\phi(0)=g_st+\int_0^1 j^*_0(\omega)d\omega$. Assuming that the initial value of $\int_0^1 j_0(\omega)d\omega$ is the same under each scenario $s=PUBBL$, $PRIV$, and $PRIVu$, we can normalise it at zero. Therefore, with no loss of generality, we can use the following simpler expression:

$$Welf_s = \int_0^\infty e^{-rt}\left[\log(\gamma)g_st+\log(x_s^\alpha M^{1-\alpha})\right]dt= \tag{44}$$

$$= \frac{\log(\gamma)g_s}{r^2}+\frac{\log(x_s^\alpha M^{1-\alpha})}{r}, \tag{45}$$

This is the expression we have used in all our numerical welfare comparisons.

As a by-product of our analysis, notice that taking the derivative of both sides of eq. (43) with respect to time gives:

$$\frac{\dot D(t)}{D(t)}=\log(\gamma)g_s,$$

which clarifies the link between the aggregate innovation rate g_s and the percapita utility growth rate.

Appendix 2

Lemma 1. *In the Public Basic Research economy there can exist no more than one steady state equilibrium.*

Proof. At the steady state, $\frac{dm_0}{dt}=0$, and hence eq. (8) can be rewritten as:

$$\left(1-m_0\right)n_A^{1-a}\lambda_1=m_0\bar L_G^{1-a}\lambda_0. \tag{46}$$

which defines m_0 as an increasing function of n_A:

$$m_0=\frac{n_A^{1-a}\lambda_1}{\bar L_G^{1-a}\lambda_0+n_A^{1-a}\lambda_1}. \tag{47}$$

From (47) it is easily seen that $\left(1-m_0\right)n_A$ is an increasing function of n_A.

Eq. (7b) implies that v_L^0 is an increasing function of v_L^1; in turn, (7c) implies that v_L^1 is a decreasing function of n_A. Therefore, also v_L^0 is a decreasing function of n_A. But then, eq. (7a) implies that w_s too will be a decreasing function of n_A.

Let us then rewrite the labour market equilibrium condition (10) as

$$(1-m_0)n_A = L - \frac{1}{w_s}\left(\frac{\alpha}{1-\alpha}\right)M - \bar{L}_G. \tag{48}$$

In light of the preceding discussion, the left side of equation (48) is an increasing function of n_A, while the right side is a decreasing function of n_A. The steady state equilibrium value of n_A will be associated with the unique intersection between the curves defined by the two sides of this equation. Since the real values of all the other endogenous variables at the steady state are pinned down by n_A, they will be uniquely determined. Therefore, if a steady state equilibrium exists it will be unique. QED.

Lemma 2. *In the Privatized Basic Research economy of Definition 2 there can exist no more than one steady state equilibrium.*

Proof. Use eq.(13a) to obtain w_s, and plug into (12) to obtain the steady state version of eq. (13b), which, solved for v_A gives:

$$v_A = \left(\frac{a}{r}\right)^a \left(\frac{1-a}{\lambda_0}\right)^{1-a} (n_B)^{1-a}\,\lambda_1(v_L^0 - v_A). \tag{49}$$

Plugging (13a) and (49) into (13b) and solving for v_L^1 gives:

$$v_L^1 = \frac{\pi}{r + \left(\frac{r(1-a)}{a\lambda_0}\right)^{1-a}(n_B)^{a(1-a)}\,\lambda_1},$$

which can be plugged into eq. (13c) to solve for v_L^0 as:

$$v_L^0 = \frac{\pi}{r + (n_B)^{1-a}\,\lambda_0}\left[1 + \frac{(n_B)^{1-a}\,\lambda_0}{r + \left(\frac{r(1-a)}{a\lambda_0}\right)^{1-a}(n_B)^{a(1-a)}\,\lambda_1}\right]. \tag{50}$$

Plugging (50) into eq. (49) and solving for v_A yields:

$$v_A = \frac{\frac{\pi}{r+(n_B)^{1-a}\lambda_0}\left[1 + \frac{(n_B)^{1-a}\lambda_0}{r+\left(\frac{r(1-a)}{a\lambda_0}\right)^{1-a}(n_B)^{a(1-a)}\lambda_1}\right]}{1 + \frac{r^a\lambda_0^{1-a}}{a^a(1-a)^{1-a}(n_B)^{a(1-a)}\lambda_1}}. \tag{51}$$

As will soon be clear, it is important to study how $\frac{v_A}{n_B^a}$ changes with n_B^a. Based on eq. (49), we can write: $\frac{d}{dn_B}\left(\frac{v_A}{n_B^a}\right) =$

$$= \frac{d}{dn_B}\left[\frac{\pi}{rn_B^a+n_B\lambda_0}\left(1+\frac{n_B^{1-a}\lambda_0}{r+n_B^{a(1-a)}\lambda_1\left(-\frac{1}{a}\frac{r}{\lambda_0}(a-1)\right)^{1-a}}\right)\frac{a^a(1-a)^{1-a}n_B^{a(1-a)}\lambda_1}{r^a\lambda_0^{1-a}+a^a(1-a)^{1-a}n_B^{a(1-a)}\lambda_1}\right] =$$

$$= -\left(\begin{array}{c} 2a^2r^3\lambda_0^2n_B^{3a^2-a-1}(1-a)^{1-a}+a^2r^4\lambda_0n_B^{3a^2-2}(1-a)^{1-a}+ \\ a^2r^3\lambda_1n_B^{2a^2+a-2}(1-a)^{2-2a}+a^2r^2\lambda_0^3n_B^{a(3a-2)}(1-a)^{1-a}+ \\ r^2\lambda_0^2\lambda_1n_B^{2a^2-1}(1-a)^{1-a}\left(-\frac{1}{a}\frac{r}{\lambda_0}(a-1)\right)^{1-a}+ \\ a\lambda_0\lambda_1^3n_B^{2a-1}(1-a)^{2-2a}\left(-\frac{1}{a}\frac{r}{\lambda_0}(a-1)\right)^{2-2a}+ \\ 2a^2r^2\lambda_1^2n_B^{a^2+2a-2}(1-a)^{2-2a}\left(-\frac{1}{a}\frac{r}{\lambda_0}(a-1)\right)^{1-a}+ \\ a^2r\lambda_1^3n_B^{3a-2}(1-a)^{2-2a}\left(-\frac{1}{a}\frac{r}{\lambda_0}(a-1)\right)^{2-2a}+ \\ a^2r\lambda_0^2\lambda_1n_B^{a(2a-1)}(1-a)^{2-2a}+(2-a)a^2\lambda_0^2\lambda_1^2n_B^{a^2}(1-a)^{2-2a}\left(-\frac{1}{a}\frac{r}{\lambda_0}(a-1)\right)^{1-a}+ \\ 2a^2r^2\lambda_0\lambda_1n_B^{2a^2-1}(1-a)^{2-2a}+2a^2r^3\lambda_0\lambda_1n_B^{2a^2+a-2}(1-a)^{1-a}\left(-\frac{1}{a}\frac{r}{\lambda_0}(a-1)\right)^{1-a}+ \\ a^2r\lambda_0^2\lambda_1^2n_B^{a^2+a-1}(1-a)^{1-a}\left(-\frac{1}{a}\frac{r}{\lambda_0}(a-1)\right)^{2-2a}+ \\ 2a^2r^2\lambda_0^2\lambda_1n_B^{2a^2-1}(1-a)^{1-a}\left(-\frac{1}{a}\frac{r}{\lambda_0}(a-1)\right)^{1-a}+ \\ a^2r^2\lambda_0\lambda_1^2n_B^{a^2+2a-2}(1-a)^{1-a}\left(-\frac{1}{a}\frac{r}{\lambda_0}(a-1)\right)^{2-2a}+ \\ ar\lambda_0^3\lambda_1n_B^{a(2a-1)}(1-a)^{1-a}\left(-\frac{1}{a}\frac{r}{\lambda_0}(a-1)\right)^{1-a}+ \\ ar\lambda_0\lambda_1^2n_B^{a^2+a-1}(1-a)^{2-2a}\left(-\frac{1}{a}\frac{r}{\lambda_0}(a-1)\right)^{1-a}+ \\ (1-a)r\lambda_0^2\lambda_1^2n_B^{a^2+a-1}(1-a)^{1-a}\left(-\frac{1}{a}\frac{r}{\lambda_0}(a-1)\right)^{2-2a} \\ (3-a)a^2r\lambda_0\lambda_1^2n_B^{a^2+a-1}(1-a)^{2-2a}\left(-\frac{1}{a}\frac{r}{\lambda_0}(a-1)\right)^{1-a} \end{array}\right) \cdot$$

$$\cdot\pi a^a\lambda_1\frac{n_B^{-4a^2+2a+1}}{(\lambda_0n_B+rn_B^a)^2\left(r^a\lambda_0^{1-a}+a^a\frac{\lambda_1}{n_B^{a(a-1)}}(1-a)^{1-a}\right)^2\left(r+\frac{\lambda_1}{n_B^{a(a-1)}}\left(-\frac{1}{a}\frac{r}{\lambda_0}(a-1)\right)^{1-a}\right)^2}$$

which is certainly negative because $0<a<1$, that is:

$$\frac{d}{dn_B}\left(n_B^{-a}v_A\right)<0. \tag{52}$$

Plugging (12) into (17), setting $\frac{dm_0}{dt}=0$, and solving for m_0 gives:

$$m_0=\frac{1}{1+\frac{\lambda_0^{2+a}}{\lambda_1}\left[\left(\frac{a}{r(1-a)}\right)^{1-a}\right]n_B^{(1-a)^2}}. \tag{53}$$

Eq. (53) shows that m_0 is a decreasing function of n_B, and therefore $1-m_0$ is an increasing function of n_B. However, notice also that m_0n_B is an increasing function of n_B.

Obtaining skilled wage from (13a) and plugging it into (14), and in light of eq.s (12) and (49), we can rewrite the skilled labour market condition (16) as:

$$m_0 n_B = L - \frac{\alpha M}{(1-\alpha)\lambda_0 n_B^{-a} v_A} - \frac{(1-m_0)\, n_B^a (1-a) r}{\lambda_0 a}. \tag{54}$$

Recalling the discussion after eq. (53), the left side of equation (54) is an increasing function of n_B. From (52) and (53), the right side of (54) is instead a decreasing function of n_B. Therefore there will exist only one intersection between the corresponding curves, and therefore a unique real value of n_B that solves equation (54). Since the real values of all other endogenous variables are uniquely pinned down by n_B, there can exist only a unique steady state equilibrium. **QED**